

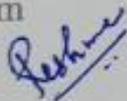


CERTIFICATE

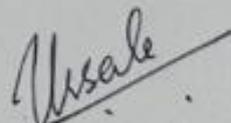
This is to certify that the dissertation entitled, **STUDY ON ONLINE CLOTH SHOPPING BEFORE AND DURING COVID 19** is a bonafide record of the work done by Ms. **ASWATHI V J** under my guidance as partial fulfilment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 04/03/2022

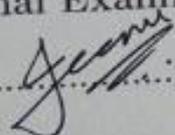
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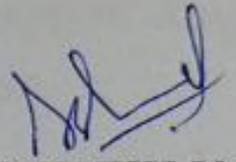
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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Dr Elizabeth Reshma M T, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

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ASWATHI V J

AB19AMAT011

14. How much do you spend on online shopping every month?

- Entire purchase
- More than half the purchase
- Limited purchase

15. Do discounts/offers play a role in your online shopping?

- Yes
- No

16. If yes, by how?

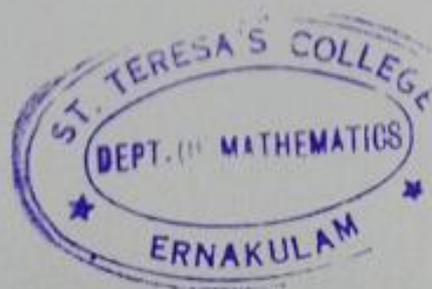
- I buy clothes whenever there is a discount sale
- I only look for discounts if I want to buy something

17. How easy was it for you to find what you were looking for online?

- Very easy
- Easy
- Neutral
- Difficult

18. On a scale of 1 to 5, how likely are you to recommend online shopping to your friend? (one being the least and five being the most)

- One
- Two
- Three
- Four
- Five



Project Report

On

**A STUDY ON THE EFFECT OF COVID-19
ON SMALL BUSINESSES**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

SHAHANAS T N

(Register No. AB19BMAT005)

Under the Supervision of

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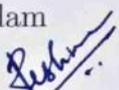


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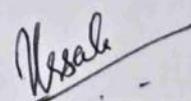
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Date: 08-03-2022

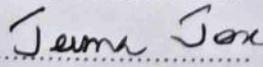
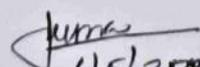
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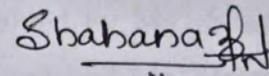
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Firstly, I thank God Almighty for His grace for being able to complete this project work successfully. I express my deep sense of gratitude to our guide Dr. Elizabeth Reshma M.T (Assistant Professor), Department of Mathematics and Statistics, St. Teresa's College, Ernakulam, for her valuable guidance and suggestions. I would like to mention the unending help and support provided by Ms. Shanty B.P (H.O.D of Statistics), Ms. Reshmy S (Assistant professor) and Ms. Rosmin Raju throughout the course of project. I also thank Dr. Ursala Paul (H.O.D of Mathematics), other teachers of the department, parents, friends, especially my group members and all those who gave me the moral support. This project would not have been possible without the support of the people mentioned above.

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Chapter 1

INTRODUCTION

Chapter 1

INTRODUCTION

COVID-19 put an immediate halt to many business activities across the globe, as several countries had shut down their ports, airports and domestic transportation while imposing nation-wide lockdowns, leading to a disturbance in business and civil life. The enormity of the lockdown affected manufacturing activities and supply chains alike, disrupting the overall economy of India. Like any other major sector, the Micro, Small and Medium Enterprise (MSME) sector also witnessed a considerable decline in economic activities and loss of jobs due to the nationwide lockdown. Declining output is not a good sign for the economy as the sector provides gainful non- farm employment to millions, especially in rural areas. MSME sector has emerged as a very important sector of the Indian economy, contributing significantly to employment generation, innovation, exports and inclusive growth of the economy. MSMEs are the backbone of the socio-economic development of our country. It also accounts for 45 percentage of the total industrial production, 40 percent of total exports and contributes very significantly to the GDP. Manufacturing segment within the MSME contributes to 7.09 percent of GDP. MSME also contributes to 30 percent of services. The total contribution of MSME to the GDP is 37.54. The announcement of country wide lockdown dragged MSME owners in unexpected times, where no one had experience to handle this kind of situation. Extended lockdown had negative impact on supply of finished goods, procurement of raw material and availability of employees to work in production and supply processes. During April to June 2020, sector faced challenges related to debt repayments, wages/salaries, statutory dues, etc. Reports have shown

that disruptions caused by the Covid-19 pandemic have impacted MSMEs earnings by 20-50 percent, micro and small enterprises faced the maximum heat, mainly due to liquidity crunch. Enterprises working in essential commodity business were better off in terms of interrupted but predictable cash flows. This study is an effort to examine how covid 19 affected the small business sectors in Ernakulam , with the help of statistical methods. Thus, the study provides deeper insights into the impacts of COVID - 19 pandemic on small business.

Definition of Small Business : A business which functions on a small scale level involves less capital investment, less number of labour and fewer machines to operates is known as a Small Business.

1.1 SIGNIFICANCE OF STUDY

The statistical study is relevant as it tries to address the significant changes in the small business sectors before and during the pandemic. We also get an insight about how some enterprises innovated their ways by shifting focus from non essential commodities towards essential commodities; like production of hand sanitizers, toiletries, PPE kits, reusable masks etc. are able to survive in tough time.

1.2 OBJECTIVES

1. To study the impact of COVID-19 on the number of small business owners.
2. To compare the sales volume of small businesses before and during COVID-19.
3. To compare the annual profit of small businesses before and during COVID-19.

Chapter 2

DATA DESCRIPTION

The data used in this study is the primary data collected directly from the small business owners/employees through an online form and also by conducting in-person surveys.

The variables under consideration are

- Period at which the business started (before /during pandemic)
- Area of business (urban/rural)
- Mode of taking orders (directly/through social media/ others)
- Whether the business closed down during pandemic.
- Monthly profit (in rupees)
- Number of orders per month
- Factors (availability of raw materials/delivery and transportation/online marketing/government restrictions) affecting the business.
- Degree of impact (Affected positively/Affected negatively/ Not affected)

Chapter 3

METHODOLOGY

This study is done with the help of primary data.

The primary data is collected through surveys. A well structured questionnaire is to be formed and circulated among the small business owners in Ernakulam using online form and door to door data collection.

The target population of the survey was the rural and urban population of the Ernakulam district. In the present situation the use of online mode for conducting surveys are found to be a great help.

Questionnaire concerned with the increase and decrease in profit and number of orders before and during the pandemic along with the factors which may be effected the business are included in the questionnaire. From the collected data the sales volume and annual income are calculated.

The survey questionnaire which was circulated among the small business owners in the Ernakulam district received 107 responses out of which 27 were found to be inappropriate and were excluded further. Collected data was interpreted using bar charts and tables and analyzed using Z-test and Chi square test.

3.1 PERCENTAGE CHANGE

A percentage analysis shows that how two items changed as percentage from one period to another period.

$\frac{\text{change in the performance}}{\text{the base value}} \times 100 \rightarrow \text{denotes the percentage change}$

3.2 TESTING EQUALITY OF MEANS BASED ON PAIRED OBSERVATION

Z-TEST

3.2.1 ASSUMPTIONS

1. Sampling distribution of the test statistic is normal.
2. Sample statistics are close to the population parameter and therefore finding standard error, sample statistics are used in place where population parameter are to be used.

3.2.2 DEFINITION

A Z-test is a statistical test to determine whether two population means are different when the variances are known and the sample size is large. A Z-test is a hypothesis test in which the Z-statistics follows a distribution. A Z-statistics, or Z-score is a number representing the result from the Z-test.

The formula is given by;

$$Z = \frac{(\bar{x}_1 - \bar{x}_2 - \Delta)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where \bar{x}_1 and \bar{x}_2 are the means of the two samples, Δ is the hypothesized difference between the population means (0 if testing for equal means), and s_1 and s_2 are the standard deviations of the two samples, and n_1 and n_2 are the sizes of the two samples.

Null Hypothesis $\rightarrow H_0 : \mu_1 = \mu_2$

Alternative Hypothesis $\rightarrow H_1 : \mu_1 > \mu_2$

where μ_1 and μ_2 are population means before and during pandemic respectively.

3.2.3 USES

1. To test the given population mean when the sample is large or when the population SD is known.
2. To determine whether two population means are different when the variances are known and the sample size is large.
3. To test the equality of two samples standard deviations when the samples are large or when the population standard deviations are known.
4. To test population proportions.
5. To test the equality of two sample proportions.
6. To test the population SD when the sample is large.
7. To test the equality of the correlation coefficients.

3.3 CHI SQUARE TEST

The test we use to measure the differences between what is observed and what is expected based on an assumed hypothesis is called the **Chi-Square Test**.

3.3.1 ASSUMPTIONS

Both variables are catagorical:

It is assumed that both both variables are categorical. That is, both variables take on value that are names or lables.

3.3.2 PROCEDURE

1. First of all set the hypotheses that the variables are independent as H_0 and that there is a relationship between the variables as alternative hypothesis H_1 .
2. Tabulation of observed values and calculation of expected values by the formula

$$E_{ij} = \frac{i^{\text{th}} \text{ row total} \times j^{\text{th}} \text{ column total}}{\text{Grand total}}$$

where E_{ij} is the expected value in the i^{th} row and j^{th} column.

3. Then the test statistic for the Chi-Square Test of Independence denotes as χ^2 is computed as:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where O is the observed value and E is the expected value.

4. Finally the calculated value of χ^2 is compared to the critical value from the χ^2 distribution table with degrees of freedom $df = (R-1)(C-1)$ (where R is the number of rows and C is the number of columns) and chosen confidence level. If the calculated χ^2 value is greater than the critical χ^2 value, then we reject the null hypothesis.

3.3.3 APPLICATIONS

1. To test the goodness of fit of distributions.
2. To test the independence of attributes.
3. Test of Homogeneity.

Chapter 4

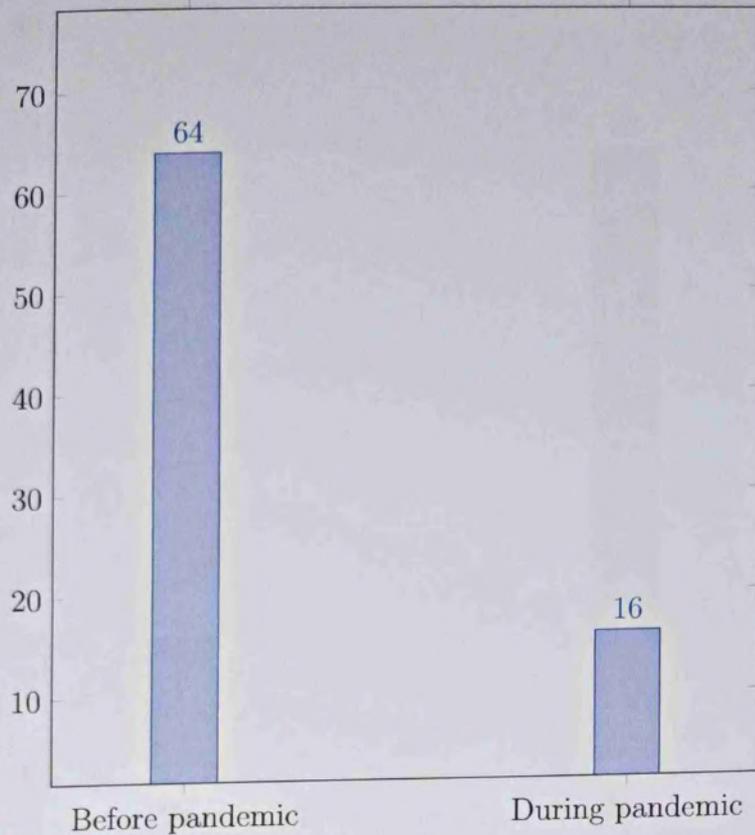
DATA ANALYSIS

4.1 GRAPHICAL ANALYSIS

Impact on the Number small Business Owners :

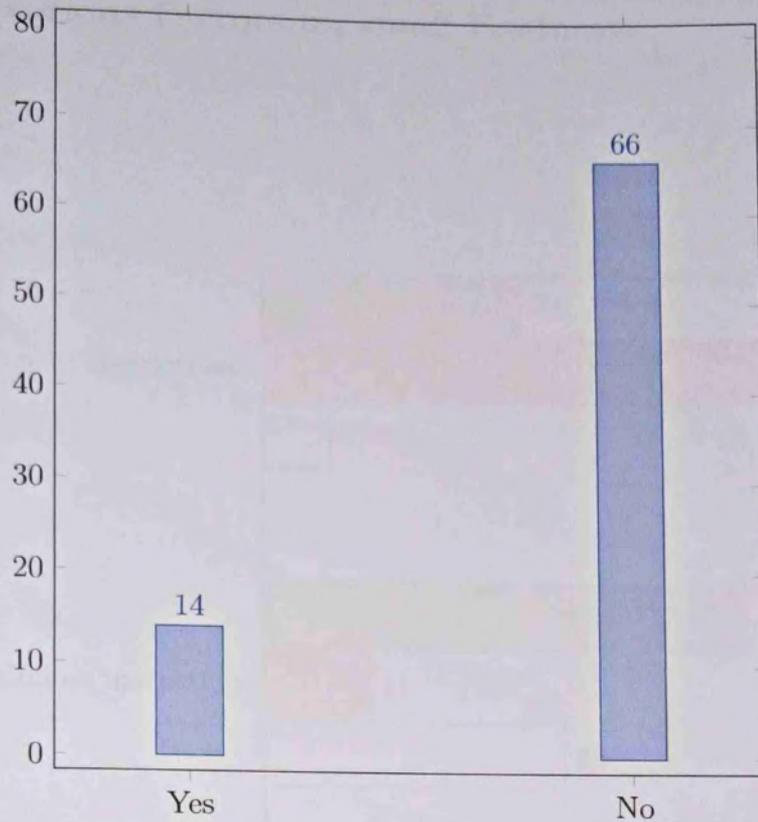
The Number of small business owners varies before and during Pandemic. It is analysed from 80 responses that, 64 small businesses started before Pandemic period and 16 small businesses started during the Pandemic period.

When was the business started	Number
Before Pandemic	64
During Pandemic	16
Total	80



The respondents were asked whether their business closed down during the pandemic period. It is observed that from 80 respondents, 14 respondents' business closed down during the pandemic period.

Did your business closed down ?	Number
Yes	14
No	66
Total	80

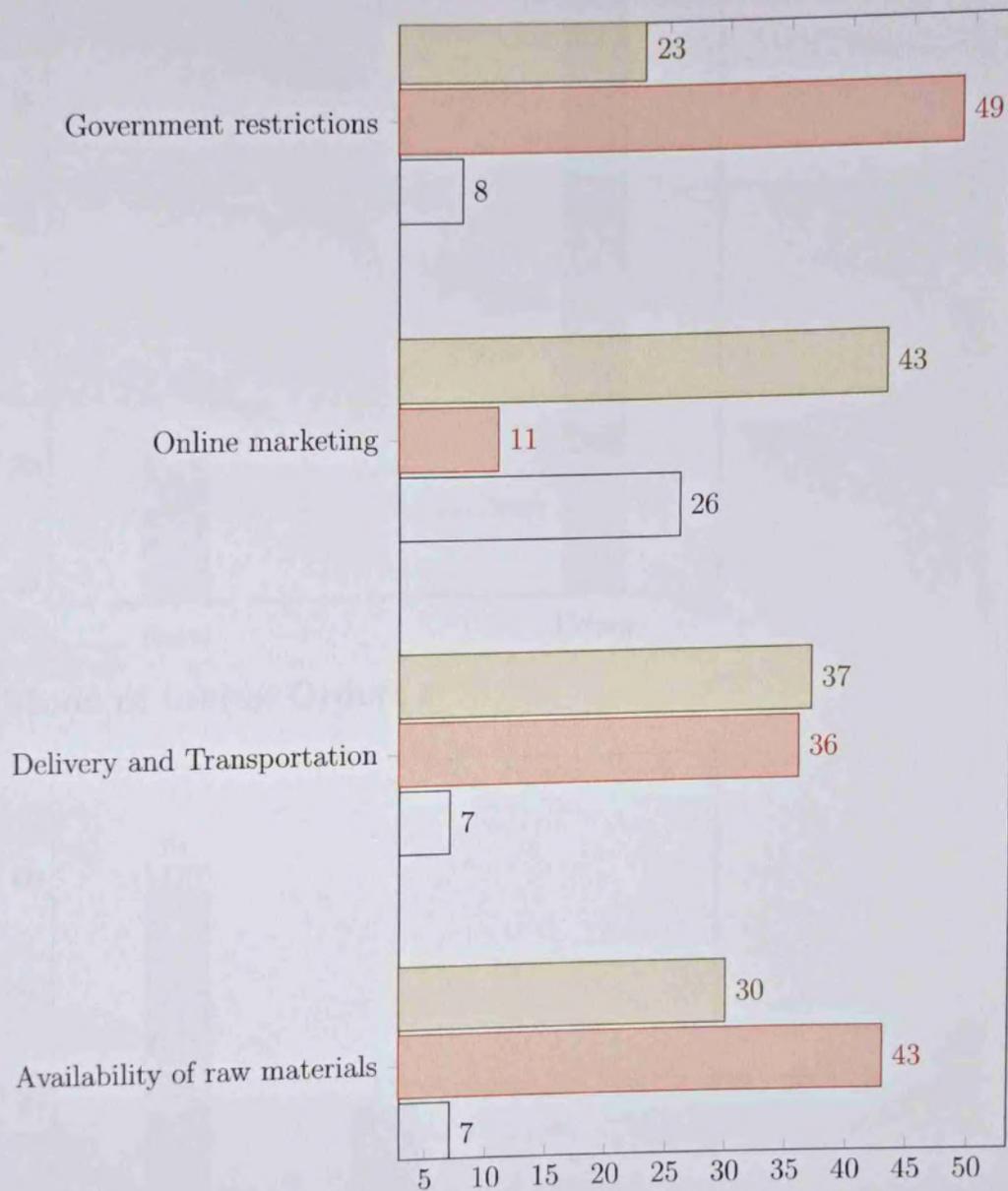


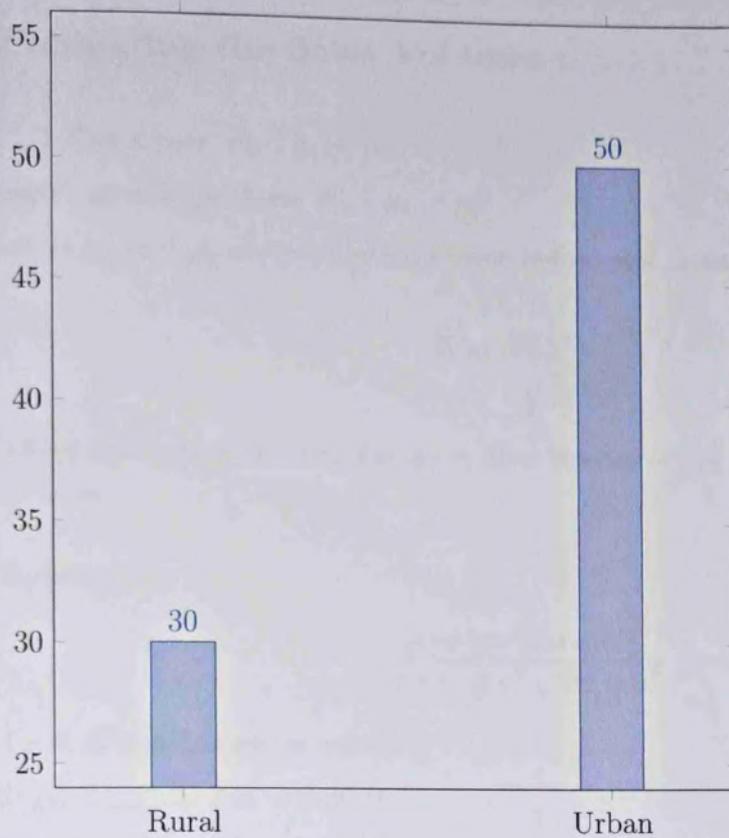
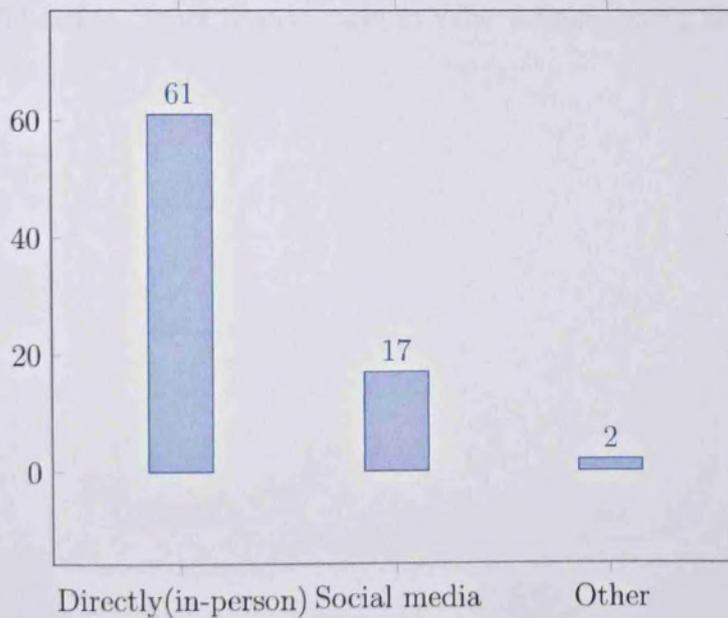
Percentage of change in Number of Owners :

Percentage change = 17.5 %

Hence, out of 80 small businesses, 82.5 % survived during pandemic.

Effect of various Factors on small Business:



Region of Business :**Mode of taking Orders :**

More than 60 % of the small business owners responded sell their products directly only.

4.2 STATISTICAL ANALYSIS

Comparing the Sales Volumes :

Null Hypothesis $H_0 : \mu_1 = \mu_2$

Alternative Hypothesis $H_1 : \mu_1 > \mu_2$

where μ_1 and μ_2 are population means before and during pandemic respectively.

$$Z = \frac{(x_1 - x_2) - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\bar{x}_1 = 4492.38$, $\bar{x}_2 = 1244.25$, $\Delta = 0$, $s_1^2 = 49505032.11^2$, $s_2^2 = 4466765.51^2$, $n_1 = 63$ and $n_2 = 80$

Substituting,

$$Z = \frac{(4492.38 - 1244.25) - 0}{\sqrt{\frac{49505032.11^2}{63^2} + \frac{4466765.51^2}{80^2}}} = 3.54$$

Level of significance, $\alpha = 0.05$

P ($Z = < z$) one tail = 0.0002

Therefore, H_0 is rejected.

Hence $\mu_1 > \mu_2$

Therefore, there is a decrease in Sales volume during pandemic.

Comparing the Annual Profits :

Null Hypothesis $H_0 : \mu_1 = \mu_2$

Alternative Hypothesis $H_1 : \mu_1 > \mu_2$

where μ_1 and μ_2 are population means before and during pandemic respectively.

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\bar{x}_1 = 293898.3$, $\bar{x}_2 = 147309$, $\Delta = 0$, $s_1^2 = 53289861751^2$, $s_2^2 = 25309730834^2$, $n_1 = 63$
and $n_2 = 80$

Substituting,

$$Z = \frac{(293898.3 - 147309) - 0}{\sqrt{\frac{53289861751^2}{63^2} + \frac{25309730834^2}{80^2}}} = 3.75$$

Level of significance, $\alpha = 0.05$

P ($Z = < z$) one tail = 0.00009

Therefore, H_0 is rejected.

Hence $\mu_1 > \mu_2$

Therefore, there is a decrease in Annual profit during pandemic.

Analysing the Effect of various factors on small business :

The following factors were studied in relation to the degree of impact they had on small Businesses during the Pandemic. The data is interpreted using a contingency table and examined using Chi-square Test.

Observed values(O_i)				
Factors / Degree of Impact	Affected positively	Affected negatively	Not affected	Total
Availability of Raw materials	7	43	30	80
Delivery and Transportation	7	36	37	80
Online Marketing	26	11	43	80
Government restrictions	8	49	23	80
Total	48	139	133	320

Expected values(E_i)				
Factors / Degree of Impact	Affected positively	Affected negatively	Not affected	Total
Availability of Raw materials	12	34.75	33.25	80
Delivery and Transportation	12	34.75	33.25	80
Online Marketing	12	34.75	33.25	80
Government restrictions	12	34.75	33.25	80
Total	48	139	133	320

Factors / Degree of Impact	$(O_i - E_i)^2/E_i$ values		
	Affected positively	Affected negatively	Not affected
Availability of Raw materials	2.0833	1.9586	0.3177
Delivery and Transportation	2.0833	0.0449	0.4229
Online Marketing	16.3333	16.2320	2.8590
Government restrictions	1.3333	5.8435	3.1598

From the Table, we get

$$\chi^2 = 52.67186852$$

$$df = 6$$

$$p\text{-value} = 1.36605 \times 10^{-9}$$

Therefore, there is a relation between factors and degree of impact. Thus the shortage of raw materials and transportation restrictions due to lockdowns have brought the small businesses down during the pandemic.

Chapter 5

CONCLUSION

The sudden rise for the price and less availability of raw materials affected the small business owner's financial recession and also lead to the close down of their businesses during the pandemic period. For the owners who had enough stocks for their business even during the pandemic time, they could not sell off their products to their usual target level as people depended online business modes more during the pandemic time.

Expecting such pandemic situations again, small businesses shall be equipped with precautions to withstand them. Expanding business to online mode and using the advantages of online marketing will be lead to development in sales. Adapting shift in business strategy (i.e., including relevant products into the existing business) might help in improving the business in difficult times.

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ANNEXURE

Questionnaire for the Study :

1. What product are you selling ?

2. When was the business started ?

- a. Before pandemic
- b. During pandemic

3. Area of business :

- a. Rural
- b. Urban

4. How do you take orders ? (multiple option selection given)

- a. Directly (in-person)
- b. Through social media (Whatsapp, Facebook, Instagram)
- c. Other (specify, if any):

5. Did your business close down during pandemic ?

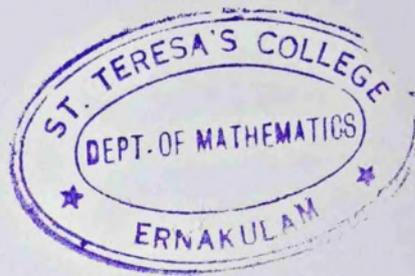
- a. Yes
- b. No

6. Specify the number: (Please only enter numbers. Put '0' if not applicable)

	Before pandemic	During pandemic
Number of orders (per month)		
Monthly profit		

7. How did the following factors affect your business during pandemic?

	Affected positively	Affected negatively	Partially affected positively	Partially affected negatively	Not affected
Availability of raw materials					
Delivery and Transportation					
Online marketing					
Government restrictions					



Project Report

On

MATHEMATICS IN FORENSIC SCIENCE

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

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CERTIFICATE

This is to certify that the dissertation entitled, **MATHEMATICS IN FORENSIC SCIENCE** is a bonafide record of the work done by Ms. **NIRANJANA MOOTHANDASSERY** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 04.03.2022

Place: Ernakulam

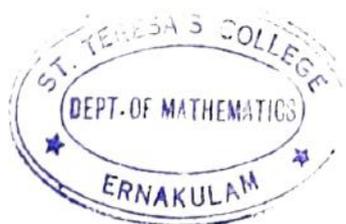

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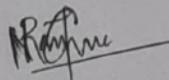
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DECLARATION

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NIRANJANA MOOTHANDASSERY

Date: 04.03.2022

AB19AMAT022

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I would like to express my special thanks to our project guide and HOD Dr. Ursala Paul for her able guidance and support in completing my project. Also I would like to thank all my teachers for the constant support. Their continuous invaluable knowledgeably guidance throughout the course of this study helped me to complete the work up to this stage

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Ernakulam.

NIRANJANA MOOTHANDASSERY

Date: 04.03.2022

AB19AMAT022

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Chapter 1

Forensic Mathematics

1.1 Introduction

Forensic science is a branch of science used to analyze crime scene evidence. All science uses mathematical concepts and equations, and forensic scientist are well educated in mathematical concepts they uses to analyze evidence from crime scenes.

Without mathematics it is impossible to analyze forensic evidence scientifically. One of the main things investigators do at the crime scene is to collect, measure and document evidence. This data help investigators to perform calculations and determine the facts of a crime. Mathematics shows proof of what ever happened during a crime in data and number.

The court ask the significance of evidence in the context of the crime and, as an expert witness the forensic scientist should be able to respond appropriately to such a challenge. Methods based on Bayesian statistic utilizing probability arguments may facilitate both the comparison of the evidence type and the weight that should be attach to each by the court.

The discussion and presentation of the data within the report submitted to the court by the expert witness must be prepared with rigour and clarity that can only come from a sound understanding of the essential mathematical and statistical method applied within forensic science.

1.2 Forensic Science

Forensic science is the application of science to criminal and civil, as governed by the legal standards of admissible evidence and criminal procedure. Forensic science is an ever growing field of science that can be further subdivided into toxicology, anthropology, serology and many others. Forensic scientist collect, preserve and analyze scientific evidence during the course of an investigation. While some forensic scientist travel to the scene of the crime to collect the evidence themselves and others occupy a laboratory role, performing analysis on object brought to them by other individuals.

1.3 Forensic Mathematics

One of the main thing crime scene investigators do is, collect, measure and document evidence. Their data help forensic scientists to perform calculation and determine the facts of a crime. Mathematics is the main key to analyze forensic evidence scientifically it is impossible to carry on without mathematics. For example, for pathologist calculus is need to estimate the time of death of victims. Overall, calculus has many application in the field forensic science. Mathematics make it possible to show the proof of what occurred during a crime in data and numbers.

1.3.1 Measurements:

One area of mathematics that is crucial for forensic science is take precise measurement at a crime scene. Knowing the exact length of shoe print from the crime scene help to rule out crime suspects whose shoe are the wrong size.

Forensic scientist need exact measurements of everything at a crime scene in order to perform scientific calculation properly. Investigators

spend a great deal of time measuring distance, weight, temperature, volume and other aspects of evidence to get the number correct.

1.3.2 Proportion:

Forensic scientist use not only measurements but proportions in their analysis. If a human leg bone is discovered in an unmarked grave forensic scientist use mathematical equations to determine what proportion or percentage of persons overall height of the leg bone would be. Once they know that they can determine how tall the person was and whether it was a child or an adult. Proportions are one way mathematics is involve in forensic science.

1.4 Applications of Mathematics in forensic Science

Forensic scientists analyze the evidence and search around crime scenes for clues pointing to possible suspects. Mathematics can be used to determine time of death, how crime are committed, when they were committed and who committed them. Some of the application of mathematics in various field of forensic science are given below :

- Psycho physical detection Monitoring pulse rate, blood pressure, and breathing patterns.
- Heights and distance Footprints in dirt and mud, length of objects.
- Bullet trajectories Geometry and trigonometry.
- Entomology Time of death.
- Trigonometry and industry physics can be used to reverse calculate height.
- To find an elevation consistent with two blood drops.
- Can be used to determine the height of the blood when it exceed to the body.

- Examining skid mark can help to reconstruct the accident. Marks are caused by the speed of the car, braking force, frictional force of the road and impacts with other vehicles.
- Newton's law of cooling describes the cooling of a warmer object to the cooler temperature of its environment.

1.4.1 Trigonometry

Trigonometry is very useful in forensic science. Knowledge about trigonometry is absolutely necessary for many crime scene reconstruction. Blood stain pattern analysis (BPA) is one of the several specialties in the field of forensic science. It involves the study and analysis of blood stain at a known or suspected violent crime scene. Blood stain evidence is most often associated with violent acts such as assault, homicide, abduction, suicide or even accidents. Pythagoras theorem, trigonometry function, trigonometric rules are application of trigonometry in forensic science. Trigonometric function related to non right angled triangles and can be used to find an unknown angle or side.

1.4.2 Probability

In forensic science, empirical probabilities are particularly important and examples may be derived from data on height, fingerprint class, blood group, allele frequencies in DNA profiles or shoe size among the population. Some examples involving the use of empirical data like the matching of hair evidence and analysis of human teeth mark are discussed later.

Forensic science uses several approaches for DNA statistics with computer programs such as match probability, exclusion probability, likelihood ratios, Bayesian approaches, and paternity and kinship testing.

Random match probabilities (RMP) are used to estimate and express the rarity of a DNA profile. RMP can be defined as the probability that someone else in the population, chosen at random, would have the same genotype as the genotype of the contributor of the forensic evidence. RMP is calculated using the genotype frequencies at all the loci, or how

common or rare the alleles of a genotype are. The genotype frequencies are multiplied across all loci, using the product rule, to calculate the RMP. This statistic gives weight to the evidence either for or against a particular suspect being a contributor to the DNA mixture.

Chapter 2

Ballistics and Ricochet analysis

2.1 Ricochet Analysis of the bullet

Ricochet occurs when the incident angle is below the critical angle for the surface, a bullet after impact bounces off a solid surface at a glancing angle after impact and then continues its trajectory, otherwise the bullet will either fragment on the impact or penetrate the solid surface.

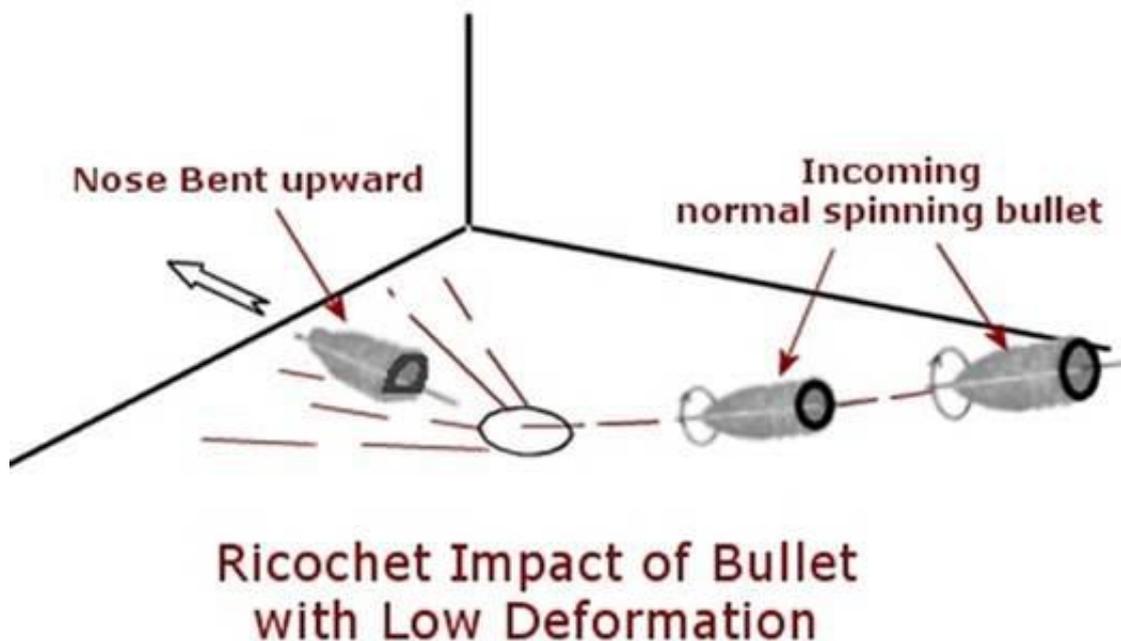


Figure 2.1:

For soil and water the critical angle is very low, at around 6–7°, whereas for hard surfaces this value will be much larger.

In almost all cases the ricochet angle θ_r at which the bullet leaves

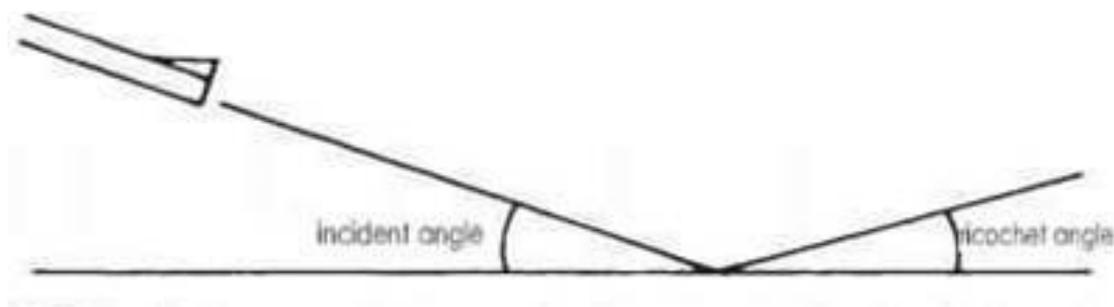


Figure 2.2: Incident and Ricochet angle

the surface is lower than the incident angle θ_i and these parameters are linked approximately by the equation

$$\tan \frac{\theta_r}{\theta_i} = C$$

where C is a constant dependent on the surface and the bullet involved.

θ_r : Ricochetangle

θ_i : Incidentangle

C :constant

2.2 Aspects of ballistics

The effect of angle and the measurement of angle are important in understanding the trajectories followed by missiles such as bullets, arrows and cricket balls etc. All missiles start their motion with some initial speed and a specific launch angle from the initial force comes from the explosive charge in a rifle, the elastic energy in a tensioned string or human arm muscles strength. Once set on its trajectory, the only force acting on a missile is the vertically downward acting gravity. This is called the vacuum trajectory assumption. Horizontal component of the velocity remains unchanged. The result is that the missile follows a curved path, reaching a maximum height when the gravitational deceleration has reduced its upward velocity component to zero. It then descends with increasing downward vertical speed and constant horizontal speed until it impacts on the ground. (Fig 3)

If it is launched horizontally, for example from an elevated window, roof or cliff-top, it follows the downward part of this trajectory. (Fig 2.2)

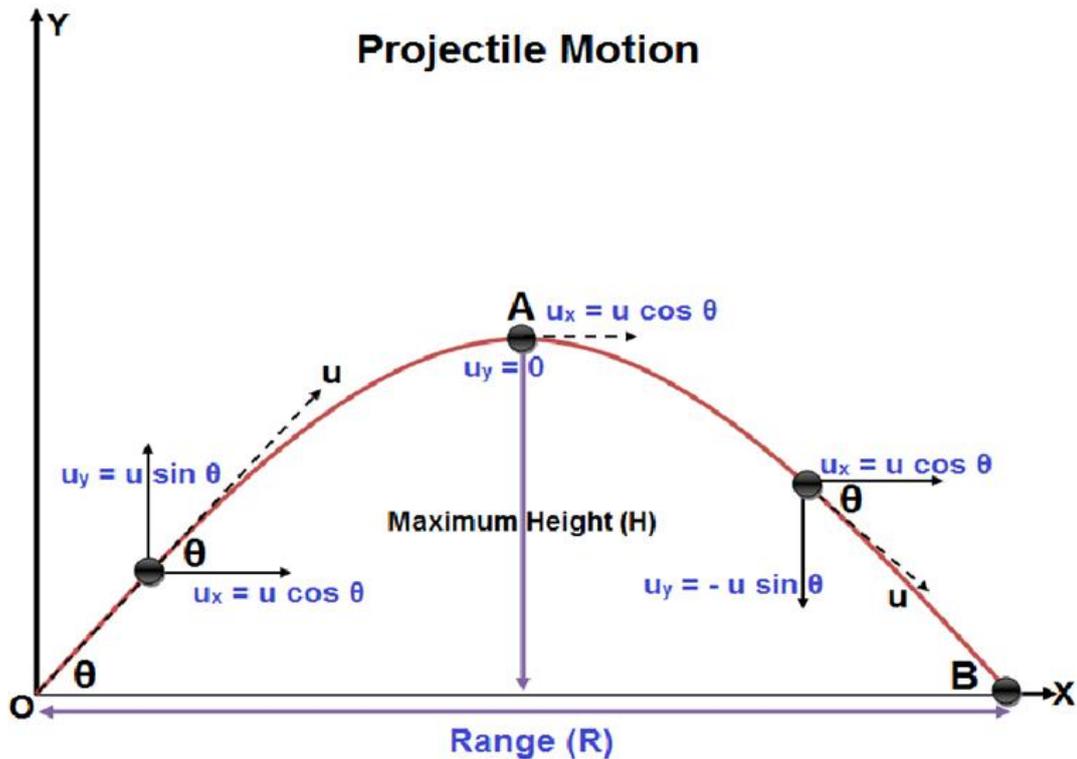


Figure 2.3:

The trajectory of a missile launched with speed u at an angle of θ to the horizontal is given by

$$y = \tan \theta x - \frac{g}{2u^2 \cos^2 \theta} x^2$$

where y is the height at any particular horizontal distance x and $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity.

In the equation ($y = (\tan \theta)x$) represents the straight line obtained in the absence of gravity by following the launch angle i.e. the line-of-sight path and the next term calculates how much the gravitational acceleration moves the projectile path downwards from this straight line. For specific initial conditions defined by u and θ , this equation is similar to the quadratic form:

$$y = Ax - Bx^2$$

This function represents a parabola with maximum altitude at $x = \frac{A}{2B}$

$$y_{max} = A\left(\frac{A}{2B}\right) - B\left(\frac{A}{2B}\right)^2 = \frac{A^2}{4B} = \frac{\tan^2 \theta}{4g} 2u^2 \cos^2 \theta = \frac{u^2 \sin^2 \theta}{2g}$$

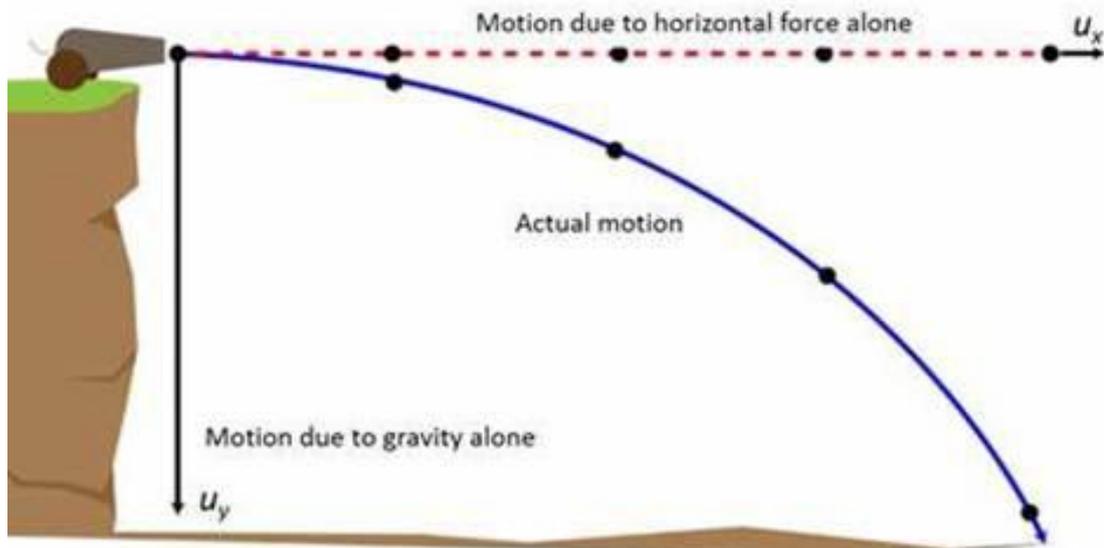


Figure 2.4:

The parabola is a symmetric shape, so if both initial and impact points are at the same height, the initial launch angle is the same as the angle at which the missile lands and also the speed with which it impacts on the same level surface is equal to the launch speed. e.g. a level surface. The horizontal range of the projectile occurs when horizontal distance

$$y = 0$$

$$(\tan \theta)x - \frac{g}{2u^2 \cos^2 \theta} x^2 = 0$$

$$x \left(\tan \theta - \frac{gx}{2u^2 \cos^2 \theta} \right) = 0$$

therefore either $x = 0$, which is the initial position, or :

$$\tan \theta - \frac{gx}{2u^2 \cos^2 \theta} = 0$$

$$x_{max} = \frac{2u^2 \cos^2 \theta \tan \theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g} = \frac{u^2 \sin 2\theta}{g}$$

Hence the maximum value of the sine function occurs when the angle is equal to 90° , the maximum range will occur here when $\theta = 45^\circ$.

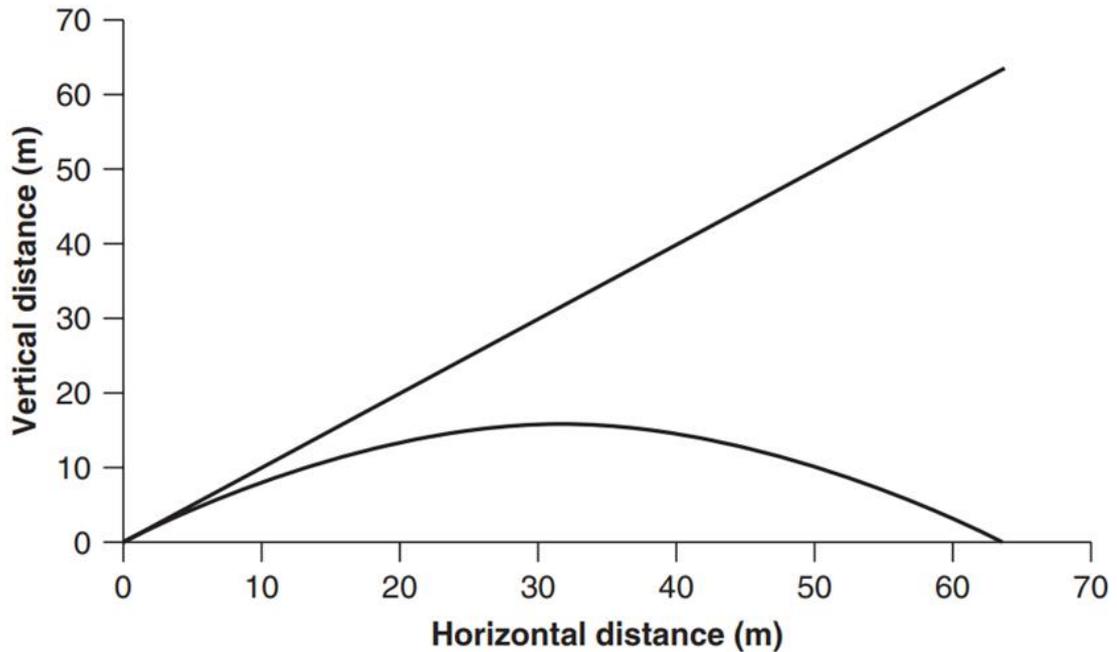


Figure 2.5:

2.2.1 Worked out example

Example :

An amateur archer who has the best launch speed of 25 m/s stands 20 m from a 5 m high wall and launches an arrow horizontally from a height of 1.6 m above the ground and at an angle of 20° horizontally, towards the wall from her standing position. Determine whether the arrow will clear the wall and if cleared by what distance? How much distance away from the other side should onlookers stand in order to be outside her range?

Solution :

Calculate the height attained over a horizontal distance of 20 m under the given launch conditions using :

$$\begin{aligned}
 y &= (\tan \theta)x - \frac{g}{2u^2 \cos^2 \theta}x^2 = (\tan 20 \times 20) - \frac{9.81 \times 20^2}{2 \times 25^2 \cos^2 20} \\
 &= 7.28 - 3.55 = 3.73 \text{ m}
 \end{aligned}$$

Since the arrow was launched 1.6 m above the ground the net height will be 5.33m and hence the height of the wall 5m is given, hence the wall will be cleared by 0.33 m.

The horizontal range is given by the sum of two calculations. The distance travelled until it is again at launch height (1.6 m) above the ground is given by :

$$x_{max} = \frac{u^2 \sin 2\theta}{g} = \frac{25^2 \sin 40}{9.81} = 41.0m$$

The remaining distance taken from a height of 1.6m, travelling at the same speed but this time on a negative launch angle of 20/degree below the horizontal, down to ground level. This distance x is given as the solution of the equation :

$$\begin{aligned} -1.6 &= (\tan(-20))x - \frac{9.81}{2 \times 25^2 \cos^2(-20)} x^2 \\ 0.00889x^2 + 0.364x - 1.6 &= 0 \end{aligned}$$

Thus we evaluate the positive root of this quadratic equation.

$$x = \frac{-0.364 \pm \sqrt{0.364^2 + 4 \times 0.00889 \times 1.6}}{2 \times 0.00889} = \frac{-0.364 \pm 0.435}{0.0178} = 4.0m$$

Hence the total range of the arrow until it hits the ground is 45.0 m, which represents 25.0 m from the wall on the far side. The negative root is not taken, as it gives the distance below ground level which is against our assumption.

Chapter 3

Determination of fall type

3.1 Introduction

In a culture that value life , explaining the death in a public forum is crucial for many reasons . The examination of a death scene and collection of potential evidential material requires special and advanced skill, knowledge, aptitude and attitude. If a body is pronounced dead at the scene, many death investigation systems require a scene investigation. Others have many protocols as to which case types absolutely require a scene investigation (whether the body is present at the scene or not). Case types that should have a scene investigation include all confirmed or suspected homicides, suicides, accidents, traffic-related deaths, in-custody deaths, and workplace-related deaths. In this chapter we are going to see whether a death is suicide ,accident ,or murder using the relationship between the height above the ground(say y) and horizontal distance travelled(say x).

3.2 Suicide,accident or murder?

Studies have shown that the distance from the wall of a building at which a body is found can help us to establish how the fall from a height originated. This is because the initial conditions of launching differ according to whether the person simply loses his balance and falls or deliberately launches himself from the tall building by jumping or even running or jumping. In the former case i.e. accidental case,

the launch angle will be zero and the initial velocity will be minimal, whereas in the latter situation the jumping action will lead to a high launch angle and larger initial speed. It has been suggested that a launch speed of 2.7 m/s or greater can be used to indicate suicide and the jump angles likely to be between 21° and 38° . For the third possible scenario of murder by pushing the victim over the edge, the launch angle may be expected to be very low with the initial velocity higher than that achieved in case of accidental fall.

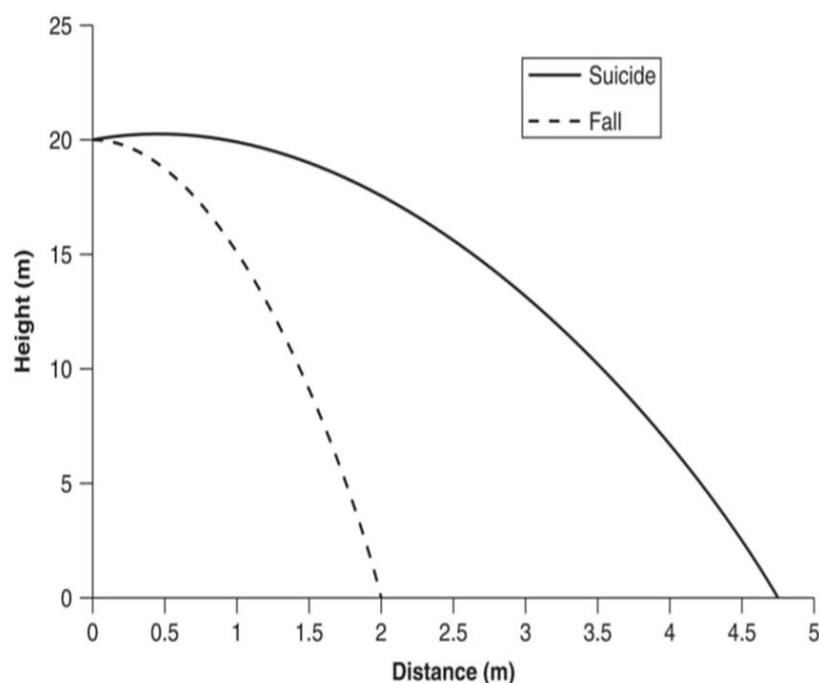


Figure 3.1:

The basic trajectory of the body is given by the same equation as before. If the ground level is set at $y = 0$ then we should simply add the fall height h , to give the relationship between y (the height above the ground) and x (horizontal distance travelled):

$$y = h + \tan \theta x - \frac{g}{2u^2 \cos^2 \theta} x^2$$

Typical trajectories, corresponding to accident or suicide, are given in the below graph. These have been calculated by assuming $\theta = 0^\circ$, $u = 1\text{m/s}$, in the former case and $\theta = 35^\circ$, $u = 3\text{m/s}$, in the latter case.

It can be seen that the impact distance for suicide is over twice that for accident. This result is supported by experimental data including an allowance for variation in the launching conditions.

3.3 Worked out example

Example : A woman jumps from the top of a vertical cliff-top 100m high

a) At what distance from the base of the building will the body land assuming initial launch conditions of $\theta = 35$ and $u = 3m/s$?

b) Compare this distance with the distance calculated on the basis of an accidental fall: $\theta = 0$ and $u = 1m/s$

Solution: a) This problem requires us to calculate x where all other parameters are known. This results in a quadratic equation:

$$0 = 100 + \tan 35x - \frac{9.8}{2 \cdot 3^2 \cdot \cos^2 35} x^2$$

$$0.812x^2 - 0.70x - 100 = 0$$

This is solved using the standard equation for the solution of quadratic equations:

$$x = \frac{0.7 + \sqrt{0.49 + 324.5}}{1.62} = 11.6m$$

b) The method is the same with these parameters; however, as the coefficient of the linear term in x is zero the solution is found more quickly:

$$0 = 100 + \tan 0x - \frac{9.8}{2 \cdot 1^2 \cdot \cos^2 0} x^2$$

$$4.90x^2 - 100 = 0$$

$$x = \sqrt{\frac{100}{4.90}} = 4.5m$$

The former result is clearly different to the latter.

3.4 Program to check the fall type

```
#import required modules
import math
class Project:
#Get user input through the constructor
def __init__(self)
self.height = float(input("Enter height of the building in meters: "))
self.theta=float(input("Enter the angle: "))
self.gravity=9.81
self.velocity=float(input("Enter the velocity in m/s: "))
print("-----")
)
# calculate the distance from the building for each type of fall
def calculateDistance(self,type):
if type=="Maximum Accidental":
self.theta=0
self.velocity=1
if type=="Minimum Suicide":
self.theta=38
self.velocity=2.7
# make the quadratic equation ax2-bx-c=0
x1 = 2 * (self.velocity * *2) * (math.cos(math.radians(self.theta)) * *2)
a=self.gravity/x1
b=math.tan(math.radians(self.theta))
c=self.height
# calculate the discriminant
d = (b * *2) + (4 * a * c)
```

```

# find the solutions
sol1 = (b - math.sqrt(d))/(2 * a)
sol2 = (b + math.sqrt(d))/(2 * a)
# print the distance calculated
if((sol1.real) > 0.0):
print(type+' distance from the building is ', format(sol1.real))
return(sol1.real)
else:
print(type+' distance from the building is ', format(sol2.real))
return(sol2.real)
# print the findings
def printFindings(self,Actual,Accidental,Suicide):
if(Actual>=Suicide):
print("The fall is SUICIDE!!")
elif(Actual>Accidental):

else:
print("The fall is ACCIDENTAL!!")
print("\n")
print("-----")
")
#creating an object with the class
obj=Project()
#call methods
Actual=obj.calculateDistance("Actual")
Accidental=obj.calculateDistance("Maximum Accidental")
Suicide=obj.calculateDistance("Minimum Suicide")
obj.printFindings(Actual,Accidental,Suicide)

```

Output of the program

Enter height of the building in meters: 100

Enter the angle: 36

Enter the velocity in m/s: 5

Actual distance from the building is 19.516519331513965

Maximum Accidental distance from the building is 4.515236409857309

Minimum Suicide distance from the building is 9.974033289332445

The fall is SUICIDE!!

Chapter 4

Case Study

4.1 Introduction

Our group visited the Harbour Janamithri Police Station, Willington Island on 20 th December 2021 to collect data on a case based on Chapter 3(determination of fall type) of our project. There we checked some case files and found a case where an employer during the construction work of Dhruv complex were found dead due to the fall of a screw jack on his head. We got the measurements such as building height and measurements giving the exact position of the body.By examining the position of the body we came to a conclusion that the screw jack fell by accident.



4.2 Case Details

FIR No : 0948

Section : 174

Name : Benoy

Age : 41

Height :169cm

Date : 30.09.2021

Place:INS Venduruthy



A man named benoy was found dead on the ground($y=0$) near the constructing Dhruv complex after a screw jack fell on his head. The body was found 4.5m (say x) away from a tall building. The screw jack was said to be in the 6 th floor of the building i.e. approximately 22.4m high. According to the witness statement the screw jack fell by accident and the police conclude the case with the same

According to our study the impact angle and initial velocity should be minimal for an accidental fall. The body was lying at a short distance , 4.5m away from the building which would only be possible when the initial conditions i.e. initial velocity u and initial launge angle θ is minimal. Hence from our studies, we concluded that the fall was accidental. i.e. $y=0$

$$h=22.4\text{m}$$

$$g=9.8$$

$$x=4.5$$

are the measurements obtained.

From the eqn

$$y = h + \tan \theta x - \frac{g}{2u^2 \cos^2 \theta} x^2$$

we usually calculate x which is used to determine the fall type provided u is minimal and $\theta = 0$ in the case of accidental fall. But here $x=4.5\text{m}$ is already obtained without the calculation which is very small which implies u and θ are minimal. Hence the fall is accidental.



Chapter 5

Bloodstain shape analysis

5.1 Introduction

Correlating the basic shape of a bloodstain with the impact conditions of the droplet involves trigonometric functions. There are two types of bloodstain formation :

1. Bloodstain formation from a stationary source
2. Bloodstain formation from a moving source

5.2 Bloodstain formation from a stationary source

Blood droplets in free-fall through the air under gravity adopt a spherical shape, surface tension forces act on the blood droplet to minimize the surface energy leading to a surface with minimum area.

On perpendicular impact of blood on a surface, it will spread out equally in all directions, then circular stain is formed. If the impact is less than 90° , the blood spreads out at the same rate in all directions. On impact the spherical blood droplet will intersect the surface in an elongated fashion in the direction of travel of the victim, then the stain is in elliptical state. The long axis (L) lies in the direction of impact along the surface and the short axis (W) is in transverse direction. By measuring the dimensions of the blood stain, the impact angle can be calculated using their ratio :

$$\sin \theta = \frac{W}{L}$$

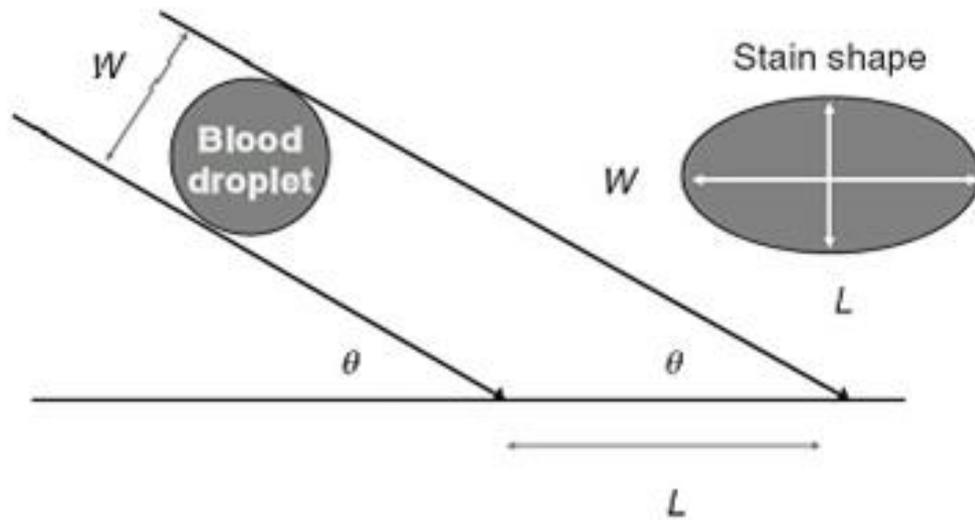
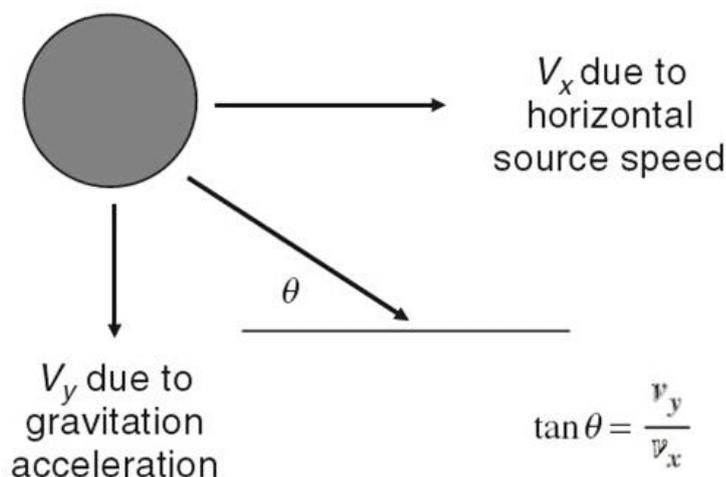


Figure 5.1: Angular impact of a blood droplet on a surface

5.3 Bloodstain formation from a moving source

Consider blood droplets falling from a moving source, the droplet under gravity starts falling vertically, it possesses a horizontal component of velocity due to the moving source and when it impacts on the ground surface, its actual impact is not 90° and it depends on the relative values of these two components.



Impact on the ground of a blood droplet from a moving source

Figure 5.2:

The above diagram shows the relation between the velocity components as impact angle vectors. These components, from a right angled triangle, which are perpendicular to each other, includes the effective impact angle, θ . Inspection of this triangle gives:

$$\tan \theta = \frac{V_y}{V_x}$$

This equation enables us to calculate the walking speed of the victim V_x , from the measurement of the impact angle θ . The vertical velocity component V_y is calculated using the drop height h :

$$V_y = \sqrt{2gh}$$

In the above equation it is assumed that the air resistance has no significant effect, which will be true for large droplets of blood falling over relatively short distance.

5.3.1 Worked out example

Example: An assailant walks away from a crime scene, blood dripping from a wound to his hand. The elliptical bloodstain has a length of 7 mm and a width of 6 mm. Calculate his walking speed assuming that his hand is moving with the same velocity as his body.

Solution impact angle is calculated using the dimensions of the stain.

$$\begin{aligned}\sin \theta &= \frac{6}{7} \\ \theta &= \sin^{-1} \frac{6}{7} = 59\end{aligned}$$

Estimation of drop distance of the blood droplet. Assuming a hand wound, this would be around 1 m; the vertical impact speed is given by:

$$V_y = \sqrt{2 \cdot 9.8 \cdot 1} = 4.43 \text{ m/s}$$

the horizontal speed is calculated using:

$$\tan 59 = 4.43/v_x$$

$$v_x = 4.43/1.664 = 2.66 \text{ m/s}$$

5.4 Program to calculate walking speed of assailant

```

#import required modules
#import math
class Project:
#get user input through the constructor
def __init__(self):
self.length=float(input("Enter the length of the bloodstain: "))
self.width=float(input("Enter the width of the bloodstain: "))
self.height=1.0
self.gravity=9.81
print("-----")
#calculate the impact angle
def calculateVelocity(self):
x=self.width/self.length
self.theta=math.asin(x)
self.theta=math.degrees(self.theta)
#calculate the velocity of the assailant from a crime scene
a=2*self.gravity*self.height
vy=math.sqrt(a)
b=math.tan(math.radians(self.theta))
vx=vy/b
return vx,vy
#print the findings
def printFindings(self,vx,vy):
print("The impact angle=",self.theta)
print("The vertical impact speed=",vy)
print("The walking speed of the assailant=",vx)
print("\n")
print("-----")
#creating an object with the class
obj=Project()
vx,vy=obj.calculateVelocity()

```


Chapter 6

Probability in forensic science

6.1 Introduction

The term probability is given to a proper measure of the certainty that a specific event or outcome will occur. In each case of probability the result's based on unbiased outcomes where every possible result's equally likely.

In forensic science, empirical probabilities are particularly important and example could also be derived from data on height, fingerprint class, blood group, allele frequency in DNA.

6.2 Calculating Probability

The most common assumption in probability is that every event has the same random chance of happening. For instance, once we toss a coin, either the tail can come up or head can. Both these events can't be predicted.

$$Probability = \frac{Number\ of\ selected\ outcomes}{Total\ number\ of\ outcomes}$$

6.2.1 RULES OF COMBINING PROBABILITIES:

We use some ground rules to combine probabilities of various situations. We can do that only when the events are independent of each other. When the outcome of one event doesn't depend upon the result of the other, the events are said to be independent.

RULE 1:

The probability of specified outcomes A and B occurring is given by:

$$P(\text{A and B}) = P(A) \times P(B)$$

RULE 2:

The probability of specified outcomes A or B occurring, where both A and B cannot occur together (mutually exclusive), is given by:

$$P(A \text{ or } B) = P(A) + P(B)$$

In some applications it is possible that both A and B occur together (e.g. A and B aren't mutually exclusive). In such cases we should always exclude this possibility to obtain:

$$P(\text{A or B}) = P(A) + P(B) - P(\text{A and B})$$

For example, some witnesses have claimed that a criminal has long, fair hair and data is out there that says that 10 cannot compute the probability of somebody having both these attributes ($P(\text{A and B})$) from this data alone as we do not know the probability of occurrence of one without the other $P(\text{A or B})$, e.g. long hair that is not fair and fair hair that is short. In other words, having long hair and having fair hair aren't mutually exclusive.

In cases where we've to seek out the probability of an event not occurring, for example, the probability that a coin wouldn't land on head when tossed. Such an event is notated by ("A'") and therefore the following applies:

$$P(A') = 1 - P(A)$$

Note that this suggests certainty – a probability of unity – that either the event will occur or it'll not!

6.2.2 Worked out examples

Example 1: A violent incident leads to a multicolored vase being broken at a crime scene into very many small pieces. Half the ceramic pieces are white and the rest are coloured either red or blue in equal proportions. A CSI is tasked with retrieving pieces of this evidence randomly. Calculate the probability of:

- (a) selecting a white piece
- (b) not selecting a red piece
- (c) selecting a white piece and a blue piece in either order
- (d) selecting one among each color in three attempts.
- (e) What assumption have you made in calculations (c) and (d)?

Solution:(a) Half the pieces are white so:

$$P(\text{white}) = \frac{1}{2} = 0.5$$

(b) A quarter of the pieces are red so:

$$P(\text{not red}) = 1 - \frac{1}{4} = 0.75$$

(c) The probability of choosing white then blue or blue then white is:

$$P(\text{w and b or b and w}) = \frac{1}{2} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{2} = \frac{2}{8} = 0.25$$

(d) Similarly, we extend the calculation to three selections, in any order. Note that there are six different orders during which the three differently coloured pieces could also be selected, e.g. white, red, blue; white, blue, red etc. Each of these has the same probability.

$$P(\text{all three colours}) = \left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}\right) \times 6 = 0.1875$$

(e) In these calculations we've assumed that the removal of a couple of pieces does not change the total number significantly, i.e. it is very

large.

Example 2:The percentage distribution of shoe sizes for 2001 is given in Table

- (a) Calculate the probability that a man selected at random will have
- a shoe size of 10
 - a shoe size of 8 or less.
- (b) If two men are randomly selected from a really large population, what's the probability
- that both will have size 9 shoes
 - that both have identical shoe size.

Table:Distribution of men's shoe sizes

SIZE	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5	11	11.5	>12
%	1	1	2	4	7	11	13	15	14	12	9	6	3	2	1

Solution : (a) .(i) Out of each hundred men, nine have this shoe size and then the proportion and thus the probability is given by:

$$P(10) = \frac{9}{100} = 0.09$$

- (ii) Here we need the number of men who have shoe sizes of 8 or less.

Using the data from the table gives:

$$P(\leq 8) = \frac{1 + 1 + 2 + 4 + 7 + 11 + 13}{100} = \frac{39}{100} = 0.39$$

(b) .(i) This probability is given by combining the probability that the first man has size 9 shoes and that the second has size 9 as well:

$$P(\text{size 9 and size 9}) = \frac{14}{100} \times \frac{14}{100} = \frac{196}{10000} = 0.0196$$

(ii) If both men have identical size then we would like to sum up, over all sizes, the individual probabilities that both have a specific size:

$$P(\text{same size}) = \frac{1}{100} \times \frac{1}{100} + \frac{1}{100} \times \frac{1}{100} + \dots + \frac{2}{100} \times \frac{2}{100} + \frac{1}{100} \times \frac{1}{100}$$

$$\begin{aligned} P(\text{same size}) &= 0.0001 + 0.0001 + 0.0004 + 0.0016 + 0.0049 + 0.0121 \\ &+ 0.0169 + 0.0225 + 0.0196 + 0.0144 + 0.0081 + 0.0036 + 0.0009 + \\ &0.0004 + 0.0001 = 0.1057 \end{aligned}$$

The probability of two men having the same shoe size, based on this data, is therefore 0.1057

6.3 Program

```
shoes=5:1,5.5:1,6:2,6.5:4,7:7,7.5:11,8:13,8.5:15,9:14,9.5:12,10:9,10.5:6,11:3,11.5:2,12:1
option=0
while(option!=3):
print("\n =====MENU=====")
print("1. Probability that a random man selected will have a given shoe
sizes ")
print("2. Probability when two men are selected ")
print("3. Exit")
print("=====")
option=int(input("Enter the option :"))
if(option==1):
sizes=map(float,input("Enter the shoe sizes :").rstrip().split())
p=0
for size in sizes:
p+=shoes[size]/100

    print("Probability is",round(p,4))
elif(option==2):
print("\n =====")
print("1.Probability that the have a given size")
print("2.Probability that they have same size")
print("=====")
option2=int(input("Enter the option :"))
if(option2==1):
size=float(input("Enter the size:"))
p=(shoes[size]/100)**2
print("Probability is",round(p,4))
elif(option==2):
p=0
```

```

percent=shoes.values()
for value in percent:
p+=(value/100)**2
print("Probability that they have same size is",round(p,4))

```

6.3.1 Output of the program

```

=====MENU=====
1. Probability that a random man selected will have a given shoe sizes
2. Probability when two men are selected
3. Exit
=====
Enter the option :1
Enter the shoe sizes :9
Probability is 0.14

=====MENU=====
1. Probability that a random man selected will have a given shoe sizes
2. Probability when two men are selected
3. Exit
=====
Enter the option :2

=====
1.Probability that the have a given size
2.Probability that they have same size
=====
Enter the option :1
Enter the size:9
Probability is 0.0196

=====MENU=====
1. Probability that a random man selected will have a given shoe sizes

```

2. Probability when two men are selected

3. Exit

=====

Enter the option :2

=====

1. Probability that they have a given size

2. Probability that they have same size

=====

Enter the option :2

Probability that they have same size is 0.1057

CONCLUSION

From the above it can be concluded that mathematics is a subject which can be incorporated in all types of sciences including the various branches of forensic science such as forensic biology, forensic chemistry, forensic physics, forensic ballistics, etc.

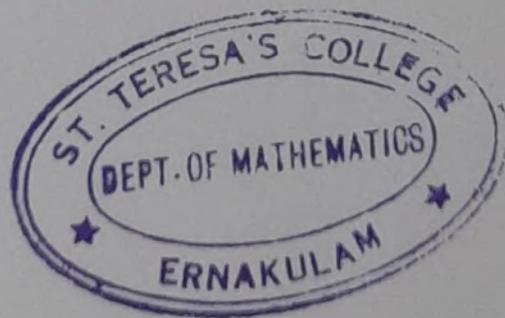
Mathematics has a wide scope in forensic science because it deals with the analysis of the evidence obtained from a crime scene followed by its interpretation and followed by mathematical calculations.

Moreover, only with the mathematics interpretation, it is possible to determine the findings such as height from which a blood drop has originated, or the angle at which a blood drop has struck a target surface resulting in the formation of bloodstains, or the probability that the blood types of any two randomly chosen individuals would match with each other and much more.

As it is clearly understood that mathematical calculations have a wide range of applications in forensic science. So, for a forensic expert, it is important to possess excellent knowledge of mathematics and statistics along with the principles and theory of different sciences to solve a crime efficiently and effectively.

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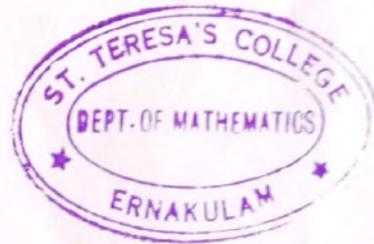


CERTIFICATE

This is to certify that the dissertation entitled, **AIR POLLUTION VARIATION DUE TO LOCKDOWN IN KOCHI** is a bonafide record of the work done by Ms. **PARVATHY RAJU** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 4/03/2022
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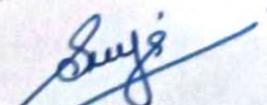
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This is to certify that the project entitled 'STUDY OF NPN TRANSISTOR CHARACTERISTICS' is an authentic work done by SANDRA JOSEPH, St. Teresa's College, Ernakulam, under my supervision at Department of Physics, St. Teresa's college partial requirements for the award of Degree of Bachelor Of science in Physics during the academic year 2021-22. The work presented in this dissertation has not been submitted for any other degree in this or any other university.

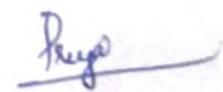
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ABSTRACT

The transistor is without a doubt one of the most important contributions to the world of electrical components. Many firms have begun large-scale programmes to create transistor circuits, and practical applications will undoubtedly become more common in the near future.

This project “**Study of NPN Transistor Characteristics**” discusses the input and output characteristics of a NPN transistor in all the three configuration- CE, CB and CC.

ACKNOWLEDGEMENT

I thank Almighty for His abundant blessing throughout this journey and to make this project a successful one. I owe a deep sense of gratitude to our project guide Dr.Sreeja V G for her immense support and valuable ideas that helped us to make this project. I would like to express my sincere gratitude to the head of the department Dr. Priya Parvathy Ameena Jose for her esteemed guidance and encouragement. I would also like to thank and express my heartfelt acknowledgement to our teachers, lab assistants and all the non-teaching staff who have always been there to help us throughout. A heartfelt thanks to my group members for being the pillar of support. Last, thanking everyone who has helped and encouraged us to make this project a reality.

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Summary And Future Prospects

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CHAPTER - 1

AN INTRODUCTION TO TRANSISTORS

INTRODUCTION

A transistor is a type of semiconductor device that can be used to conduct as well as insulate electric current or voltage. It can act as both a switch and amplifier by providing a small signal voltage. A voltage or current applied to one pair of the transistor's terminals changes the current through another pair of terminals. They are the key components in most modern devices.

Transistors can be classified into :

- ❖ Bipolar junction transistors (BJTs)
- ❖ Field-Effect Transistors (FETs)
- ❖ Insulated Gate Bipolar Transistors (IGBTs)

Bipolar junction transistors are of two types :

- ❖ npn Transistor
- ❖ pnp Transistor

Field effect transistors are of two types :

- ❖ JFET
- ❖ MOSFET

1.1 BIPOLAR JUNCTION TRANSISTORS

A bipolar junction transistor or BJT is a three terminal semiconductor device consisting of two p-n junction diodes that can amplify signals. It uses both electrons and holes as charge carriers.

A typical transistor has three terminals that help to make connections to external circuits and carry the current. The three terminals are:

1. Emitter
2. Base
3. Collector

Emitter :

The emitter section supplies the charge. Hence it is heavily doped so that it can inject a large number of charge carriers into the base. The size of the emitter is always greater than the base but less than the collector. It is the negative lead of the transistor.

Base :

The base is the middle layer. The size of the base is very small. It is less than the emitter and collector. The size is kept small so that the charge that is coming from the emitter and entering the base do not recombine in the base region and is transferred to the collector region.

The base is lightly doped and it is used to activate the transistor.

Collector :

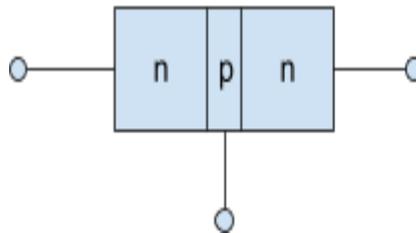
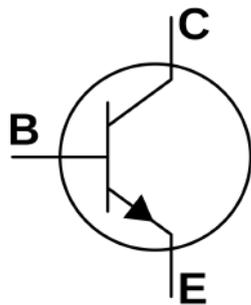
The function of the collector is to collect the charge carriers. It is moderately doped and the size is slightly large when compared to the emitter and base. It is because all the charges coming from the emitter recombine at base and heat is released in this process. Hence the collector terminal must be large so that it can dissipate the heat and the device is not burnt.

Bipolar junction transistors can be classified into two types:

- NPN Transistor
- PNP Transistor

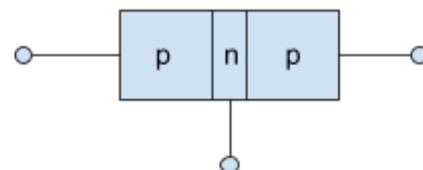
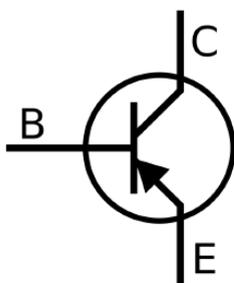
1.1.1 NPN Transistor

When a P type semiconductor layer is sandwiched between two N-type semiconductor layers the transistor is said to be a NPN transistor.



1.1.2 PNP Transistor

When an N-type semiconductor layer is sandwiched between two P-type semiconductor layers the transistor is said to be a PNP transistor



1.2 WORKING OF A TRANSISTOR

The transistor mainly works in three regions:

1. **Active Region** :-

When the emitter junction is forward biased and the collector junction is reverse bias the transistor is said to be in active mode. This region is used for amplification purpose. In the active region collector current is β times the base current i.e,

$$I_c = \beta I_B$$

Where,

I_c = Collector current

β = current amplification factor

I_B = base current

2. **Saturation region** :-

When both the emitter and collector junctions are forward biased the transistor will work in the saturation region. In this region the transistor is used for switching operation. The transistor act as an ON switch. In the saturation region

$$I_c = I_E$$

Where,

I_c = Collector current

I_E = Emitter current

3. **Cut-off region** :-

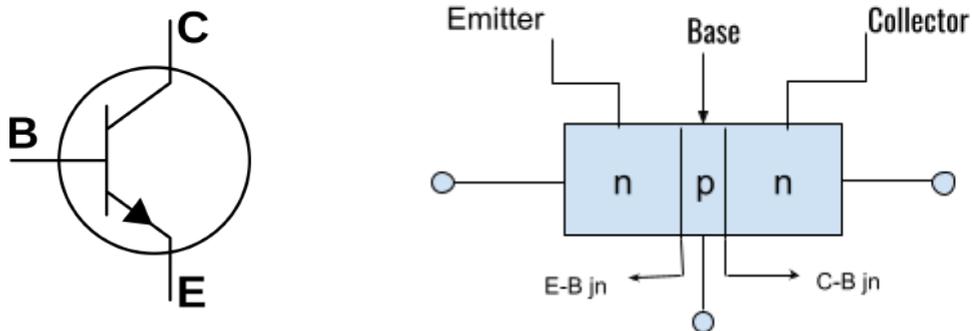
The transistor works in the Cut-off region when both the emitter and collector are reverse biased. Therefore,

$$I_E = I_c = I_B$$

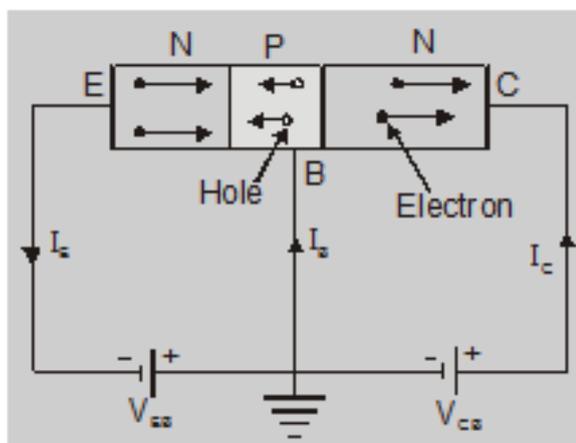
4. **Reverse Active** :-

In this the emitter base junction is reverse biased and collector base junction is forward biased.

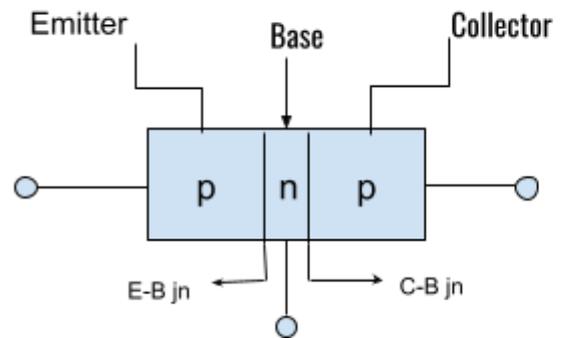
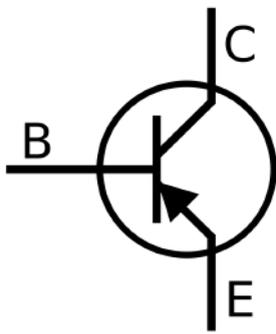
NPN Transistor



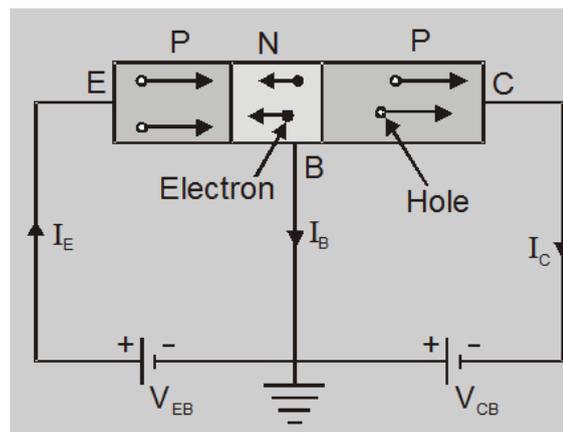
Since the Emitter-Base junction in an npn transistor is forward biased, a lot of electrons from the emitter enter the base region. Base is lightly doped with p-type impurities (Holes). Due to this there is very less electron-hole recombination i.e, very less electrons combine to constitute the base current (I_B). The remaining electrons cross over the collector region to constitute the collector current (I_C).



PNP Transistor



In a pnp transistor the emitter base junction is forward biased and collector base junction is reverse biased. Due to this a large number of holes flow from the emitter to the base and the electrons from base to the emitter region. The base is lightly doped with n-type impurities and hence number of electrons in base is very small. Due to this electron-hole recombination is less and a very few holes combine with the electrons to create the base current (I_B). The remaining holes cross over the collector region to create the collector current (I_C).



1.3 ADVANTAGES OF USING TRANSISTORS

Transistors have been proven as a very important invention in science. It has many uses and advantages:

- It is small in size and is very cost-efficient.
- It needs very low voltage to function.
- It has a long life and requires no power to operate.
- A single integrated circuit can be developed using the transistor.
- Current switches fast in the terminals.

1.4 LIMITATIONS OF TRANSISTORS

Even though transistors are extremely efficient, there are some limitations to its uses:

- Transistors get damaged very easily due to changes in electrical and temperature conditions.
- They lack higher electron mobility.
- They can get affected by radiation.

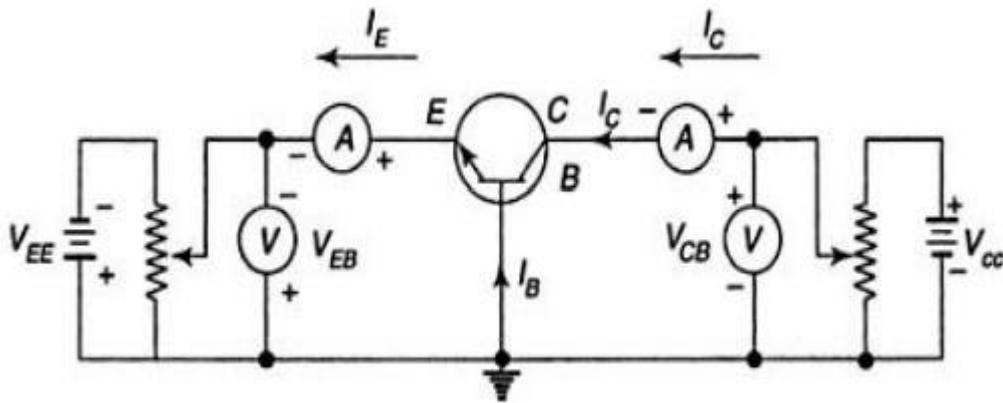
1.5 TRANSISTOR CONFIGURATIONS

Three types of configurations in a transistor are :

- Common Base configuration (CB)
- Common Emitter configuration (CE)
- Common Collector configuration (CC)

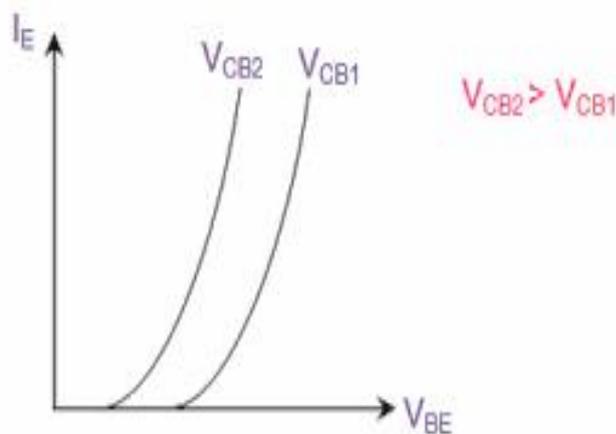
1.5.1 COMMON BASE CONFIGURATION

In the Common base configuration (CB) the Base is grounded and used as a common terminal for both input and output. It is also known as grounded base configuration. Here, the emitter is the input terminal and the collector is the output terminal.



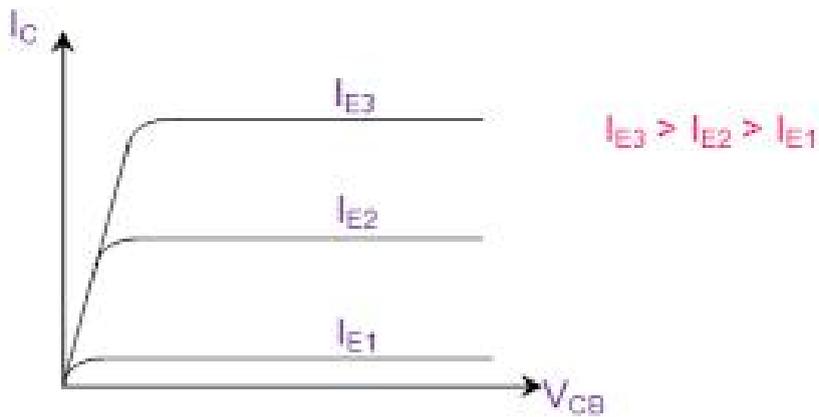
Input Characteristics

It is defined as the characteristic curve drawn between the input voltage to input current keeping output voltage constant. To determine the input voltage the collector base voltage (V_{CB}) is kept constant at zero and the emitter current I_E is increased from zero by increasing V_{EB} . This is repeated for higher fixed values of V_{CB}



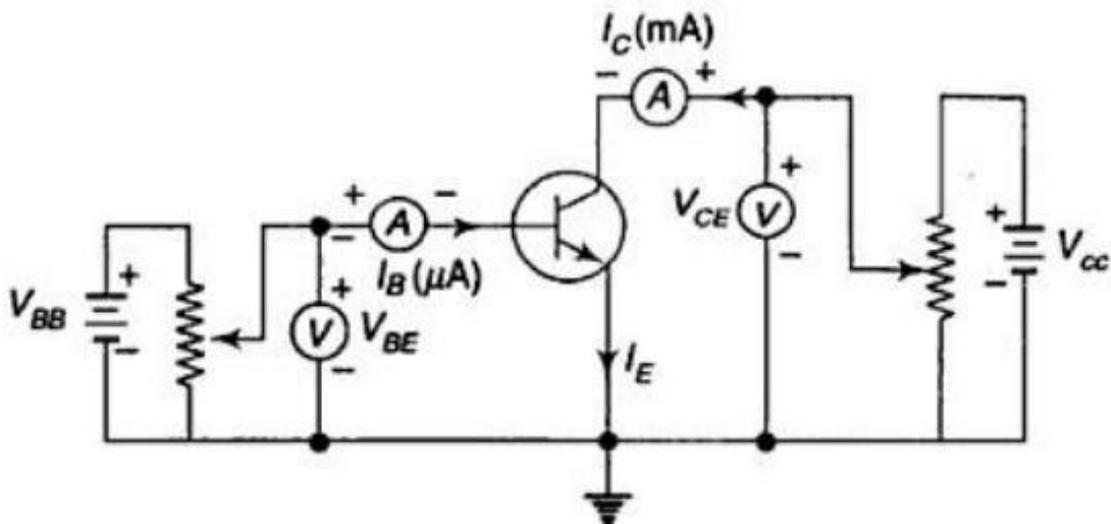
Output characteristics

It is defined as the characteristic curve drawn between output current and output voltage keeping input current constant. To determine the output characteristics the emitter current is kept constant at a particular value and the collector current (I_C) is increased from zero to higher values by increasing V_{CB} . This is repeated for higher fixed values of I_E . It is seen that for a constant value of I_E , I_C is independent of V_{CB} and the curves are parallel to X-axis of V_{CB}



1.5.2 COMMON EMITTER CONFIGURATION

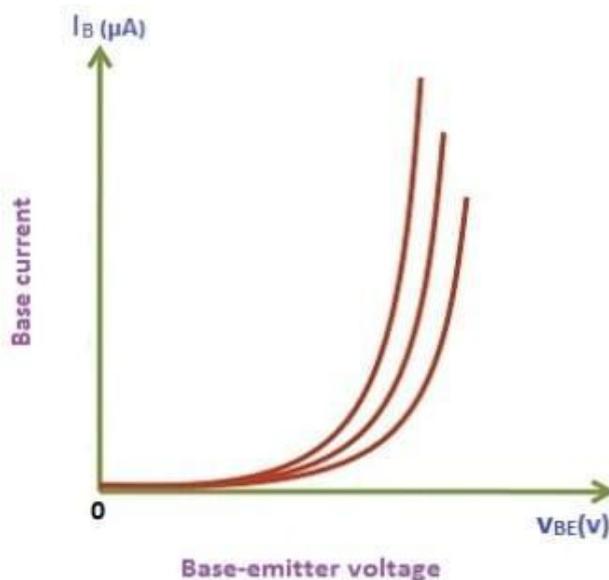
In common Emitter configuration (CE) the emitter is grounded and is a common terminal for both the input and output. It is also known as grounded emitter configuration. Here base is used as the input terminal and collector as the output terminal.



Input Characteristics

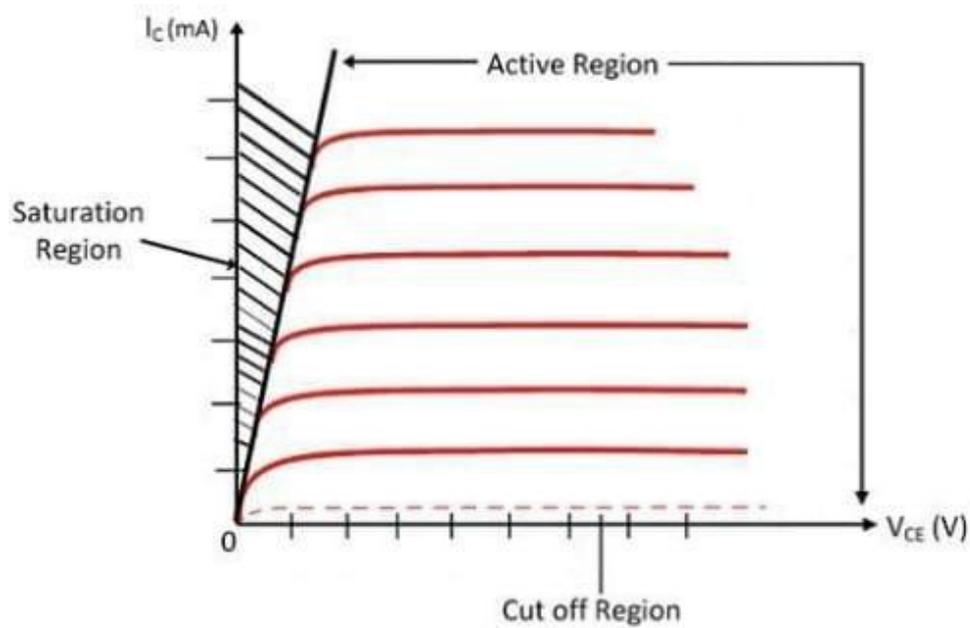
It is defined as the curve drawn between the input voltage and the input current at constant output voltage.

To determine the input characteristics the collector base voltage is kept constant V_{CB} at zero and base current I_B is increased from zero by increasing V_{BE} . This is repeated for higher values of V_{CE} .



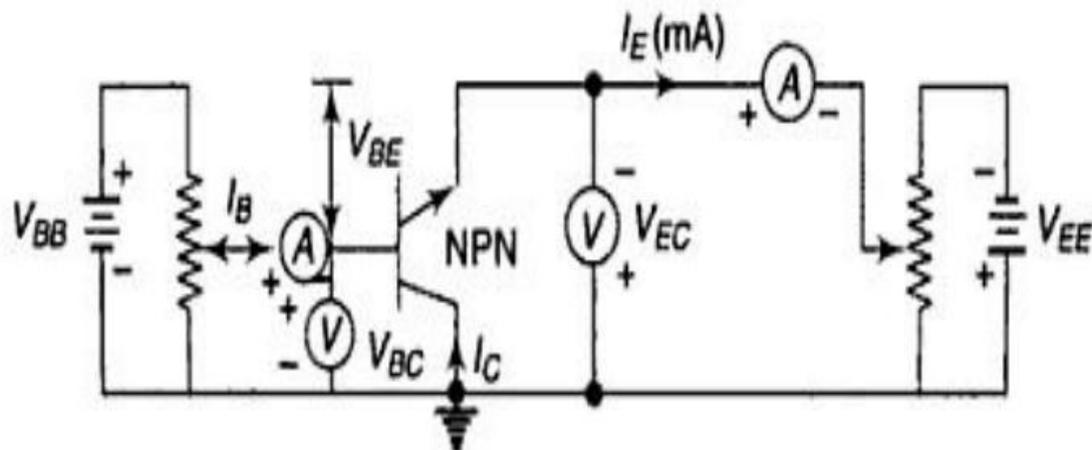
Output Characteristics

It is defined as the characteristic curve drawn between the output voltage to the output current at constant input current. To determine output characteristics, the base current I_B is kept constant at zero and collector current I_C is increased from zero by increasing V_{CE} . This is repeated for higher fixed values of I_B .



1.5.3 COMMON COLLECTOR CONFIGURATION

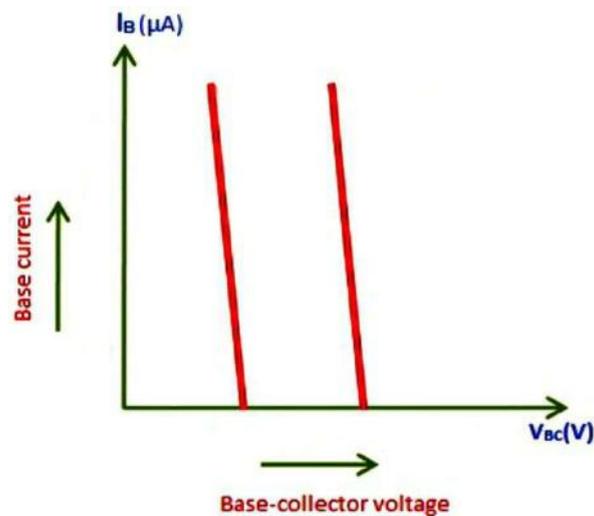
In a common collector configuration the collector is grounded and is used as the common terminal for both the input and the output terminal. It is also known as grounded collector configuration. Here the base is the input terminal and the emitter is the output terminal.



Input Characteristics

It is defined as the characteristic curve drawn between input voltage to input current whereas output voltage is constant.

To determine input characteristics, the emitter base voltage V_{EB} is kept constant at zero and base current I_B is increased from zero by increasing V_{BC} . This is repeated for higher fixed values of V_{CE} .

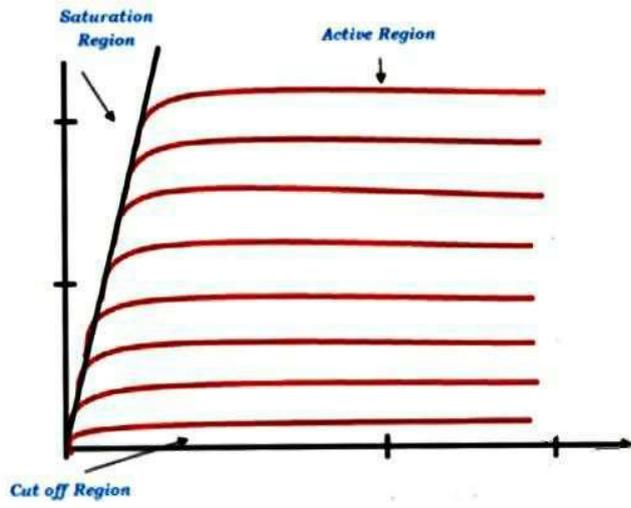


Output characteristics

It is defined as the characteristic curve drawn between output voltage to output current whereas input current is constant.

To determine output characteristics, the base current I_B is kept constant at zero and emitter current I_E is increased from zero by increasing V_{EC} . This is repeated for higher fixed values of I_B .

From the characteristic it is seen that for a constant value of I_B , I_E is independent of V_{EB} and the curves are parallel to the axis of V_{EC} .



CONCLUSION

Thus, a transistor is an electronic device made of three layers of semiconductor material that can act as an insulator and a conductor. It is an essential part of many technological advances and devices. This chapter helps to give an idea about the basic theory of transistors. It contains a quick view on the construction, working, types of transistors (npn and pnp transistors) and advantages and limitations. It also covers the three configurations of transistors - Common Emitter Configuration (CE), Common Base Configuration (CB) and Common Collector Configuration (CC) and their respective input and output characteristics.

CHAPTER 2

BASIC REQUIREMENTS

(Details of Components and Devices)

INTRODUCTION

The experiment works with a npn transistor in all the three configuration i.e. CE, CB and CC configuration, to find the input and output characteristics. The basic devices used are transistor, voltmeter, ammeter, rheostat, battery eliminator, bread board and connection wires. This chapter contains the details regarding the components and devices used for the experiments.

2.1. COMPONENTS AND DEVICES

(i) Transistor BC 107

The BC107 is a small single NPN Transistor available in TO-18 metal can package. These transistors are age old and have been used in low noise and low signal designs. Today a lot of new transistors have come as replacement for BC107, but still the transistor can be found in the market for its legacy.



The **BC107** is a low signal NPN which is known for its low noise operations making it famously used in signal processing circuits and television receivers.

The transistor is still available in the market due to its legacy but you will find better modern transistors as replacement for BC107.

Applications:

- Driver Modules like Relay Driver, LED driver etc..
- Amplifier modules like Audio amplifiers, signal Amplifier etc..
- Darlington pair

(ii) Voltmeter:

It is an instrument that measures voltages of either direct or alternating electric current on a scale usually graduated in volts, millivolts (0.001 volt), or kilovolts (1,000 volts). Many voltmeters are digital, giving readings as numerical displays.



Analog voltmeters move a pointer across a scale in proportion to the voltage measured and can be built from a galvanometer and series resistor. Metres using amplifiers can measure tiny voltages of microvolts or less. Digital voltmeters give a numerical display of voltage by use of an analog-to-digital converter.

(iii) Ammeter

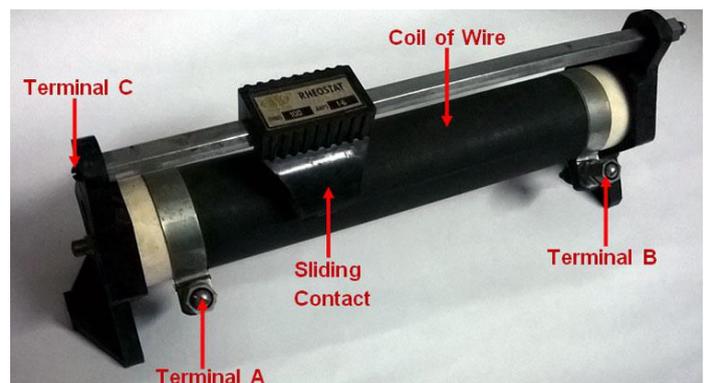
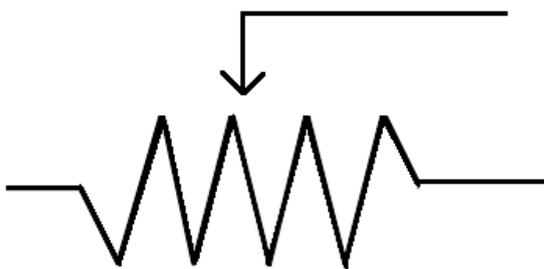
It is an instrument for measuring either direct or alternating electric current, in amperes. An ammeter can measure a wide range of current values because

at high values only a small portion of the current is directed through the meter mechanism; a shunt in parallel with the meter carries the major portion.



(iii) Rheostat

A rheostat is defined as variable resistor which is used for controlling the flow of electric current either by increasing or decreasing the resistance. The term rheostat was coined by the English scientist Sir Charles Wheatstone and is derived from the Greek word “rheos” and “statis” which means current controlling device.



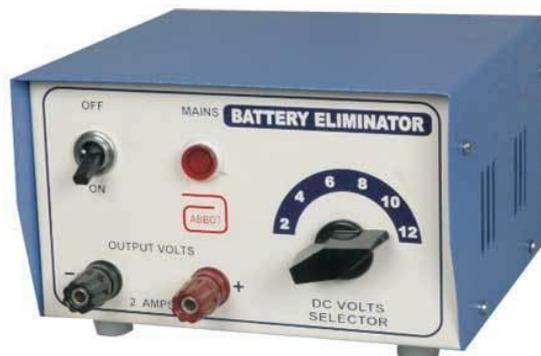
For any given rheostat, we can change its resistance. We know that resistance is dependent on three factors:

- Length
- Areas of cross-section
- Type

In order to change the resistance of the rheostat, the effective length needs to be changed with the help of sliding contact. The effective length is defined as the length between the fixed terminal and the position of the sliding terminal. As the effective length changes, the resistance of the rheostat changes.

(iv) **Battery Eliminator:**

A battery eliminator is a device powered by an electrical source other than a battery, which then converts the source to a suitable DC voltage that may be used by a second device designed to be powered by batteries. A battery eliminator eliminates the need to replace batteries but may remove the advantage of portability. A battery eliminator is also effective in replacing obsolete battery designs.



2.2 PROCEDURE

Connections are made as shown in the circuit diagrams. The rheostat Rh1 is used to vary base voltage (input voltage) V_{BE} and it is read from voltmeter V1. The base current (input current) I_B is measured using a microammeter (μA). The collector voltage (output voltage) V_{CE} is varied using the rheostat Rh2 and readings are noted from voltmeter V2. The collector current (output current) I_C is measured by the milliammeter (mA).

CONCLUSION

The chapter briefs the details, uses and types of various components and devices used in the experiment to find out the input and output characteristics of the transistor. The experiment is, thus, done with the npn transistor - Transistor BC 107, ammeter - to measure the current, voltmeter - to measure the voltage drop, rheostat - used as a variable resistance, battery eliminator - for supply of voltage, bread board and connecting wires.

CHAPTER-3

CHARACTERISTICS OF COMMON EMITTER CONFIGURATION

INTRODUCTION

In this chapter we try to find the input and output characteristics of common emitter configuration experimentally and compare it with the theoretical values.

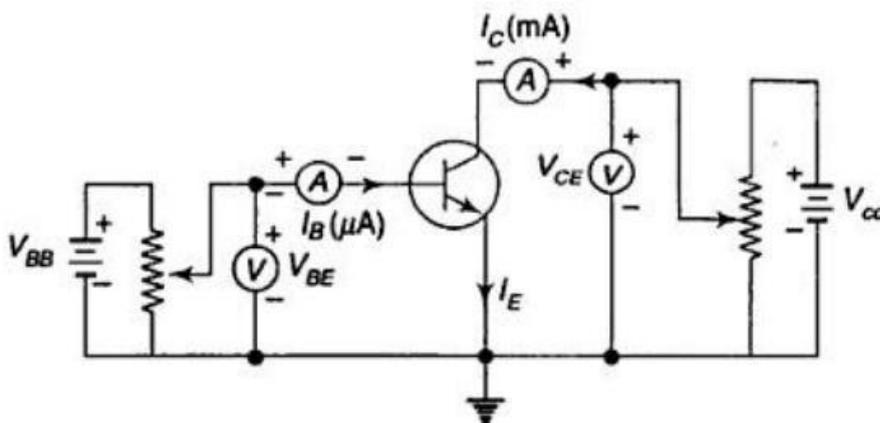
Aim

To study the input and output characteristics of a transistor in common emitter (CE) configuration.

Apparatus

Transistor BC107, Breadboard, Rheostat, Analogue Ammeter, Analogue voltmeter

Circuit diagram



3.1 Procedure

a) To find input characteristics:

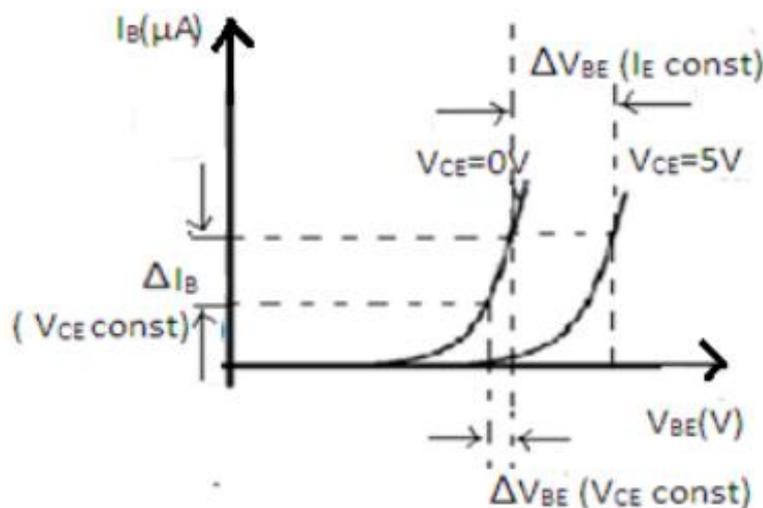
Connect the circuit diagram as shown in the circuit diagram. Keep output voltage $V_{CE}=0V$ by varying V_{CC} . Varying V_{BB} gradually, note down the base current I_B and base emitter voltage V_{BE} . Step size is not fixed because of linear curve. Initially vary V_{BB} in steps of 0.1 V. Once the current starts increasing vary V_{BB} in steps of 1V up to 12 V. Repeat the above procedure.

b) To find output characteristics:

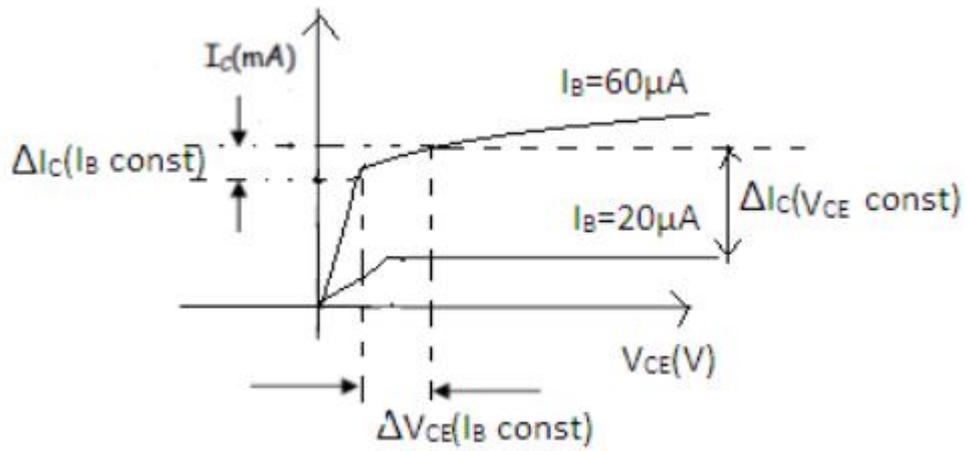
Connect the circuit diagram as shown in the figure. Keep base current (I_B) constant by varying the rheostat. Varying the rheostat gradually note down collector current (I_C) and collector emitter voltage (V_{CE}) Repeat the above experiment.

Plot the graph of input characteristics by taking V_{BE} on X axis and I_B on Y axis at a constant V_{CE} as a constant parameter. Plot the graph of output characteristics by taking V_{CE} on X axis and taking I_C on Y axis taking I_B as a constant parameter.

Graph



Input Characteristics



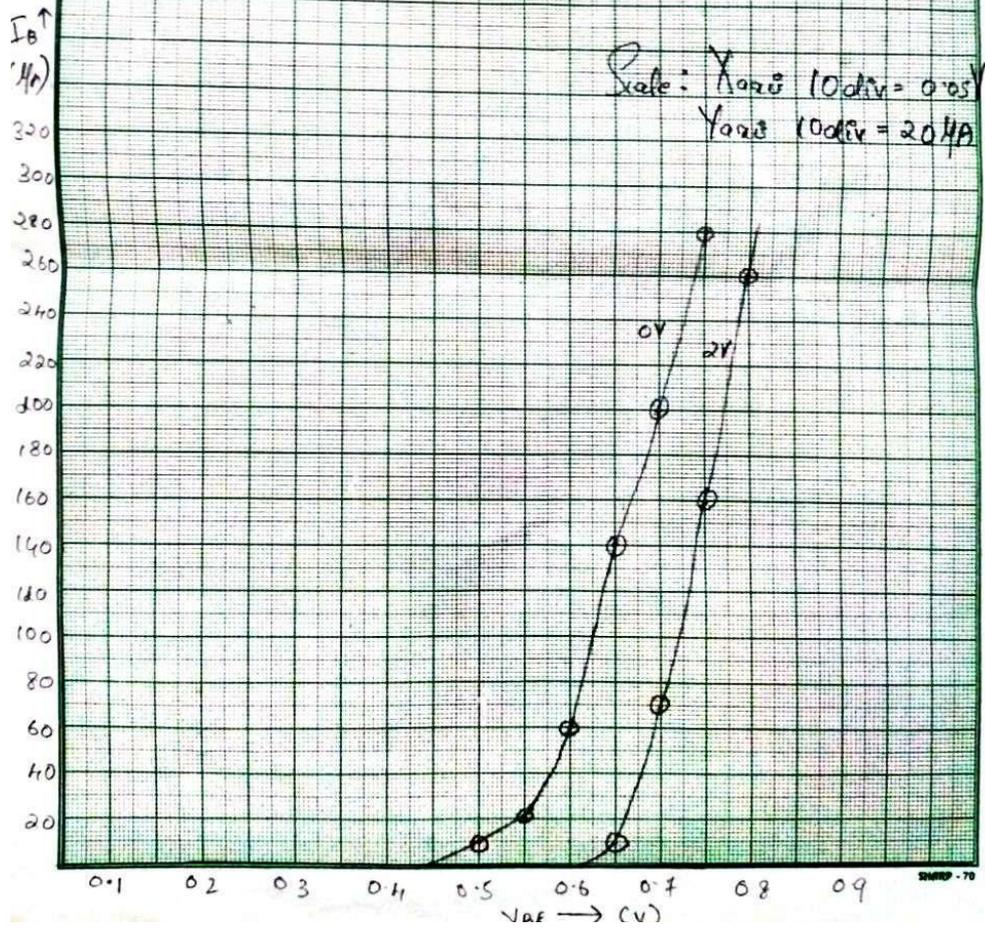
Output Characteristics

3.2 Observations

INPUT CHARACTERISTICS

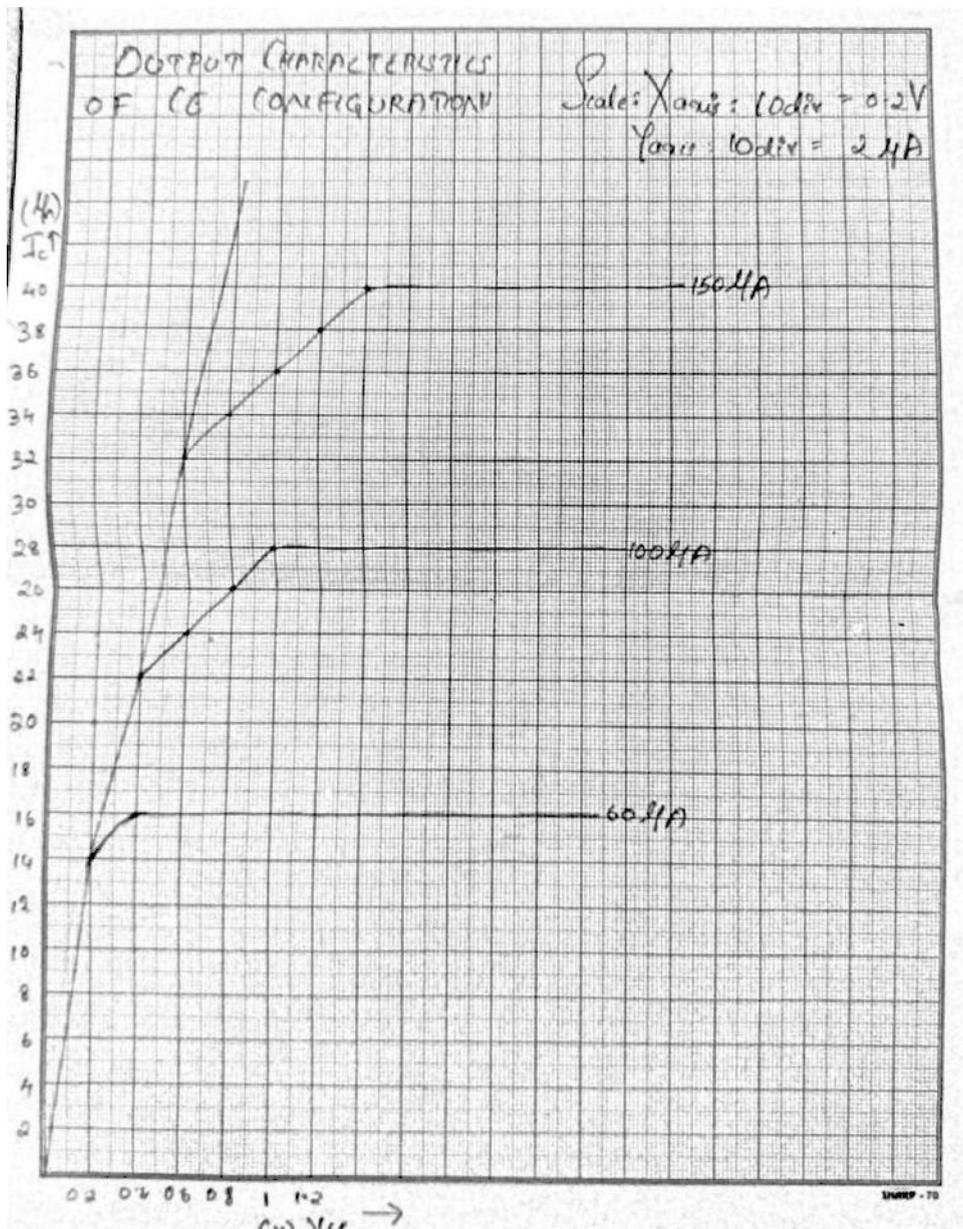
$V_{CE} = 0V$		$V_{CE} = 2V$	
$V_{BE}(V)$	$I_B(\mu A)$	$V_{BE}(V)$	$I_B(\mu A)$
0.5	10	0.65	10
0.55	20	0.7	70
0.6	60	0.75	160
0.65	140	0.8	260
0.7	200	0.85	370
0.75	280	0.9	460

INPUT CHARACTERISTICS OF CE CONFIGURATION



OUTPUT CHARACTERISTICS

$I_B=0 \mu\text{A}$		$I_B=100 \mu\text{A}$		$I_B=150 \mu\text{A}$	
V_{CE} (V)	I_C (mA)	V_{CE} (V)	I_C (mA)	V_{CE} (V)	I_C (mA)
0	0	0	0	0	0
0.2	14	0.2	20	0.2	24
0.4	16	0.4	22	0.4	28
0.6	16	0.6	24	0.6	32
0.8	16	0.8	26	0.8	34
1	16	1	28	1	36
1.2	16	1.2	28	1.2	38
1.4	16	1.4	28	1.4	40
1.6	16	1.6	28	1.6	40
1.8	16	1.8	28	1.8	40



Calculations from the Graph

To obtain input resistance find ΔV_{BE} and ΔI_B

For a constant V_{CE} on one of the input characteristics;

$$\text{Input impedance} = \Delta V_{BE} / \Delta I_B = (0.6 - 0.5) / (60 - 10)$$

$$= 0.1 / 50 = 0.002 \Omega$$

$$\text{Reverse voltage gain} = \Delta V_{EB} / \Delta V_{CE} = (0.6 - 0.5) / (0.4 - 0.2)$$

$$= 0.1 / 0.2 = 0.5$$

To obtain output resistance find $\Delta I_c / \Delta V_{CB}$ at a constant I_B .

$$\begin{aligned}\text{Forward current gain} &= \Delta I_c / \Delta I_B = (16 - 14) / (20 - 10) \\ &= 2 / 10 = 0.2\end{aligned}$$

3.3 Results

- a. The input resistance = 0.002Ω
- b. The reverse voltage gain = 0.5
- c. The forward current gain = 0.2

CONCLUSIONS

In this Chapter we studied the input and output characteristics of an NPN transistor in CE configuration. We have also included the observations and graphs of the same.

4.1 Procedure

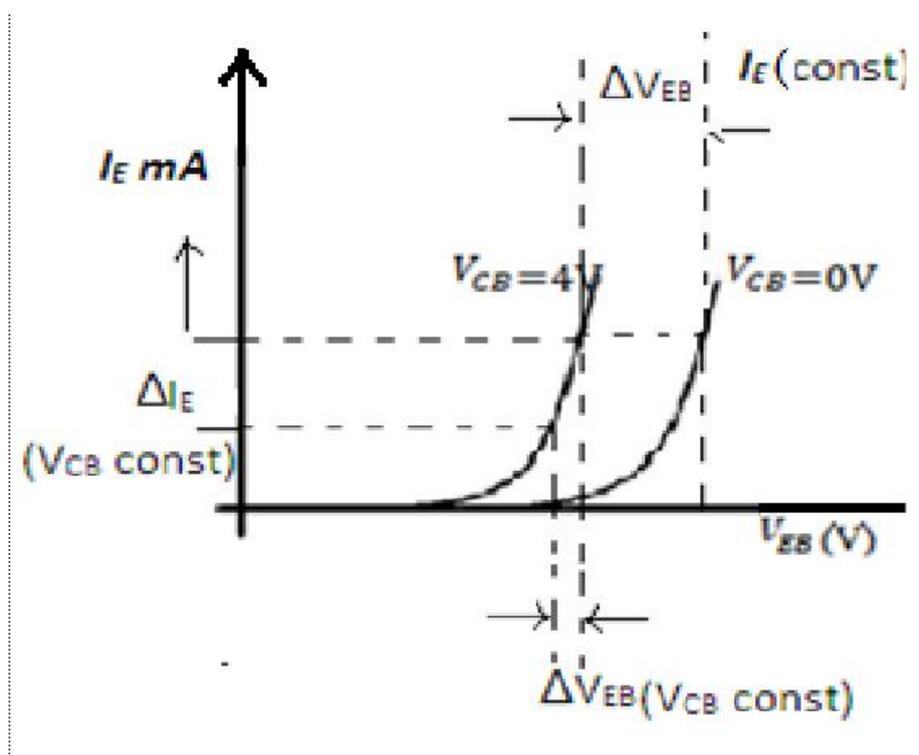
a) To find Input Characteristics:

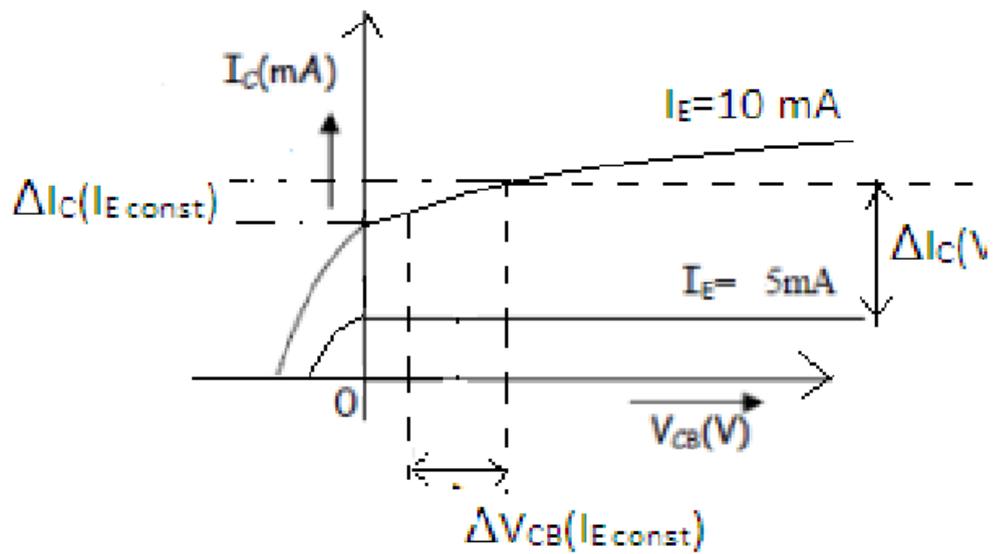
Connect the circuit as shown in the circuit diagram. Keep output voltage V_{CB} constant by varying rheostat. Varying rheostat gradually, note down emitter current (I_E) and emitter-base voltage (V_{EB}). Step size is not fixed because of the nonlinear curve. Repeat above procedure for different constant value V_{CB}

b) To find Output Characteristics:

Connect the circuit as shown in the circuit diagram. Keep emitter current (I_E) by varying resistance in rheostat and note down collector current I_C and collector-base voltage (V_{CB}). Repeat the procedure for different constant values of I_E Plot the input characteristics for different values of V_{CB} by taking V_{EB} on X-axis and I_E on Y-axis taking V_{CB} as constant parameter. Plot the output characteristics by taking V_{CB} on X-axis and I_C on Y-axis taking I_E as a constant parameter.

Graph



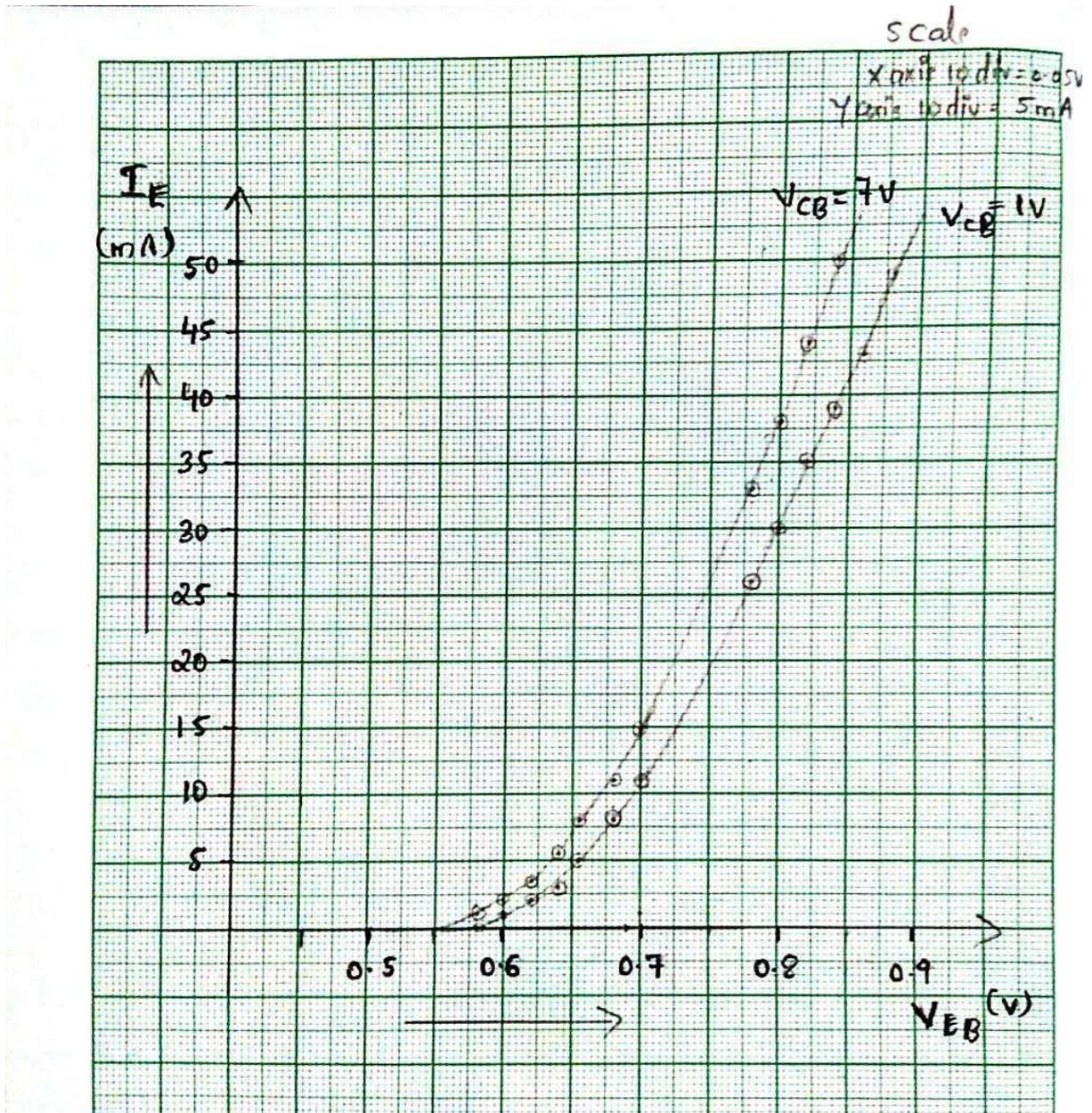


4.2 Observations

INPUT CHARACTERISTICS

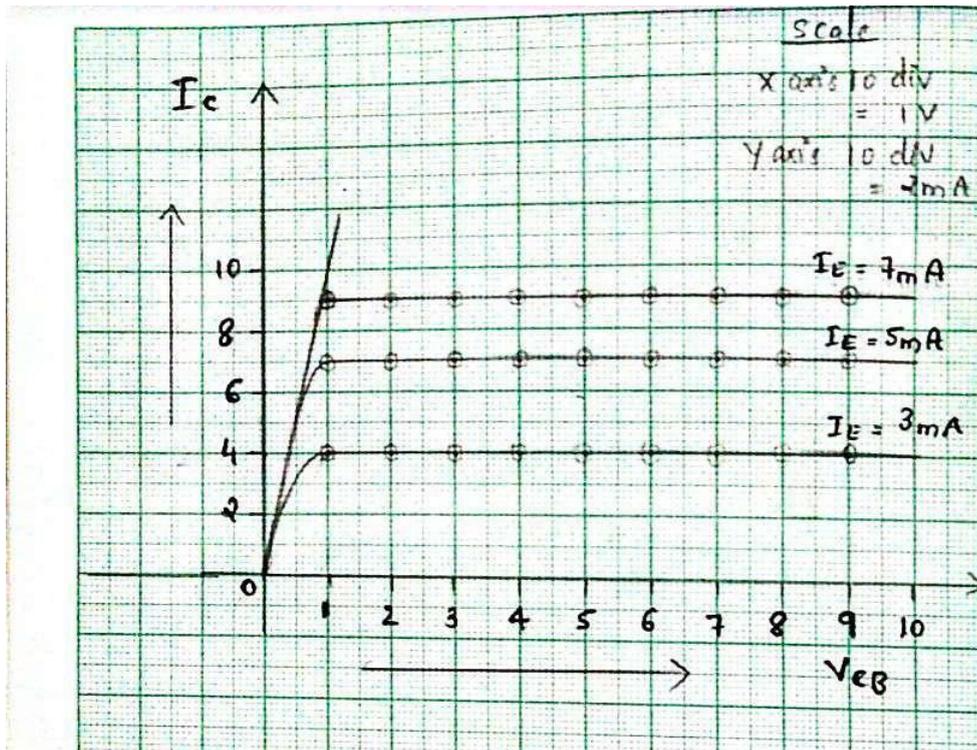
$V_{CB} = 1 \text{ V}$		$V_{CB} = 7 \text{ V}$	
$V_{EB} \text{ (V)}$	$I_E \text{ (mA)}$	$V_{EB} \text{ (V)}$	$I_E \text{ (mA)}$
0.58	0	0.58	1
0.6	1	0.6	2
0.62	2	0.62	3.5
0.64	3	0.64	5.5
0.66	5	0.66	8
0.68	8	0.68	11
0.7	11	0.7	15
0.72	14	0.72	18
0.74	18	0.74	23
0.76	22	0.76	28

0.78	26	0.78	33
0.8	30	.8	38
0.82	35	.82	44
0.84	39	.84	50
0.86	43	.86	Out of range
0.88	49		
0.9	Out of range		



OUTPUT CHARACTERISTICS

$I_E = 3 \text{ mA}$		$I_E = 5 \text{ mA}$		$I_E = 7 \text{ mA}$	
$V_{CB} \text{ (V)}$	$I_C \text{ (mA)}$	$V_{CB} \text{ (V)}$	$I_C \text{ (mA)}$	$V_{CB} \text{ (V)}$	$I_C \text{ (mA)}$
0	4	0	7	0	9
1	4	1	7	1	9
2	4	2	7	2	9
3	4	3	7	3	9
4	4	4	7	4	9
5	4	5	7	5	9
6	4	6	7	6	9
7	4	7	7	7	9
8	4	8	7	8	9
9	4	9	7	9	9



Calculations from the Graph

1) Input characteristics:

$$\text{Input Impedance} = \Delta V_{EB} / \Delta I_E = (0.88 - 0.84) / (50 - 49) \\ = 0.25 / 1 = 0.25 \Omega$$

$$\text{Reverse voltage gain} = \Delta V_{EB} / \Delta V_{CB} = (0.64 - 0.62) / (3 - 2) \\ = 0.02 / 1 = 0.02$$

2) Output Characteristics:

$$\text{Forward Current gain} = \Delta I_C / \Delta I_E = (9 - 7) / (7 - 5) = 2 / 2 = 1$$

4.3 Results

- The Input resistance = 0.25Ω
- The Reverse Voltage gain = 0.02
- The Forward Current gain = 1

CONCLUSIONS

In this Chapter we studied the input and output characteristics of an NPN transistor in CB configuration. We have also included the observations and graphs of the same.

CHAPTER-5

CHARACTERISTICS OF COMMON COLLECTOR CONFIGURATION

INTRODUCTION

In this chapter we will be discussing about the input and output characteristics of Common Collector Configuration

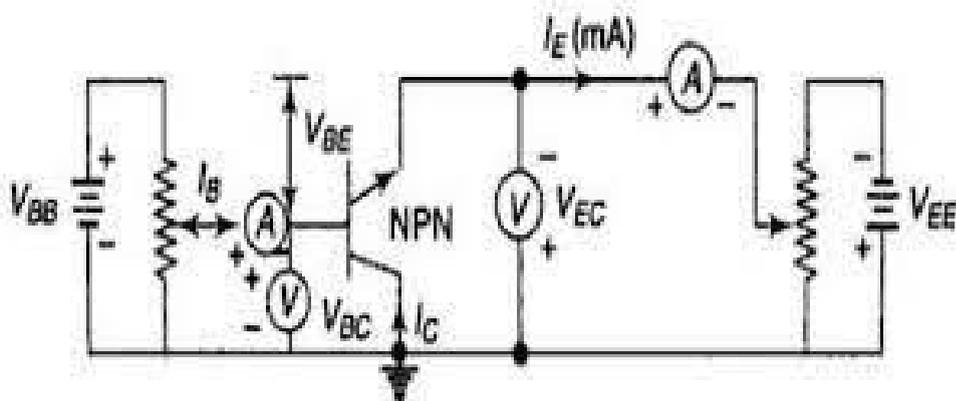
Aim

To study the input and output characteristics of a transistor in a common collector configuration(CC).

Apparatus

Transistor BC 107, Rheostat, Analogue ammeter, Analogue voltmeter, Connecting wires

Circuit Diagram



5.1 Procedure

a) Input Characteristics:

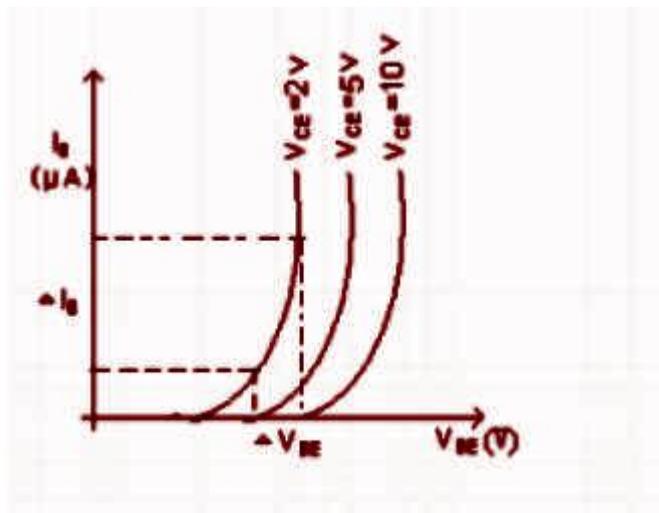
Connect the circuit as shown in the circuit diagram. Keep output voltage V_{CE} as constant rheostat. Varying gradually, note down base current I_B and emitter-base voltage (V_{BE}). Step size is not fixed because of the nonlinear curve. Repeat the experiment for different constant values of V_{CE} .

b) Output Characteristics:

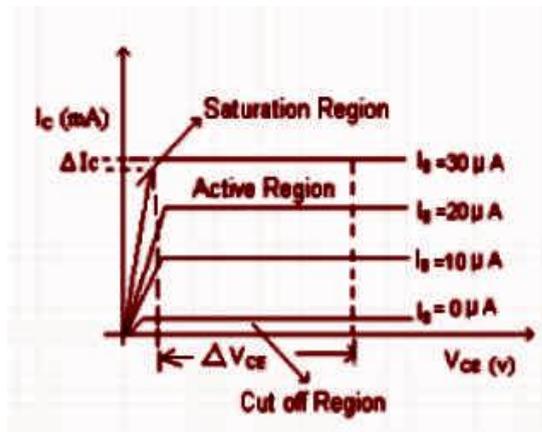
Fix base current, I_B at constant value. Vary the output voltage in steps. Measure the voltage V_{CE} and current I_C for different values. Repeat the experiment for different constant values of I_B . Draw output static characteristics for tabulated values.

Plot the input characteristics for different values of V_{CE} by taking V_{BE} on X-axis and I_B on Y-axis taking V_{CC} as constant parameter. Plot the output characteristics by taking V_{CE} on X-axis and I_C on Y-axis taking I_B as a constant parameter.

Graph



Input Characteristics



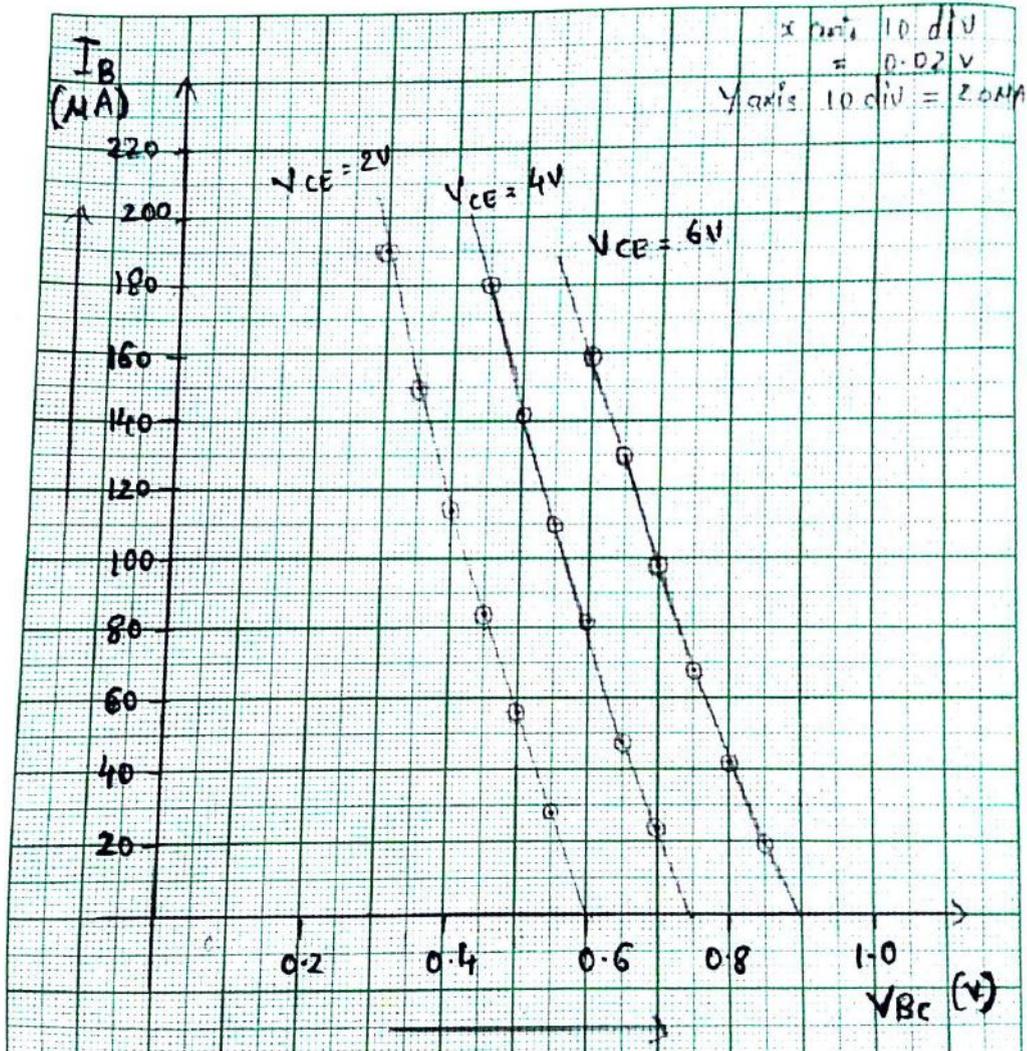
Output Characteristics

5.2 Observations

INPUT CHARACTERISTICS

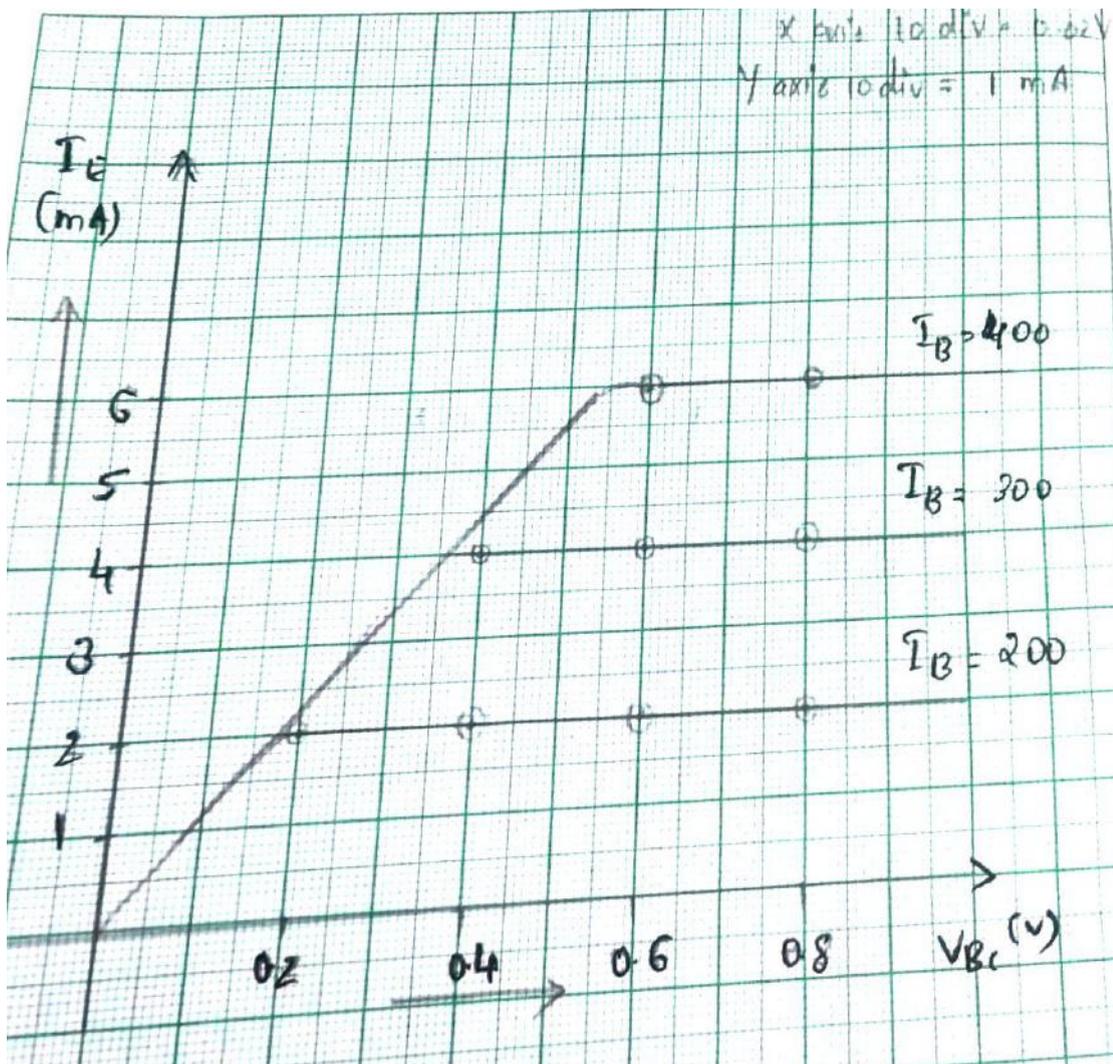
$V_{EC} = 2V$		$V_{EC} = 4V$		$V_{EC} = 6V$	
V_{BC} (V)	I_B (μA)	V_{BC} (V)	I_B (μA)	V_{BC} (V)	I_B (μA)
0.9	0	0.75	0	0.6	0
0.85	20	0.7	24	0.55	28
0.8	42	0.65	48	0.5	56
0.75	68	0.6	82	0.45	84
0.7	98	0.55	110	0.4	114
0.65	130	0.5	142	0.35	150
0.6	160	0.45	180	0.3	190

Scale



OUTPUT CHARACTERISTICS

$I_B = 200 \mu A$		$I_B = 300 \mu A$		$I_B = 400 \mu A$	
$V_{BC} (V)$	$I_E (mA)$	$V_{BC} (V)$	$I_E (mA)$	$V_{BC} (V)$	$I_E (mA)$
0	0	0	0	0	0
0.2	2	0.2	2	0.2	2
0.4	2	0.4	4	0.4	4
0.6	2	0.8	4	0.6	6
0.8	2	2	4	0.8	6



Calculations from the Graph

a. Input resistance = $\Delta V_{BC} / \Delta I_B = 0.5/20 = 0.025\Omega$

b. Reverse voltage gain = $\Delta V_{BC} / \Delta V_{EC} = 0.15/2 = 0.075$

c. Forward current gain = $\Delta I_E / \Delta I_B = 2/100 = 0.02$

5.3 Results

a. Input Resistance = 0.025Ω

b. Reverse Voltage Gain = 0.07

c. Forward Current Gain = 0.02

CONCLUSIONS

In this Chapter we studied the input and output characteristics of an NPN transistor in CC configuration. We have also included the observations and graphs of the same.

CHAPTER-6

SUMMARY AND FUTURE PROSPECTS

6.1 SUMMARY

In this project we conducted a detailed study of characteristics of the transistor in different configurations i.e, common emitter configuration, common base configuration and common collector configuration.

In Common emitter configuration, the emitter is grounded and it is the common terminal for both the input and output. The base is used as input terminal and collector is used as output terminal. In common emitter configuration current gain and voltage gain are calculated.

In Common base configuration the base is grounded and it acts as the common terminal. Here emitter is the input terminal and collector is the output terminal. In common base configuration output voltage gain is calculated .

In common collector configuration the collector is grounded and it is the common terminal for both the input and output. Here base is input terminal and emitter is the output terminal.

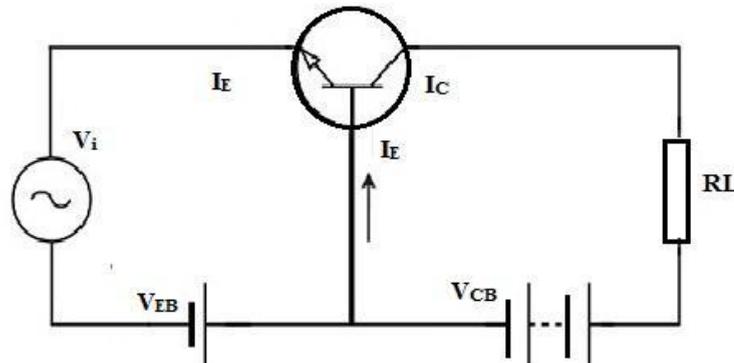
Common Emitter (CE)	Common Base (CB)	Common Collector (CC)
Emitter is grounded.	Base is grounded.	Collector is grounded.
Current gain (β) in CE configuration is given by $\beta = \Delta I_C / \Delta I_B$ We got $\beta = 0.2$	Current gain (α) in CB configuration is given by: $\alpha = \Delta I_C / \Delta I_E$ Here $\alpha = 1$	The current gain is calculated as , $\gamma = \Delta I_E / \Delta I_B$ We got $\gamma = 0.02$
Input resistance is obtained 0.002Ω	Input resistance is obtained as 0.25Ω	Input resistance is obtained as 0.025Ω
The reverse voltage gain is given by: $\Delta V_{EB} / \Delta V_{CE} = 0.5$	The reverse voltage gain is given by: $\Delta V_{EB} / \Delta V_{CB} = 0.02$	The reverse voltage gain in CC configuration is given by $\Delta V_{BC} / \Delta V_{EC} = 0.07$
It is used as audio frequency	It is used for amplification purposes.	. It is used for impedance matching

6.2 FUTURE PROSPECTS OF PRESENT WORK

- ❖ **Transistor as an Amplifier:** An amplifier circuit can be defined as a circuit that amplifies a signal. The amplifier's input is either a voltage or a current, and the output is an amplifier input signal. A transistor amplifier is an amplifier circuit that uses a transistor . Transistor amplifier circuits are used in a variety of applications, including audio, radio, and optical fibre communication.
- ❖ **Transistor as an Amplifier Circuit:** A transistor can be used as an amplifier by boosting the strength of a weak signal. One may

gain a sense of how a transistor circuit operates as an amplifier circuit by looking at the transistor amplifier circuit below.

The input signal can be applied between the emitter-base junction and the output across the R_c load in the collector circuit in the circuit below.



Always remember that the input is forward-biased, while the output is connected reverse-biased for proper amplification. Thus, we apply DC voltage (V_{EE}) to the input circuit in addition to the signal, as indicated in the above diagram. Generally, the input circuit often has low resistance; a small change in signal voltage at the input causes a considerable change in the emitter current. Because of the transistor's action, a change in the emitter current will result in a change in the collector circuit. The flow of collector current through a R_c currently generates a huge voltage across it. As a result, the weak signal applied at the input circuit will be amplified at the collector circuit in the output. The transistor acts as an amplifier in this manner.

- ❖ **Microphone:** Our voice or sound wave is converted to an electronic signal by the microphone, which is a transducer. The magnitude of the sound wave varies with time according to our voice because it does not have a constant value. Because of the small alternating voltage created by the microphone, the electrical output of the microphone varies in response to sound waves, as the base current I_b varies. This means

that a slight change in I_b can induce a huge change in I_c . When the microphone output is fed into the transistor as an input, the variable collector current I_c flows into the loudspeaker, and we know that if the transistor's input changes, the output of the transistor will fluctuate dramatically. As a result, the transistor enhances the microphone's electronic signal. Although the frequency remains constant, the amplitude of the sound wave from the loudspeaker is greater than that of the sound waves fed into the microphone.

❖ **Transistor Used as a Switch:** BJT Transistors can be used to manage DC power to a load by acting as a switching device. The controlling current flows between the emitter and the base, whereas the switched (controlled) current flows between the emitter and the collector.

To brief other practical application of transistors, it includes:

- Transistors are used in oscillators and modulators as amplifiers.
- Transistors are used in Radio-frequency circuits for wireless systems.
- Transistor switches are used in Burglar alarms, industrial control circuits, memories and microprocessors.
- They are used in Sub Wordline Driver (SWD) to produce low frequency currents.
- MOSFETs are used in Chopper circuits.
- JFET, MOSFET can act as a passive element like Resistor.

The first announcement of the transistor's development was made with absolutely no fanfare. Originally, the integrated circuit was supposed to be solely valuable in military applications. Investors in the microprocessor pulled out before it was built, believing it would be a waste of money. The transistor and its descendants have always been discounted, despite the fact that they have shown to be more capable than anyone anticipated. Today's predictions

also suggest that the transistor's capabilities are limited. This time, the expectations are that transistors will not be able to shrink any farther than they are now. Then again, in 1961, scientists predicted that no transistor on a chip could ever be smaller than 10 millionths of a metre -- and on a modern Intel Pentium chip they are 100 times smaller than that.

With the benefit of hindsight, such predictions appear absurd, and it's easy to imagine that today's predictions will sound just as absurd thirty years from now. However, recent size limit predictions are based on some very basic physics, such as the size of the atom and the electron. Because transistors operate on electric current, they must constantly be large enough to allow electrons to pass through.

On the other hand, all that's really needed is a single electron at a time. It would be phenomenally small for a transistor to operate with only one electron, but it is theoretically possible. Future transistors could make contemporary circuits appear as large and bulky as vacuum tubes are now. The trouble is that once gadgets get that small, everything moves according to quantum mechanics' principles, which allows electrons to perform some strange things. In such a small device, the electron behaves more like a wave than a single particle. It would smear out in space as a wave, and it may even tunnel through the transistor without actually acting on it.

Despite this, researchers are actively working on new ways to produce such tiny devices, forsaking silicon and all current production methods. Single electron transistors are what they're called, and depending on whether or not they're keeping an electron, they're termed "on" or "off." (At this level, transistors are only employed as binary coding switches, not as amplifiers.) In reality, the quantum weirdness of the ultra-small might be exploited by such a minuscule device. Instead of merely "on" or "off," the electron might be configured to have three positions: "somewhere between on and off." This would pave the way for whole new types of computers. However, there are currently no practical single electron transistors.

Miniaturisation is possible even without new technology. Present transistors are expected to be at least twice as small by 2010 if current manufacturing procedures are improved. Intel's latest processor has nearly a billion transistors, implying that four times as many transistors on a chip are theoretically achievable. Computers with chips like this would be far "smarter" than they are now.

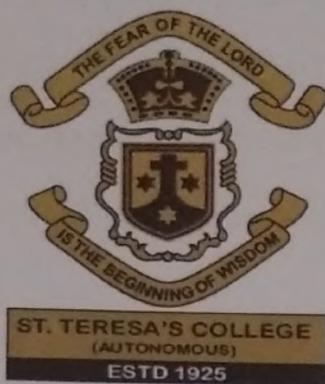
Transistors are the most basic electronic components that can be found in every device. They've grown in size and performance over the last 30 years. Their capacity to behave as a switch or to amplify current or voltage has allowed them to be used in a wide range of applications, including logic gates in computer processors and sound amplifiers. The ability to hold more electronics in one's hand than could be contained in a large building in the days when vacuum tubes were the only active devices available is perhaps transistors' most important contribution. They have made it possible to hold more electronics in one's hand than could be contained in a large building in the days when vacuum tubes were the only active devices available. As a result, complex functionality may now be packed into small packages—computers, cell phones, automotive engine controllers, and a variety of other devices.

A continuous process of research and development is underway to improve transistor performance parameters and to identify new semiconductor materials other than silicon. Transistors are being created to assist rapid technological advances such as wireless charging and energy conversion. Transistors are still being studied extensively around the world, as reducing the size and power consumption of individual transistors on a chip can result in quick profits. Researchers have already produced extremely small transistors made of only a few molecules in the lab, including one that uses only a single electron. They've also shown that transistors constructed of plastic are feasible, and that they could be even cheaper and more shock-resistant than conventional electronics.

Increasing transistor densities on chips (which manufacturers desire) is expected to be achieved in the near future via improving fabrication techniques for classic semiconductor devices.

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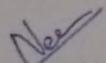


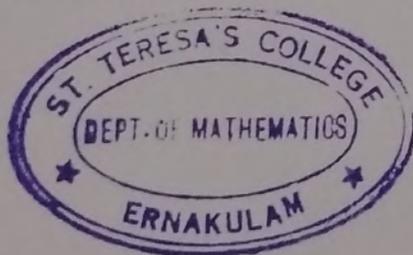
CERTIFICATE

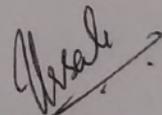
This is to certify that the dissertation entitled, **SPHERICAL GEOMETRY** is a bonafide record of the work done by Ms. **ANAGHA FLORY** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date:

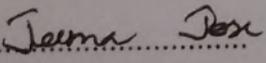
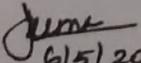
Place: Ernakulam


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External Examiners

1: 

6/5/2022

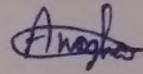
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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Neenu Susan Paul, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date:09-03-2022

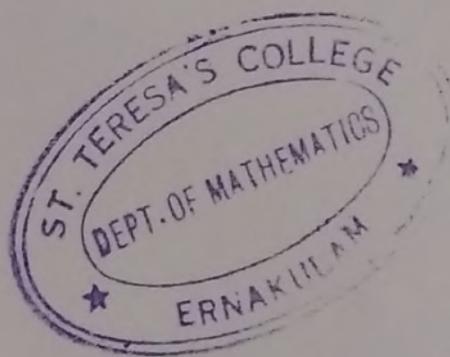


ANAGHA FLORY

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Project Report

On

SPHERICAL GEOMETRY

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

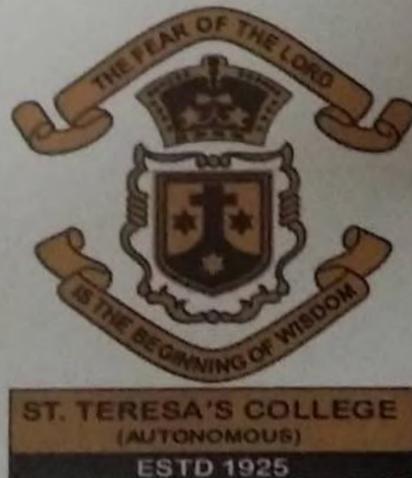
by

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Under the Supervision of

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DEPARTMENT OF MATHEMATICS

ST. TERESA'S COLLEGE (AUTONOMOUS)

ERNAKULAM, KOCHI - 682011

APRIL 2022

Project Report

On

SPHERICAL GEOMETRY

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

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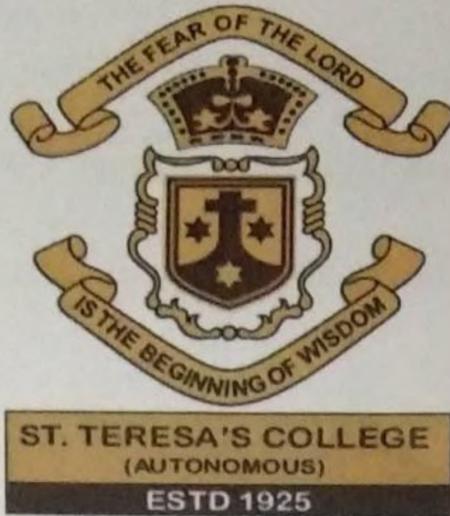


DEPARTMENT OF MATHEMATICS
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ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM



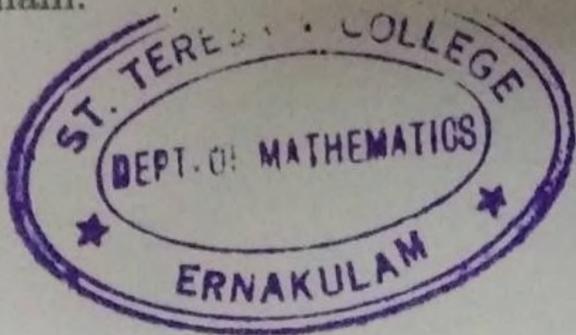
CERTIFICATE

This is to certify that the dissertation entitled, **SPHERICAL GEOMETRY** is a bonafide record of the work done by Ms. **ANAKHA LEON PETER** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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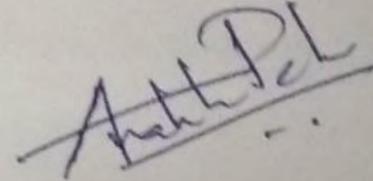
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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Neenu Susan Paul, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date:09-03-2022



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Date:09-03-2022

ANAKHA LEON PETER

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Chapter 1

EUCLIDEAN GEOMETRY AND SPHERICAL GEOMETRY

1.1 Introduction

Mathematics was commonly regarded as a "sit down" science, a subject where problems are often solved while sitting down in a classroom or office, and applications often related to theory, finance or business. However during the days of exploration, it was discovered that the world was indeed round and not flat. Spherical geometry was integral in mapping out the world. As you all know a sphere is defined as a set of all points equidistant from a given point. Spherical geometry is defined as the study of figures on the surface of sphere. Also spherical geometry is considered as the three dimensional, spherical analogue of planar geometry.

There has been much debate over the past several hundred years about who really discovered spherical geometry. The mathematician Bernard Riemann is credited with the development of spherical geometry. However, some influential thinkers in the area of spherical geometry include:

- Menelaus of Alexandria, a Greek mathematician and astronomer.
- Ibn Mu adh Al-Jayyani, an islamic mathematician and astronomer.

- Bernard Riemann, a German mathematician

Spherical geometry was studied by early Greek mathematicians such as Theodosius of Bithynia, a Greek astronomer and mathematician who wrote the 'Sphaerics', a book on the geometry of the sphere.

At the time when Earth was discovered to be round rather than flat, spherical geometry began to emerge to aid navigators in mapping the land and water. However, even before Columbus, ancient Greek and Phoenician mariners used the ideas of spherical geometry in naval explorations of the world they knew. Navigation is the process of directing the course of a craft such as boat, ship, plane to a destination. It requires, among other things, ongoing knowledge of the position, direction and distance of vessel and understanding of astronomy.

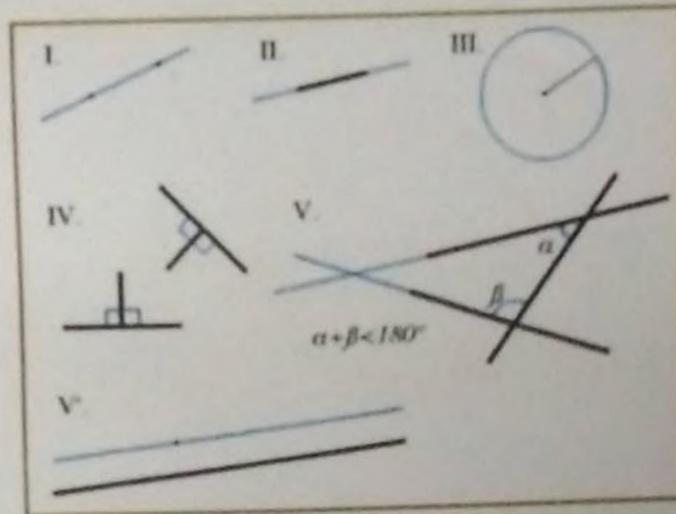
Today the concepts of spherical geometry are implemented in air and space travel, naval cruises and much more. Spherical geometry is used by pilots and ship captains as they navigate around the world. Spherical geometry also aided navigators in mapping out the land and water.

1.2 Euclidean geometry

Euclidean geometry is a mathematical system attributed to Alexandrian Greek mathematician Euclid, which he described in his book 'Geometry: The Elements'. Euclidean geometry deals with points, lines and planes and how they interact to make complex figures. Euclidean geometry marks the beginning of axiomatic approach in studying mathematical theories. Euclid's method consists of assuming a small set of intuitively appealing axioms and deducing many other postulates from these axioms. Euclid's postulates are,

1. A straight line segment can be drawn by joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segments, a circle can be drawn having the segment as radius and one end point as center.

4. All right angles are congruent.
5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the parallel postulate.

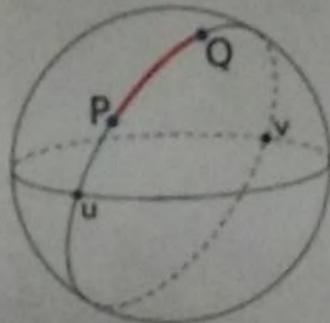


Euclid's postulates

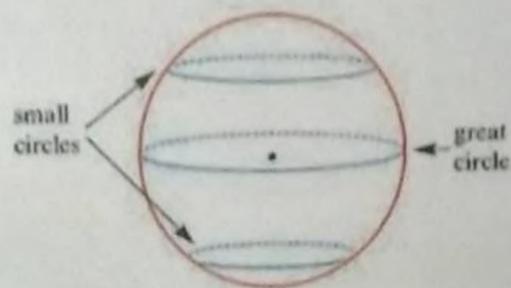
1.3 Spherical geometry

Spherical geometry is the study of figures on the surface of a sphere. Spherical geometry belongs to the non-Euclidean geometry, as it opposed to the type of geometry studied in the Euclidean geometry. It violated the parallel postulate (5th postulate) of Euclid. Now let's check how Euclid's postulates are occupied in spherical geometry,

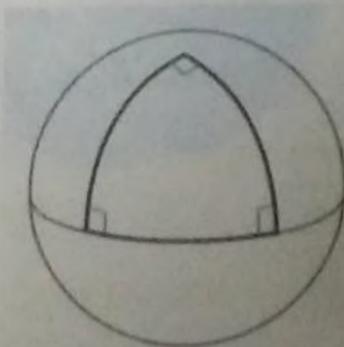
1. Choose any two points on the surface of the sphere, and draw a straight line between them. However, in spherical geometry, a straight line is a great circle.



2. A straight line will go around the sphere and connect to itself. A line of infinite length would go around the sphere an infinite amount of times. That is, straight lines in spherical geometry is finite according to their length.
3. Choose a center and find all the points in a given distance (radius) from the center. Such a set of points forms a circle. In spherical geometry, a great circle is both a line and circle. In globe, latitudes are actually not lines, they are circles in spherical geometry.



4. Right angles can be found on the surface of the sphere. Perpendicular lines intersect at two points on the surface of the sphere and the perpendicular lines form eight right angles. Consider the equator and any longitude in the globe, they are always perpendicular.

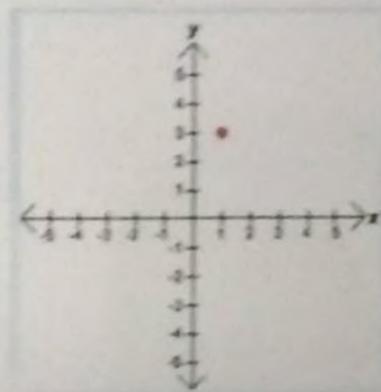
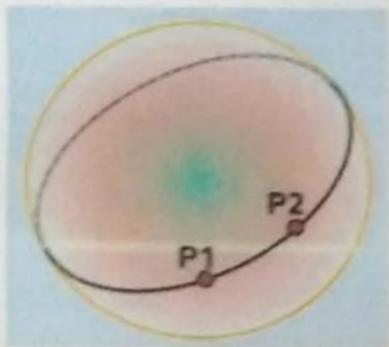


5. Parallel lines are defined as never intersecting. Then there are no parallel lines in spherical geometry. Any two lines (Great circles) drawn on the sphere will intersect in two points. It is a violation to Euclid's parallel postulate.

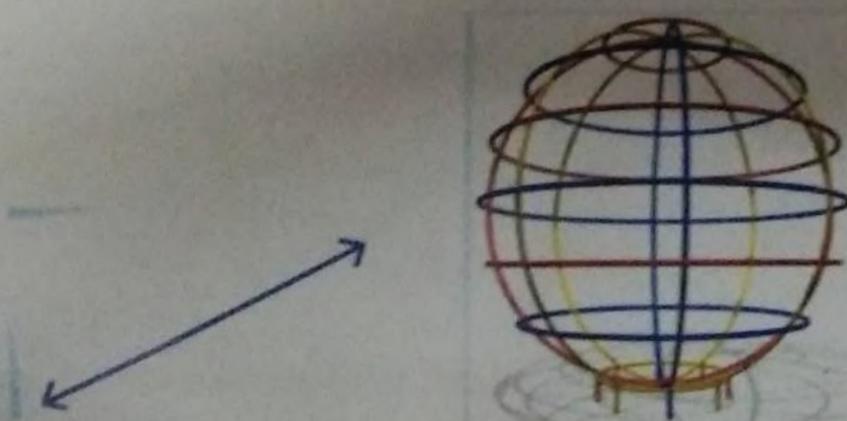
1.4 Comparison between Euclidean geometry and spherical geometry

1.4.1 Points, lines and planes

On the sphere, points are defined as having no parts. Points on a sphere and points on a plane are the same. These points have no dimension and have no volume, area, or mass. In Euclidean geometry a point lies on a line or plane. While, in spherical geometry a point lies on the surface of a sphere.



The equivalents of lines are not defined in the usual sense of "straight line" in Euclidean geometry but in the sense of "the shortest paths between points". On the sphere the shortest path between two points are the arc of a great circle connected by these points. In Euclidean geometry, a straight line is infinite whereas in spherical geometry a great circle is finite and returns to its original starting points. In spherical geometry straight lines are considered as "great circles" which wrap around the sphere in which the length is finite. These great circles divide the sphere into two hemispheres.

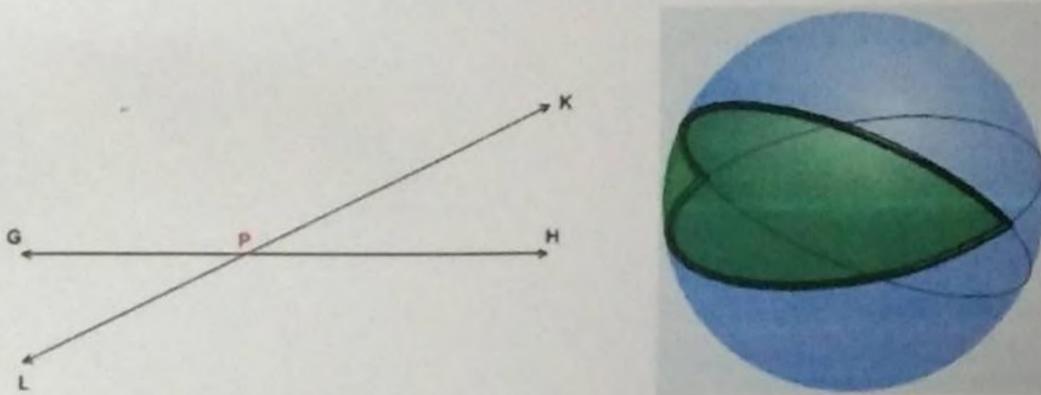


In Euclidean geometry, a plane extended in two dimensions and can be extended without end so that it is infinite. In spherical geometry

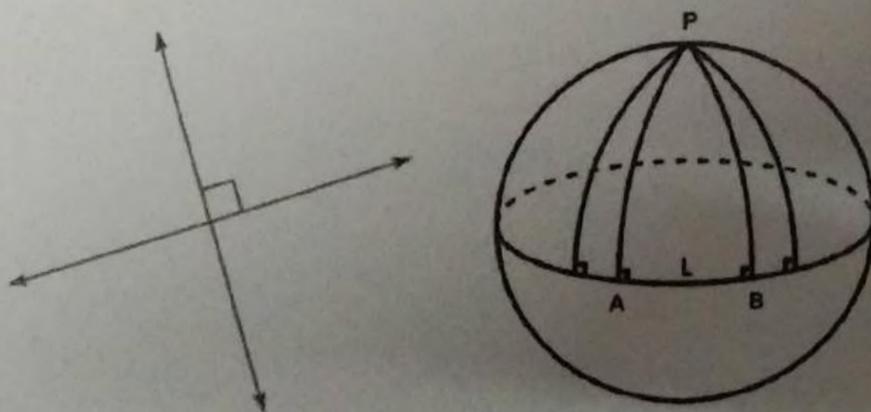
a plane and sphere meets in a circle. Also, in spherical geometry the angles are defined as the angle between two great circles. In Euclidean geometry, the sum of angles of a triangle equals 180° . But in spherical geometry the sum of angles of a triangle can be greater than 180° .

1.4.2 Intersecting, perpendicular and parallel lines

In Euclidean geometry, the intersection of two lines can be at a point or a line and parallel lines never intersect. Perpendicular lines intersect at one point to form four right angles. In spherical geometry two lines (Great circles) intersect exactly at two points.



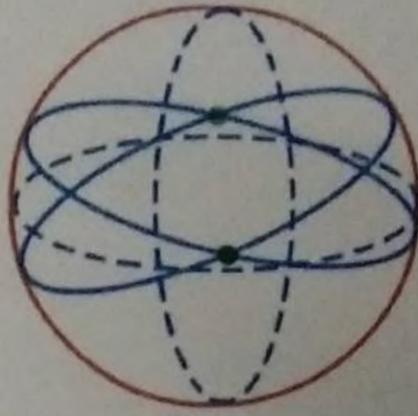
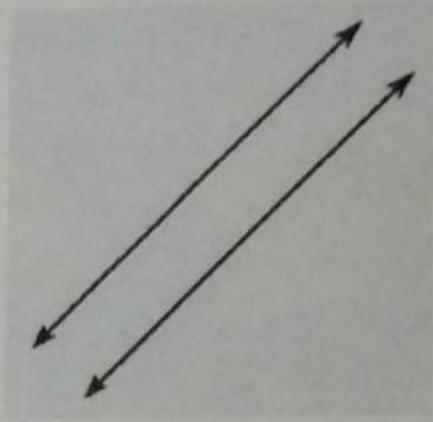
In Euclidean geometry, perpendicular lines are formed when two lines are placed perpendicularly to each other. Perpendicular lines form four right angles and intersect at exactly one point. In spherical geometry, perpendicular lines form to make eight right angles and intersect at two points.



In Euclidean geometry, parallel lines are existent. However, parallel lines are defined as never intersecting. Therefore in spherical geometry there are no parallel lines. Because, any two lines (Great circles) drawn

1.4. COMPARISON BETWEEN EUCLIDEAN GEOMETRY AND SPHERICAL
GEOMETRY *Spherical Geometry*

on the sphere will intersect at two points.



Chapter 2

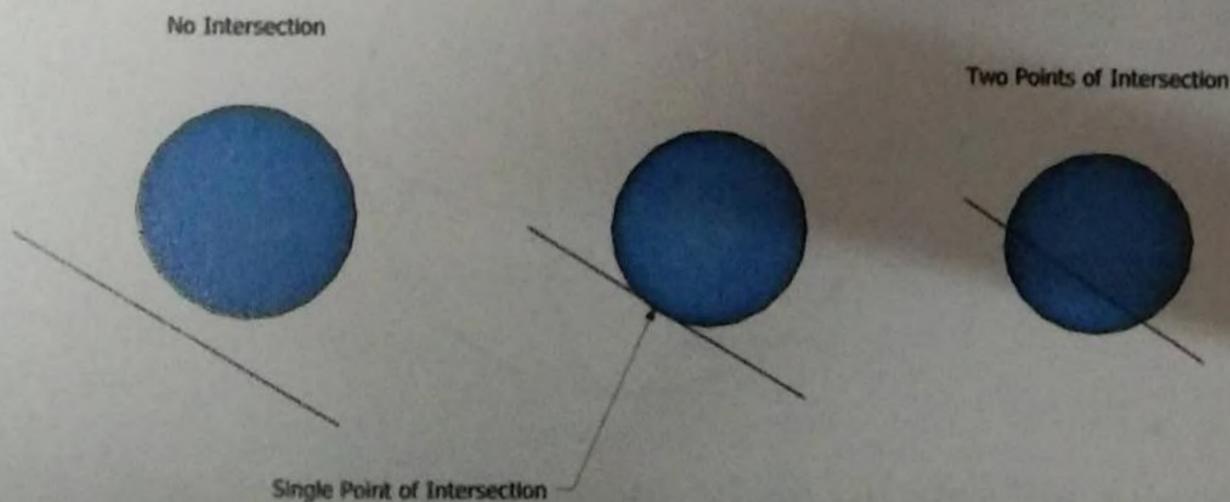
GREAT CIRCLES

2.1 Lines, Plane and Sphere

A sphere is defined as a closed surface in three dimensional space formed by a set of points on equal distance from the centre of the sphere. An arbitrary straight line and a sphere in three dimensional space can either

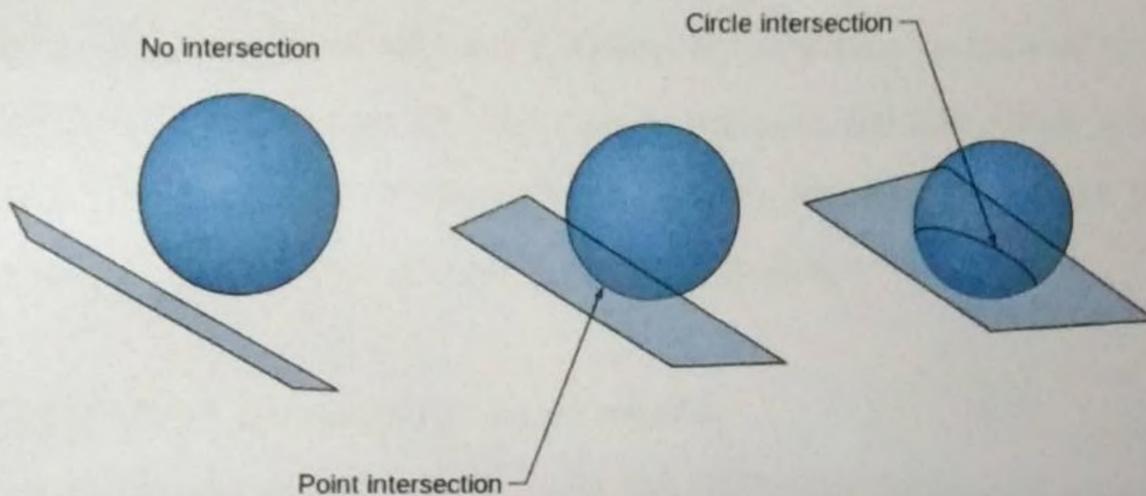
- (a) not intersect at all.
- (b) intersect at one point on the sphere, when the line is tangent to the sphere at the point of intersection.
- (c) intersect at precisely two points, when the line passes through the sphere.

In the third case, the line passes through the centre of the sphere and intersects the sphere's surface in two points, the points of the intersection form the *antipodes* of the sphere. The north and south poles are examples of the antipodes on the globe.



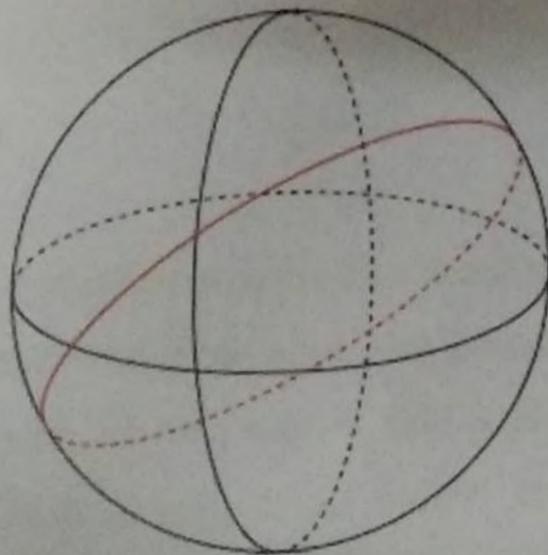
Like lines and spheres, an arbitrary straight plane and sphere in three dimensional space can have

- (a) no intersection
- (b) one point of intersection, when the plane is tangent to the sphere at that point.
- (c) An infinite number of points of intersection, when the plane cuts through the sphere and forms a circle of intersection.



2.2 Great circles and small circles

A plane section of a sphere is a circle. When the plane passes through the center of the sphere, the section is a great circle, otherwise a small circle. Great circles are defined as those circles of intersection which share the same radius and same center as the sphere it intersects. Great circles are the largest circles of intersection one can obtain by passing a straight plane through a sphere.



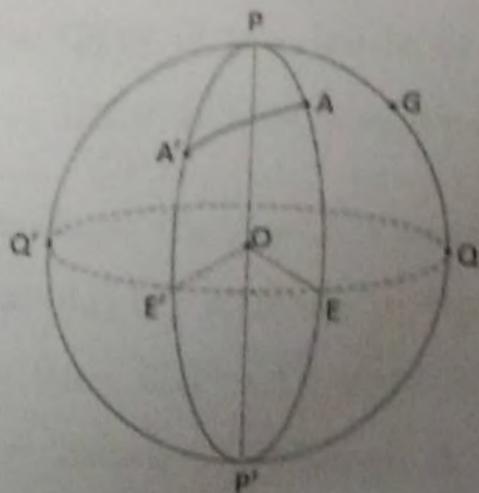
A great circle is circle on a sphere which divides the sphere in two equal hemispheres. It contains the diameter of the sphere and hence

the center as well. It is the largest possible circle that can be drawn around a sphere. On a sphere we can draw infinite number of great circles.

Any two great circles intersect in two diametrically opposite points and bisect each other. The two points on the sphere which is equidistant from all the points of a circle on the sphere are called the *poles* of the circle. A great circle is 90° distant from each of its poles. In spherical geometry the distance between any two points is measured in degrees or in radians. The length of the arc formed by any two points of the surface of the sphere is measured by the angle formed by the lines connecting the points to the center of the sphere. Their lengths in linear units can be obtained if the radius of the sphere is given.

2.3 Spherical geometry and earth

As the shape of the earth is roughly approximated as a sphere, the properties and applications of spherical geometry has a large impact in navigation. To illustrates some of the properties we shall relate them to the surface of the earth considered as a sphere with radius $R=3963$ miles.



The earth's axis of rotation meets the surface at two points, P and P', ie, the north geographical pole and the south geographical pole. A plane through the center of the earth and perpendicular to the axis PP' cuts the surface in a great circle called the equator. A plane perpendicular to axis PP' at any point between P and P' other than the midpoint cuts the surface in a small circle called a parallel of latitude.

The tropics (cancer and Capricorn) and the two arctic circles are such parallels. Any plane which contains the axis of rotation PP' meets the surface in a great circle called a meridian. Any meridian cuts the equator in two diametrically opposite points. For the 'prime meridian' (meridian of Greenwich, PGQ), these are the points on the equator of 0° longitude and 180° longitude, Q and Q' respectively. All meridians are great circles, but all latitudes other than the equator are small circles. Because their radii measures less than the earth's radius R .

If A is a station on the earth's surface on meridian PAE ,

$$\text{Arc } EA = \text{latitude of } A$$

and

$$\text{Angle } QPA = \text{longitude of } A$$

Latitude is counted positive when point A is north of the equator and counted negative when point A is south of the equator. Arc PA is the north polar distance of A and is counted from 0° to 180° . It is the complement of the latitude, and is greater than 90° when the latitude is negative. If A' is a second station,

$$\text{Arc } E'A' = \text{latitude of } A'$$

$$\text{Angle } QPA' = \text{longitude of } A'$$

$$\text{Angle } A'PA = \text{the difference of longitude of } A \text{ and } A'$$

If a plane be passed through the earth's center O and points A and A' , the plane will cut the earth's surface in the great circle AA' . Any other plane containing points AA' will cut the earth's surface in a small circle. The shortest distance between A and A' is the distance measured along the great circle joining the points.

If the latitudes and longitudes of A and A' are given, we know also their polar distances, that is the sides AP and $A'P$ of the spherical triangle APA' . The difference of the longitudes is the angle APA' included between these sides. The determination of the remaining parts of triangle APA' , when two sides and the included angle are given, constitutes a basic problem of spherical trigonometry. If an airplane is to fly from

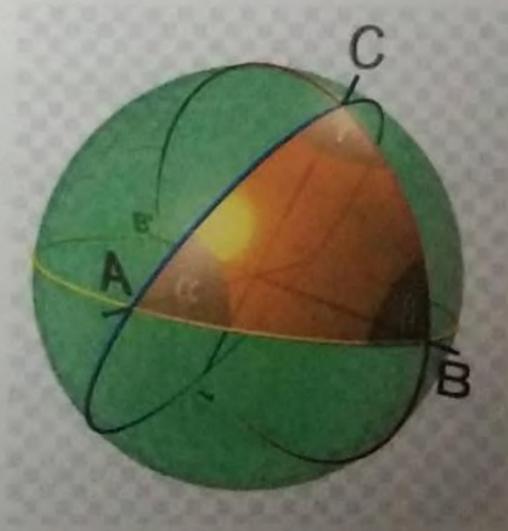
A to A' by the shortest route, it would have to start from point A at an angle PAA'.

Chapter 3

SPHERICAL TRIANGLE

In chapter 2 we discussed about the great circles. Now, when the arc of three great circles intersect on the surface of a sphere, the lines enclose an area known as spherical triangle. The Spherical triangle is the spherical analogue of the Planar triangle.

Angles of spherical triangle are measured by angle between the corresponding great circles. Angles between great circles are measured by calculating the angle between planes on which great circles themselves lie. The Spherical angle formed by two intersecting arcs of great circle is equal to the angle between the tangent lines formed when the great circles planes touch the circle at their common point.



3.1 Properties of spherical triangle

- The sides of Spherical triangle are measured in degrees, minutes and seconds
- No sides of a spherical triangle can therefore exceed 180°

3.2 TYPES OF SPHERICAL TRIANGLE

- The maximum value of an angle of a spherical triangle is 180°
- The sum of any two sides of spherical triangle is greater than the third side
- If two sides of spherical triangle are equal, the angle opposite to them are also equal to each other

3.2 Types of spherical triangle

There are different types of spherical triangle some of them are

3.2.1 Right angle Spherical triangle

A right angled Spherical triangle is one in which an angle equals to 90° . In a Spherical triangle, it is possible for more than one angle to be equal to 90°

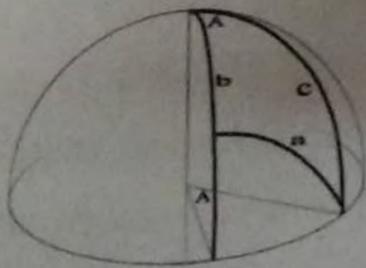


3.2.2 Quadrantal Spherical Triangle

A quadrantal Spherical triangle is one in which one side equals to 90° .

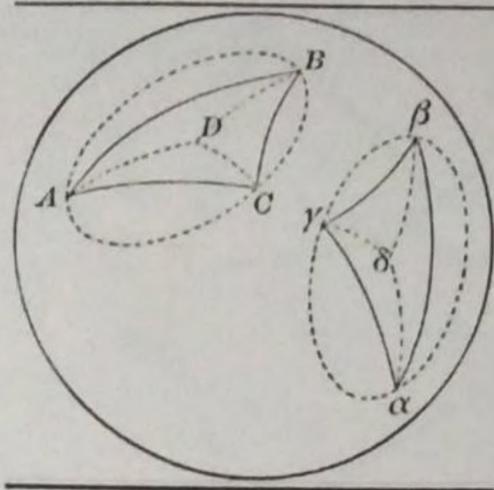
3.2.3 Oblique spherical triangle

A Spherical triangle which is not right angled or a quadrantal one is called an oblique Spherical triangle



3.2.4 Symmetric spherical triangle

Two Spherical triangle are said to be symmetrically equal when each of the six elements (that is three sides and three angles)of one are equal in the value to each of six of other

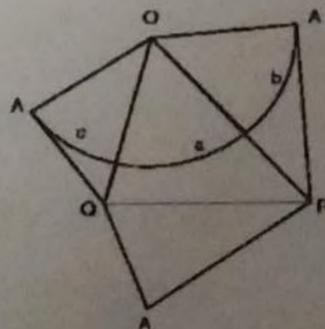


3.3 Spherical law of cosines

Spherical law of cosines states that The cosine of a side of a spherical triangle is equal to the product of the cosine of the other two sides plus the product of the sine of those two sides multiplied by the cosine of their included angle that is,

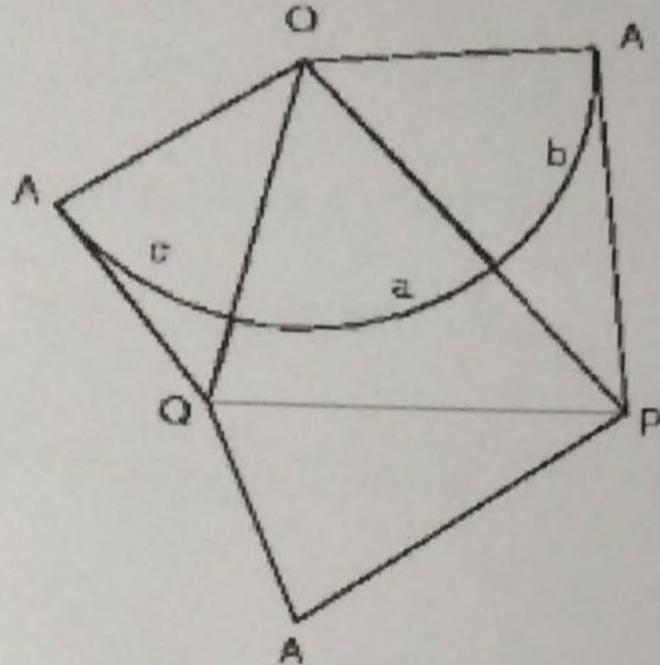
$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Proof:



The Spherical triangle ABC is formed by the intersection of great circles with planes intersecting at OA,OBQ and OCP.The plane PQA is partly composed of two tangent lines:AQ tangent to c and AP tangent to b,and will be referred to as our tangent plane . Therefore $\angle OAQ$ and $\angle OAP$ are right angles and $\angle PAQ$ equals to the angle A opposite to side a.Extracting the tetrahedron enclosed by the Planes and lying

if flat on a plane as a net, we examine the four composite triangles, $\triangle OAQ, \triangle OAP, \triangle QOP, \triangle QAP$.



Triangle OAQ and OAP are right angled triangles, so using Pythagoras theorem

$$PO^2 = AO^2 + PA^2$$

$$AO^2 = PO^2 - PA^2 \quad (3.1)$$

$$QO^2 = AO^2 + QA^2$$

$$AO^2 = QO^2 - QA^2 \quad (3.2)$$

The other two triangles QAP and QOP are plane triangles, so using cosine law for flat triangles ($a^2 = b^2 + c^2 - 2.b.c. \cos a$)

we can see that

$$PQ^2 = PO^2 + OQ^2 - 2PO.OQ. \cos a \quad (3.3)$$

$$PQ^2 = PA^2 + QA^2 - 2PA.QA. \cos A \quad (3.4)$$

subtracting (3.3) and (3.4) we get

$$(PO^2 - PA^2) + (QO^2 - QA^2) - (2.PO.OQ. \cos a - 2.PA.QA. \cos A) = PQ^2 - PQ^2$$

$$(PO^2 - PA^2) + (QO^2 - QA^2) - 2 \cdot PO \cdot QO \cdot \cos a + 2 \cdot PA \cdot QA \cdot \cos A = 0 \quad (3.5)$$

substituting AO^2 for $(PO^2 - PA^2)$ and $(QO^2 - QA^2)$
we get

$$2AO^2 + 2PA \cdot QA \cdot \cos A = 2PO \cdot QO \cdot \cos a \quad (3.6)$$

dividing both sides by $2PO \cdot QO$ we get

$$\cos a = \frac{AO}{PO} \cdot \frac{AO}{QO} + \frac{PA}{PO} \cdot \frac{QA}{QO} \cdot \cos A \quad (3.7)$$

but we know that,

$$\frac{AO}{PO} = \cos \angle POA$$

$$\frac{AO}{QO} = \cos \angle QOA$$

$$\frac{PA}{PO} = \sin \angle POA$$

$$\frac{QA}{QO} = \sin \angle QOA$$

$$\text{so, } \cos a = \cos \angle POA \cdot \cos \angle QOA + \sin \angle POA \cdot \sin \angle QOA \cdot \cos A$$

Finally substituting the side b for angle POA and c for angle QOA
we get,

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Therefore the formula for the third side a of spherical triangle, given
two sides b and c and their included angle A is,

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

Chapter 4

APPLICATIONS OF SPHERICAL TRIGONOMETRY

4.1 Terrestrial triangle

We shall consider the earth as a sphere with a radius of 3963 miles. Longitudes are to be reckoned from Greenwich the prime meridian, 180° to the west or east. The direction will be indicated by a letter W or E for west and east respectively. Then the longitude of a given place is measured by the arc of the equator contained between the meridian of Greenwich and the meridian of the place, and it is also measured by the angle at the pole between those two meridians.

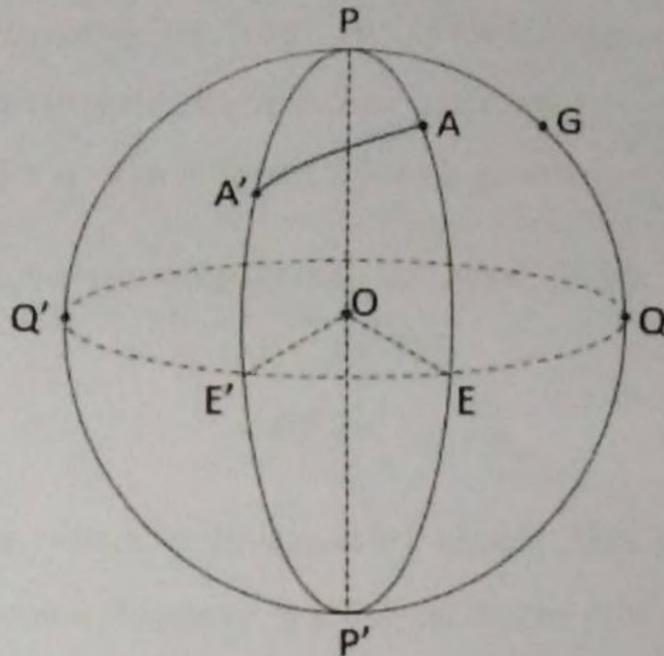
Then latitude is counted positive to the north and negative to the south of the equator. We shall denote distance from the north pole by p . This polar distance will be the complement of the latitude.

$$p = 90^\circ - \text{Latitude}$$

A triangle whose vertices are the north pole (or the south pole) and two points on the earth's surface will be called a *Terrestrial Triangle*.

In the figure, let P be the earth's north pole, G Greenwich, A and A' two stations, stations A' lying to the west of station A . Then the triangle APA' is a terrestrial triangle. Two sides of this triangle are the polar distances of the two stations, or the complements of their latitudes, and the third side is the great circle arc between the two stations. The angle

at the pole $\angle APA'$ is the difference of longitude of the two stations.



The other two angles of the terrestrial triangle APA' are the angles which the great circle arc AA' makes with the meridian at the respective stations. To sail a ship, or fly an aeroplane from A to A' , the navigator would wish to know the length of the journey if the great circle arc AA' were followed.

4.2 The cross continental application of the solution of a spherical triangle

The great circle routes are the shortest routes between various places on the earth as we can connect any two places on the earth's surface by the curvature line or arc of the great circle and this curvature is the smallest possible route between those two places, because this curvature directly connects those places or points.

New York belongs to the North American continent and London belongs to the Europe continent.



We have to find the best route from New York to London.

New York is geographically located at along the great circle of longitude $74^{\circ}0'W$ and approximately $40^{\circ}42'$ latitude north of the equator. Latitude is the complement of polar distance.

Polar distance of New York from North pole,

$$\begin{aligned} P_N &= 90^{\circ} - \text{Latitude of New York} \\ &= 90^{\circ} - 40^{\circ}42' \\ &= 49^{\circ}18' \end{aligned}$$

On the other hand, London is located along the great circle of longitude $0^{\circ}5'W$ and approximately $51^{\circ}32'$ latitude north of the Equator. Polar distance of London from the North Pole,

$$\begin{aligned} P_L &= 90^{\circ} - \text{Latitude of London} \\ &= 90^{\circ} - 51^{\circ}32' \\ &= 38^{\circ}28' \end{aligned}$$

Now we will use the formula,

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \quad (\text{we have studied in the previous chapter.})$$

The side a is the required, shortest route between New York and London. The sides b and c are given by the length of the arcs from the North Pole to the New York and London respectively.

$$\text{So, } b = 49^{\circ}18' \text{ and } c = 38^{\circ}28'$$

The angle A is the angle between the respective longitudes of London and New York. Both New York and London are located at the west of the Greenwich. Therefore the angle A is given by the difference in longitudes of the two cities New York and London.

$$A = 74^{\circ}0'W - 0^{\circ}5'W = 73^{\circ}55'$$

Applying these values in the formula,

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

$$\begin{aligned} \cos a &= \cos 49^{\circ}18' \cdot \cos 38^{\circ}28' + \sin 49^{\circ}18' \cdot \sin 38^{\circ}28' \cdot \cos 73^{\circ}55' \\ &= (0.6521 \times 0.7830) + (0.7581 \times 0.6221 \times 0.2770) \end{aligned}$$

$$\cos a = 0.6412$$

$$a = \cos^{-1}(0.6412) = 50.1186^\circ \text{ or } 50^\circ 7'$$

This means that the great circle distance between New York and London is approximately $50^\circ 7'$. We know that the distance around the earth at the equator, i.e., its circumference is 24901 miles.

Then one degree of the equator is approximately,

$$\frac{24901}{360} = 69 \text{ miles}$$

$$\text{Hence, } a = 50.1151^\circ = 50.1151 \times 69 \text{ miles} = 3458 \text{ miles}$$

Therefore the shortest distance between New York and London is 3458 miles.

Tokyo belongs to Asia and St. Louis belongs to North America. The best route or the shortest distance between these two places is the great circle route. St. Louis is geographically located at longitude $90^\circ 15' W$ and $38^\circ 40'$ latitude north of the equator.

Polar distance of St. Louis from North pole

$$P_S = 90^\circ - 38^\circ 40' = 51^\circ 20'$$

Then Tokyo is geographically located at longitude $139^\circ 44' E$ and $35^\circ 39'$ latitude north of the equator.

Therefore, Polar distance of Tokyo from North pole

$$P_T = 90^\circ - 35^\circ 39' = 54^\circ 21'$$

Therefore, $b = 51^\circ 20'$ and $c = 54^\circ 21'$

and $A =$ angle between the longitudes of the two places.

Here, St. Louis is located at the West of the Greenwich and Tokyo located at the East of the Greenwich.

$$\text{Therefore, } A = 90^\circ 15' + 139^\circ 44' = 229^\circ 59'$$

Then, $\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$

$$= \cos(51^\circ 20') \cos(54^\circ 21') + \sin(51^\circ 20') \sin(54^\circ 21') \cos(229^\circ 59')$$

$$= [(0.624) \times (0.582)] + [(0.78) \times (0.812) \times (-0.6430)] = -0.044$$

Therefore, $a = \cos^{-1}(-0.044) = 92.52^\circ$

$$a = 92.52^\circ$$

$$= 92.52 \times 69 \text{ miles}$$

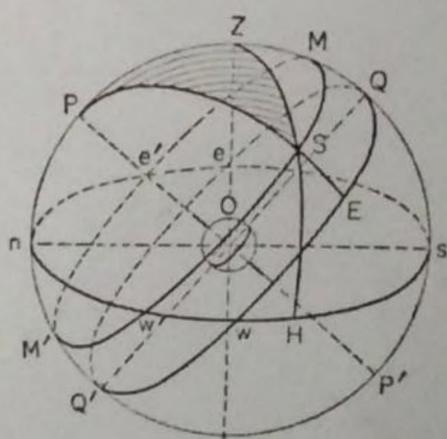
$$\approx 6384 \text{ miles.}$$

Therefore, Shortest distance between St.Louis and Tokyo is 6384 miles.

Great circle routes are mainly used for navigation. They are most commonly used for long distance travel by ships and aircrafts. Commercial vessels such as ships and aircrafts follow great circle routes as far as possible to make their journeys economical. Long distance air traffic uses great circle routes to save time and fuel as it is the shortest distance.

4.3 Celestial sphere and astronomical triangle

In astronomy and navigation, the celestial sphere is an indefinitely large sphere that has an arbitrarily large radius and is concentric to earth. All objects in the sky can be conceived as being projected upon the inner surface of the celestial sphere, which may be centered on earth or the observer. The celestial sphere is a practical tool for spherical astronomy allowing astronomers to specify the apparent positions of objects in the sky if their distances are unknown or irrelevant.



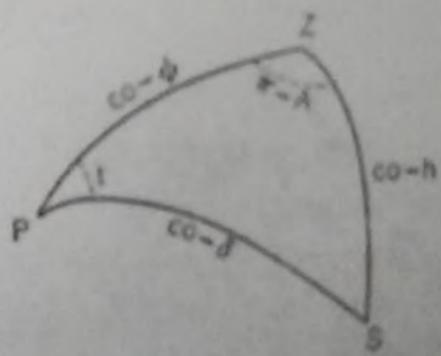
The celestial sphere possess all the properties of normal sphere which are explained in the previous chapters. The projections of earth's pole, equator, longitudes and latitudes on the surface of the celestial sphere are named respectively, celestial poles (P and P'), the celestial equator

(QwQ'e), hour circles (PSE) and parallels of declination (MSM').

Suppose an observer situated at a point O on the surface of the earth will have his zenith at the point Z. If the line at the point O is extended indefinitely would meet the celestial sphere at the point Z. The great circle S'wne is the horizon of the observer and the point Z is the pole of the great circle S'wne. Then the great circle nPZQS' is meridian of the observer which meets the horizon in the north and south points.

Then, let S be a point on the surface of the celestial sphere, as the center of the sun or a star or any other celestial objects. Also, S will appear to describe the parallel e'MSW'M'e', rising at e' and setting at w', because of the rotation of the earth. Suppose, S has the position shown in the figure, then HS will be its altitude denoted by h (i.e, height above the horizon) and the angle $\angle S'ZH$ (measured by arc S'H) will be its azimuth denoted by A. The arc ZS or $90^\circ - h$ is the zenith distance of S and denoted by z. Therefore, h and A or z and A completely define the position of S, the celestial object, with reference to horizon and zenith.

With respect to the equator and pole of the celestial sphere, ES is called the declination of S, denoted by δ and the angle $\angle QPE$ which hour circle PS of S makes with meridian PQ is called its hour angle, denoted by t. Then the arc $PS = 90^\circ - \delta$ is called the polar distance of S, denoted by p. Therefore the position of S can be defined by δ and t or by p and t.



From the figure $\triangle PZS$ is an *Astronomical Triangle*. Its parts except the angle at S are,

$$PZ = 90^\circ - \phi = 90^\circ - \delta$$

where ϕ is the latitude of O and

$$PS = p = 90^\circ - \delta$$

$$ZS = z = 90^\circ - h$$

$$\angle ZPS = t$$

$$\angle PZS = 180^\circ - A$$

4.3.1 How to determine latitude by altitude of the sun

To determine the latitude of the observer, if the altitude of the sun (or a star) when it crosses his meridian is known.

The known meridian altitude of the sun (or a star) gives the arc S'M in the figure. Subtracting the sun's declination, i.e., arc QM from S'M (or adding it if the sun is south of the equator), gives arc S'Q. The complement of arc S'Q is the required latitude.

Q) The meridian altitude of the sun was observed to be $61^\circ 27'$, the sun's declination was $12^\circ 15'$. Find the latitude.

Solution:

$$\text{Meridian altitude of sun } S'M = 61^\circ 27'$$

$$\text{Sun's declination } QM = 12^\circ 15'$$

$$S'Q = S'M - QM = 61^\circ 27' - 12^\circ 15' = 49^\circ 12'$$

$$\text{Latitude of the observer} = \text{Complement of } S'Q$$

$$= 90^\circ - 49^\circ 12'$$

$$= 40^\circ 48'$$

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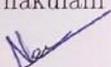


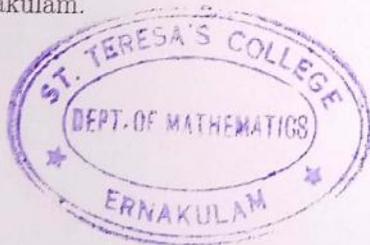
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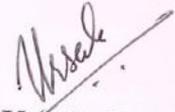
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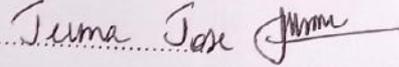
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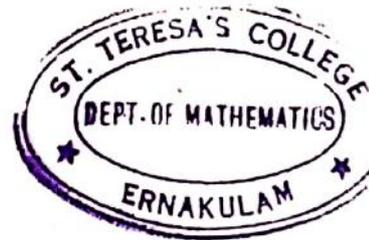


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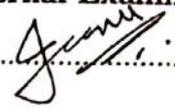
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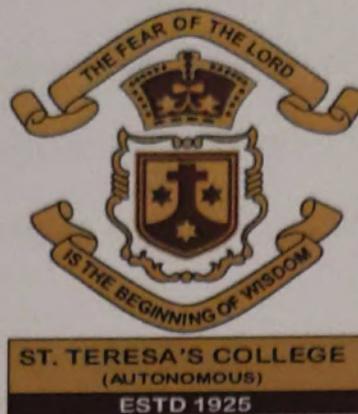



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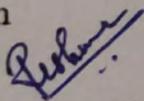
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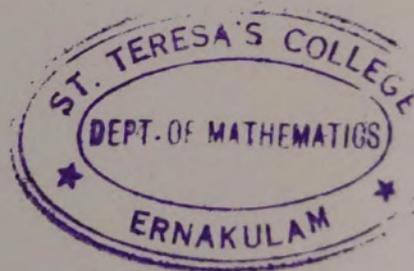


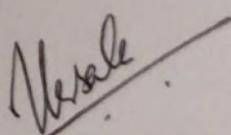
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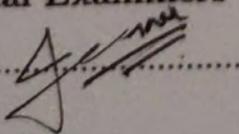
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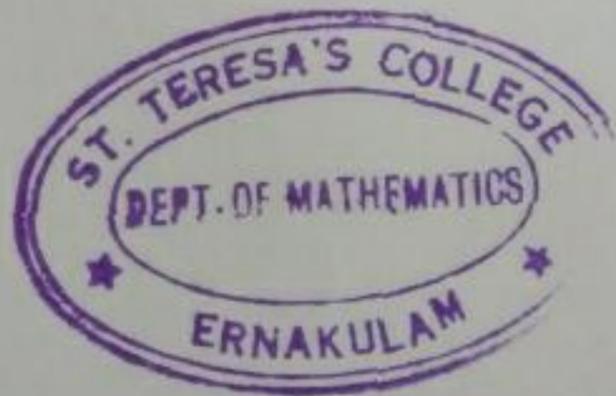
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Project Report

On

**MATHEMATICS IN CRIME SOLVING AND
CRIME CONTROL**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

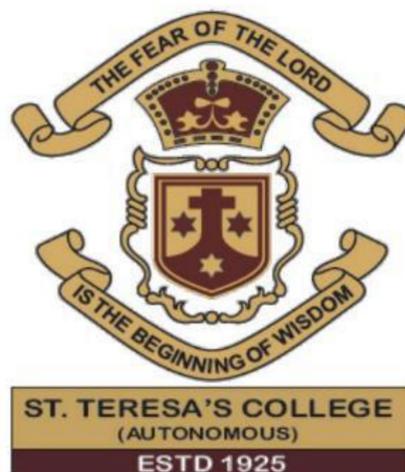
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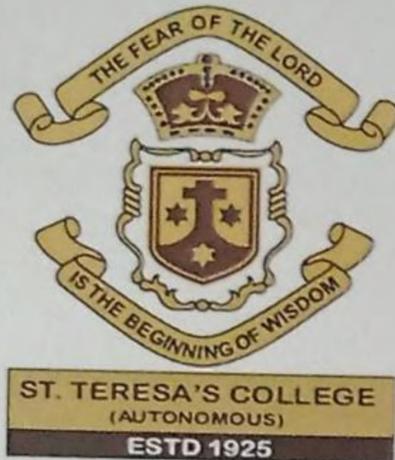


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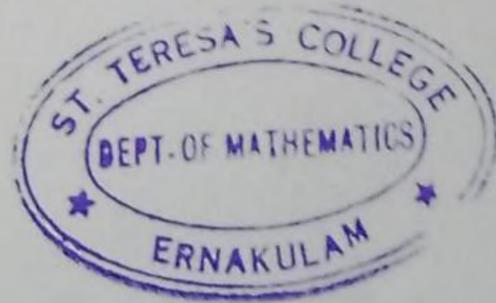


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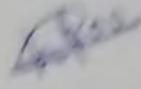
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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Dr. Susan Mathew Panikkar, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

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Chapter 1

PREREQUISITES

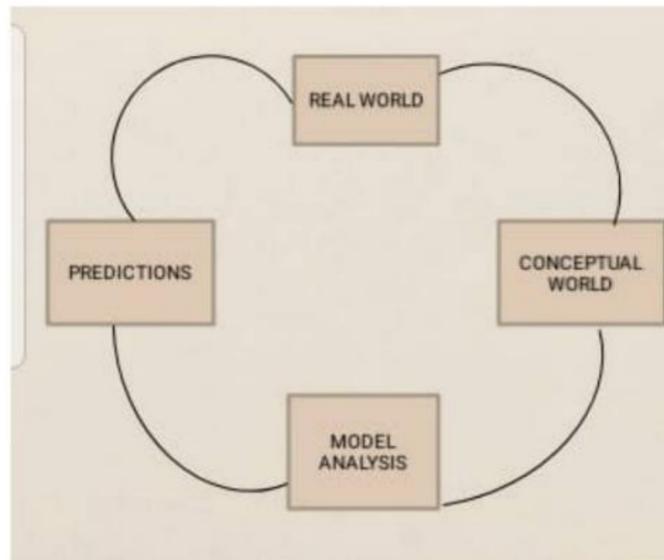
1.1 MATHEMATICAL MODELLING

1.1.1 INTRODUCTION

Mathematical Modelling is an important branch of mathematics. 'Model' is basically an imitation of real world objects . Modelling is a miniature representation of some object or a demonstration for reference of an object to be made. There are two types of models prototypic(physical model) and schematic(charts,graph,diagram etc). The manipulation of modelling and implementing it in mathematics helped in the betterment of various fields.

1.1.2 DEFINITION

Representation of real objects and phenomenon in mathematical language is stated as mathematical modelling. The subjects that cannot be studied or is difficult to study using normal theoretical methodology can be done using mathematical modelling



1.1.3 ADVANTAGE AND APPLICATION

Mathematical modelling comes to light when the theoretical method become :

- Time consuming
- Expensive
- Often dangerous
- Simply impossible

Mathematical modelling basically deals with analysing, predicting and insight of real world knowledge. As mentioned earlier it's application is beyond limit it is applicable in the field of Epidemiology, biological transpose , vehicular traffic etc.Consider the current situation of covid 19, it forced us to trace multiple things and track down and even predict the spread and new waves . Evidently, this won't be possible by simple experimental method as the real world object that we are dealing with is quite large.Hence, modelling is an unavoidable and powerful tool.

1.1.4 STEPS INVOLVED

- A problem from the real world is taken in account and is studied to make certain assumptions involving the parameters affecting the considered problem.
- Analytical and numerical analysis of the assumptions are made.
- A model is predicted for the same

- Both experimental and Intuitive validation is done.
- If valid, the model is accepted or else the process is repeated

Mathematical modelling is used in the project to study an existing model named Rossmo Model introduced by **Dr.Kim Rossmo**. The basic mathematical concept used in the generation of this model is **probability**. This was used to hunt down a serial rapist bringing Dr.Rossmo to limelight.

1.2 GRAPH THEORY

1.2.1 INTRODUCTION

In mathematics, Graph theory is a special branch of mathematics that deals with the pairwise relation of one object to other. A graph consists of vertices also known as nodes which are interconnected by edges. Graphs is a crucial field of study in mathematics.

1.2.2 DEFINITION

Graph Theory is used to study the relationship between different objects. A Graph $G(V,E)$ denotes the collection of graph with vertex $V(G)$ and edge $E(G)$.

1.2.3 GRAPH THEORY IN CRIME

We can analyse a crime using three elements of graph theory such as the vertex set, the edge set, and the incidence function that relates edges to vertices. The mathematics of graph theory explains that an edge can have an interconnection from one vertex to another if there is any connection between the entities. The prior step in crime solving is to locate different elements involved in the incident. These elements will be represented as vertices. When any two elements have any connection that connection is represented as an edge. The edge not only connects the vertices but acts as the initiator of that connection which results in graph with parallel edges.

1.2.4 APPLICATION OF GRAPH COLOURING

In graph theory, graph colouring is used to label the graph. The vertices, Edges and faces are denoted using colours without the adjacent of the above being labelled with same colours. This method of colouring is stated as *vertex colouring*, *edge colouring* and *face colouring* respectively. In crime solving this method can be used to interpret the connection between the collected evidences. An illustration of the same is explained later in this paper.

1.3 FORENSIC STATISTICS

1.3.1 INTRODUCTION

Forensic statistics refers to the application of probability, mathematical techniques and statistical concepts to the forensic data and the scientific evidence collected. Mathematics is an unavoidable part of Forensic science. The application of mathematics and statistics makes the scientific analysis of evidences methodical. Forensic investigators collect the evidences, analyse and document it using statistical methods.

Biological evidences such as DNA evidence, blood samples, hair samples, fingerprints, etc found at the crime scene are examined using various probability models to draw inferences from them. DNA profiling, fingerprint analysis, blood sample analysis, and statement analysis are major functionalities of forensic science that are deterministic in solving a crime. Forensic Statistics is widely used by Forensic Investigators and Legal practitioners to analyse statements and test the significance of evidences.

1.3.2 CONCEPTS INVOLVED

There are different mathematical and statistical techniques used to solve crimes. Some of the main mathematical principles used in crime solving are:-

- TRIGONOMETRY
- EXPONENTIAL & LOGARITHMIC FUNCTIONS
- CONDITIONAL PROBABILITY
- BAYES' THEOREM
- LIKELIHOOD RATIO

TRIGONOMETRY

Trigonometry studies the relationship between the angles and the length of sides of a right angled triangle. In Crime solving trigonometry is mainly used in Blood Spatter Analysis and it also helps in Ballistic Analysis.

BLOOD SPATTER ANALYSIS

Blood Spatter Analysis deals with the study of bloodstains. The bloodstains found at the crime scene are analysed by experts. Bloodstains develop a pattern due to the impact of multiple bloodstains originating from a single source, that is, from a gunshot or a stab wound. Working on these patterns, the investigators can trace back to the position of the victim when the crime occurred or the nature of the blood source. Blood Spatter Analysis is conducted in three steps using trigonometry.

1. Calculate the angle of impact after identifying a proper set of bloodstains.
2. The point of convergence of the bloodstains within the plane of the pattern is determined.
3. The point of convergence of the source is determined.

There are two scenarios under consideration to analyse the shape of the blood stain and calculate the angle of impact; first, the impact from a stationary source and second, the impact from a moving source.

STATIONARY SOURCE: Bloodstains take a spherical shape under a perpendicular impact. If the angle of impact is less than 90° then the blood drop is going to be elongated and elliptical in shape. The angle of impact θ is calculated using the formula

$$\sin \theta = \frac{\text{width}}{\text{length}}$$

MOVING SOURCE: Blood drops falling from a moving source like blood falling from the assailant or the moving wounded victim hits the ground with an impact

less than 90° . The components of velocity affect the angle of impact. The effective angle of impact θ is given by,

$$\tan \theta = \frac{V_g}{V_h}$$

Where, V_h is the horizontal component of velocity due to speed. V_g is the velocity component due to gravitational acceleration and is given by, $V_y = \sqrt{2gh}$ when g is the acceleration due to gravity and h is the drop height.

BALLISTIC ANALYSIS

Firearms involved in a crime are examined to discover evidences in forensic ballistic analysis. A variety of instruments and devices are also used in Ballistic Analysis. Trigonometry appears in the non-mechanical part of Ballistic Analysis. Bullet trajectories refer to the path travelled by the bullet after firing a gun. Studying the bullet trajectories the investigators can determine the position of the criminal at the time of crime and the height of the criminal.

EXPONENTIAL & LOGARITHMIC FUNCTIONS

An Exponential function $f(x) = a^x$ shows the relationship between 2 variables where a constant change in the independent variable induces a proportional change in the other variable. Logarithmic functions denoted by $f(x) = \log_a x$ are the inverse of exponential functions.

The exponential and logarithmic functions are two mathematical functions that play a significant role in forensic science. Rates of heating or cooling, or the rate of metabolizing of alcohol and drugs help to reach conclusions about the time elapsed since death.

CONDITIONAL PROBABILITY

The probability of an outcome or event happening, given another event has already occurred is defined as Conditional Probability. It is simply the relationship between the occurrence of two events. The theory of conditional probability studies the odds of an event occurring when another event has happened and the chances that both the events are connected. The formula for calculating the conditional probability of two events A and B is,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$P\left(\frac{A}{B}\right)$ is pronounced as Probability of the occurrence of event A when B has already happened.

The theory of Conditional Probability plays a crucial role in Crime Solving. It has a wide range of application including studying the Witness reliability, Statement analysis and many more. Misinterpreting conditional probabilities can lead to statistical fallacies.

BAYES' THEOREM

Bayes' theorem is a special case or an application of Conditional Probability. Named after Thomas Bayes, the theorem explains the odds of an event happening in relation to any given conditions. The Bayes' formula can be used to solve complex problems in a well constructed and precise way whereas the formula for conditional probability can be used only for relatively simpler problems. Bayes' formula is given by,

$$P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right) P(A)}{P(B)}$$

Bayes Formula helps to reach precise conclusions regarding the efficiency of evidences, strength of statements provided by the witness, and to check the validity of the assumptions used in solving a case.

LIKELIHOOD RATIO

In crime solving there are a few terms to be familiar of to understand Likelihood ratio. H is called Hypothesis, mainly two types here; H_p or Prosecution Hypothesis, and H_d or Defence Hypothesis and E is the evidence. Likelihood Ratio is defined as the ratio of the probability of E occurring when H_p is true, to the probability of occurrence of E given H_d .

$$LR = \frac{P\left(\frac{E}{H_p}\right)}{P\left(\frac{E}{H_d}\right)}$$

If $LR > 1$, it is in support of Prosecution Hypothesis, $LR < 1$, it is in support of Defence Hypothesis and when $LR = 1$ it is considered inadmissible. One of the major errors caused due to misinterpreting LR and Conditional probability is Prosecutor's Fallacy.

Chapter 2

CRIME SOLVING USING MATHEMATICAL MODELLING

2.1 ROSSMO MODEL

2.1.1 INTRODUCTION

Rossmo Model is developed by Dr. Kim Rossmo and is an application of mathematical modelling. The mathematical concept involved in the prediction is Probability. This is a blend of two entirely different subjects, mathematics and criminology. Here, the real world problem that we are dealing with is complex i.e., we have to hunt down a serial killer which is a social threat where the simple methods can end up in vain. A criminology concept lies the foundation on which the model is built, which is:

Criminals neither go too far to commit crime and also they never commit crime near their locality.

With this assumption he developed a formulae that sums up the probabilities of where the criminal is likely to live. A heat map was produced based on this inference which forms a heat zone of the probable residence of prime suspect. This was used and was succeeded in early 90's to hunt down a serial rapist. Since, we are challenging human being with a theoretical concept it has its drawbacks.

2.1.2 ROSSMO FORMULA

$$P_{ij} = k \sum_{n=1}^{(totalcrimes)} \left[\frac{\phi_{ij}}{(|X_i - x_n| + |Y_j - y_n|)} + \frac{(1 - \phi_{ij})(B^{g-f})}{(2B - |X_i - x_n| - |Y_j - y_n|)^g} \right]$$

P_{ij} : Probability of the culprit to reside in the considered square

f,g : Constants chosen from past crime to work better

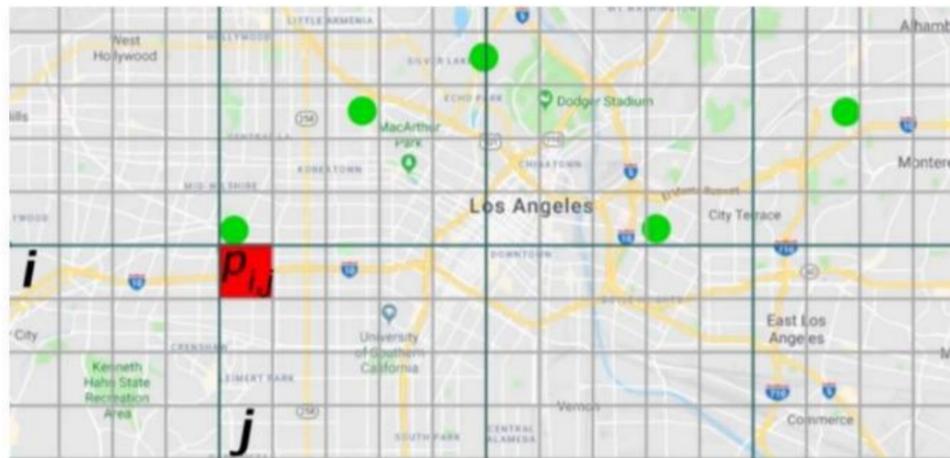
2B : Buffer zone formulated from past crimes.

B : Radius of the buffer zone.

ϕ_{ij} : Constant that is used to add weight to the formulae.

K : Empirically determined constant.

A grid map is taken for study. Here, the map is of Los Angeles. The crime spots are then marked in green. Assume a square in the grid as the residence of the culprit and mark it in Red. The column where the assumed square of the grid is present is marked as 'j' and the row in which it lies is marked as 'i'. P_{ij} is the probability of criminal to reside in the assumed square.

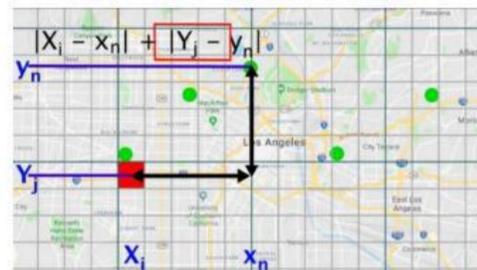


source:<http://brilliant.org>

The Rossmo formula is the summation is of N past crimes that occurred in the (x_n, y_n) coordinates in the past.



source:<http://brilliant.org>



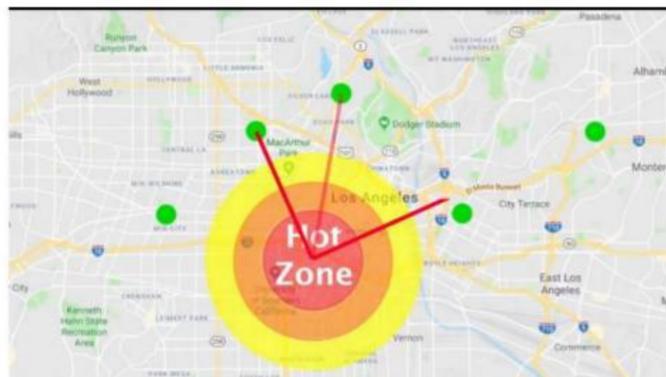
source:<http://brilliant.org>

Necessity of the two terms in the formula:

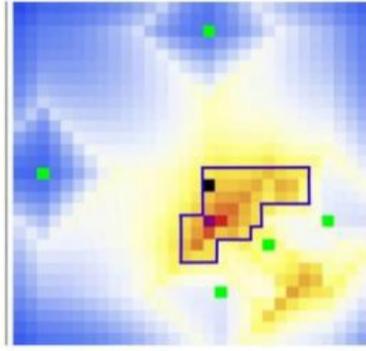
By criminology concept we already assumed that the criminal will not commit crime too close nor too far to the place where he is residing. So, a balance should be maintained in the formula.

- $|X_i - x_n|$: distance of crime scene from square considered in accordance with x axis.
- $|Y_i - y_n|$: distance of crime scene from square considered in accordance with y axis.

Let us consider the denominator of the first term in right hand side. This term is basically the distance between the assumed square and the crime spot. In this term as the distance increases the probability decreases. When we consider the second term, the distance is subtracted from the buffer zone. Therefore as denominator decrease probability of residence increase. This variation maintains a balance in accordance with the assumption made. The steps are repeated for every square in the grid. A heat map is developed with the help of the obtained data.



source:<http://brilliant.org>



source:<http://brilliant.org>

Here, a heat map is given which is developed using Rossmo Formula. This helped in finding out the residence of a 70's serial killer Richard Trenton Chase.

2.1.3 DRAWBACKS AND SUGGESTED MODIFICATION

Challenging human beings is indeed a big deal Here a theoretical concept is manipulated to do the same. Therefore it will have drawbacks. The aspects of human behaviour is complex and uncertain . Rossmo formula is used to track the residing place of the killer. We cannot be cent percent sure about the person to be present there. This increased the chances of modification.

Modification was proposed by Feroz shah Syed , Didong Li , Xun Zhang and Zhenhong Guo. This was about calculating **escape route of criminal** and the **Time and location of next crime**. This could help in strengthening of existing model.

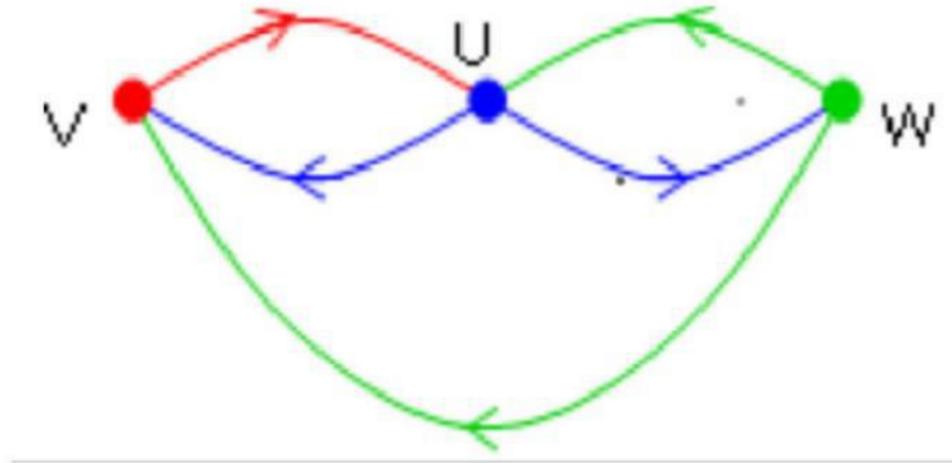
Chapter 3

CRIME PROBLEM USING GRAPH THEORY AND FORENSIC STATISTICS

3.1 GRAPH THEORY

Consider the scenario involving a crime where a jewellery shop has been subjected to robbery .Let us assume the suspects U , V and W as vertices . Assign the colours blue , red and green to the vertices. Suspect U says he is not the robber then lines to be drawn from U to V and W and assign colour blue . Suspect V says U is the robber then there is a line connecting from V to U and assign colour red . Suspect w says he is not the robber then lines to be drawn from W to U and V and assign colour green.

SUSPECTS	INVESTIGATION
U	I am not the robber
V	U is the robber
W	I am not the robber



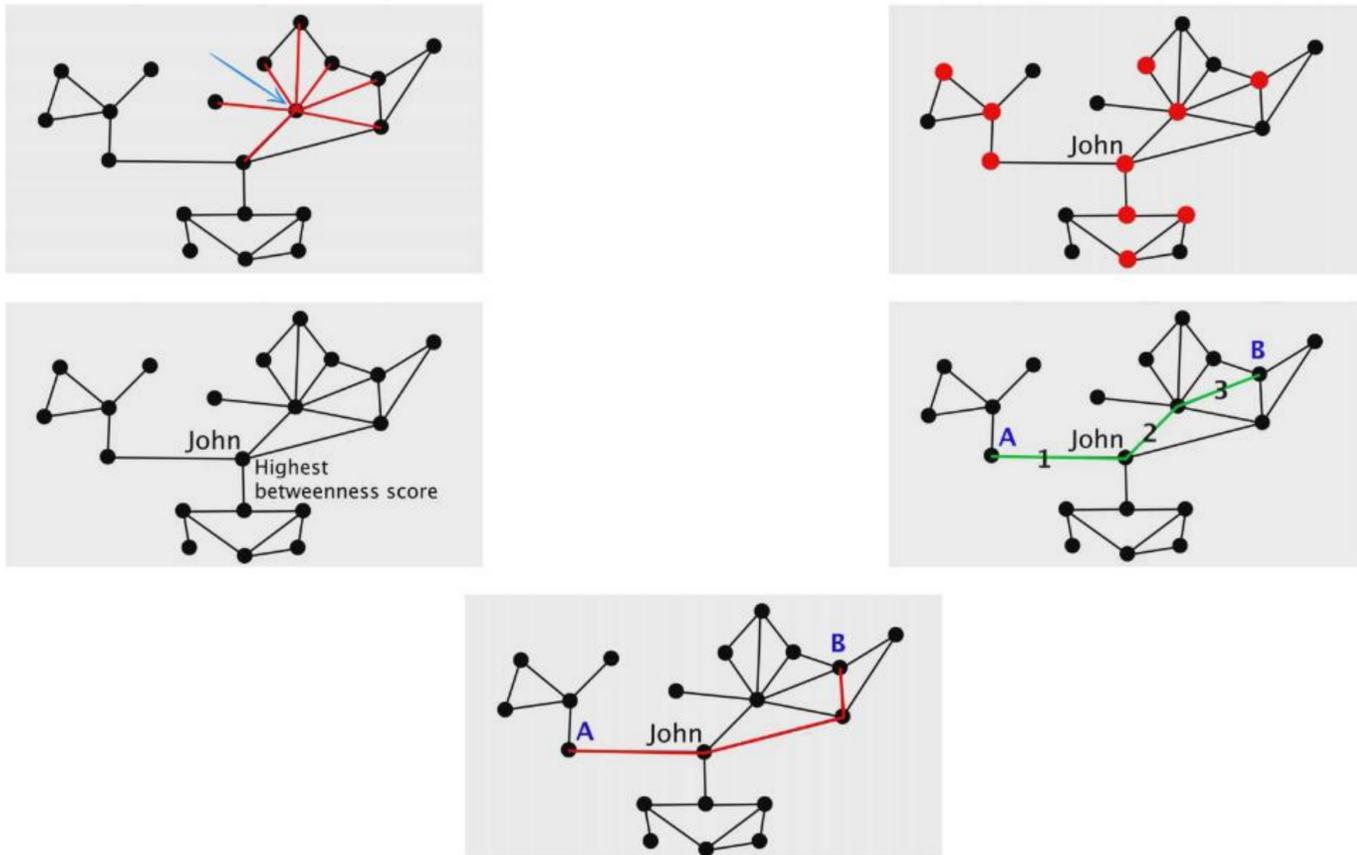
source:international journal of recent technology and engineering

This is the graph showing the connection between the attackers of the 9/11 attack through FBI investigation. What they did to prioritize people by law enforcement can be expressed in a simpler way using graphs.



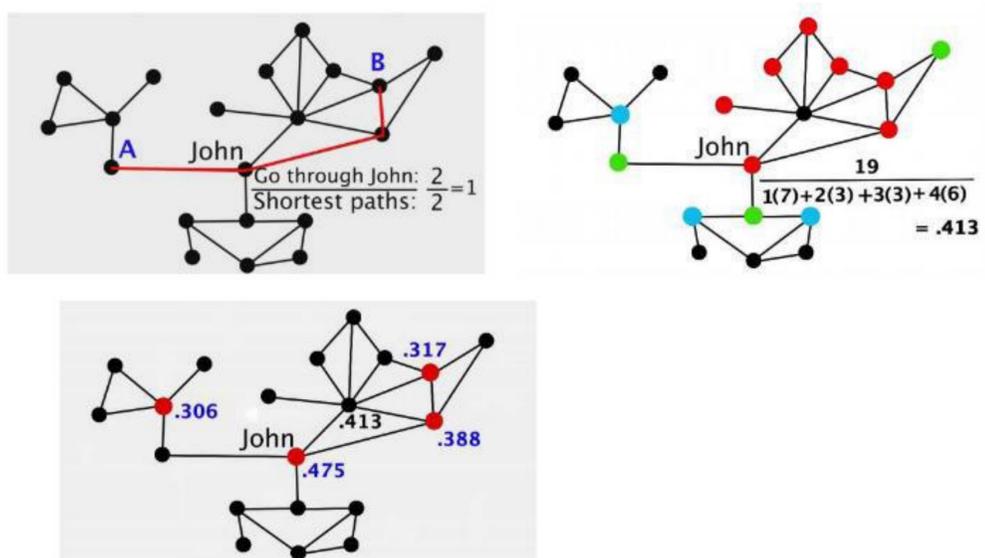
source:<http://brilliant.org>

Let's take the example of a school. All the students are connected in one way or the other whether it be social media, classes they share, close friendship and so on. For any two people let's say they are connected if they are close friends or in close contact with each other.



source:<http://brilliant.org>

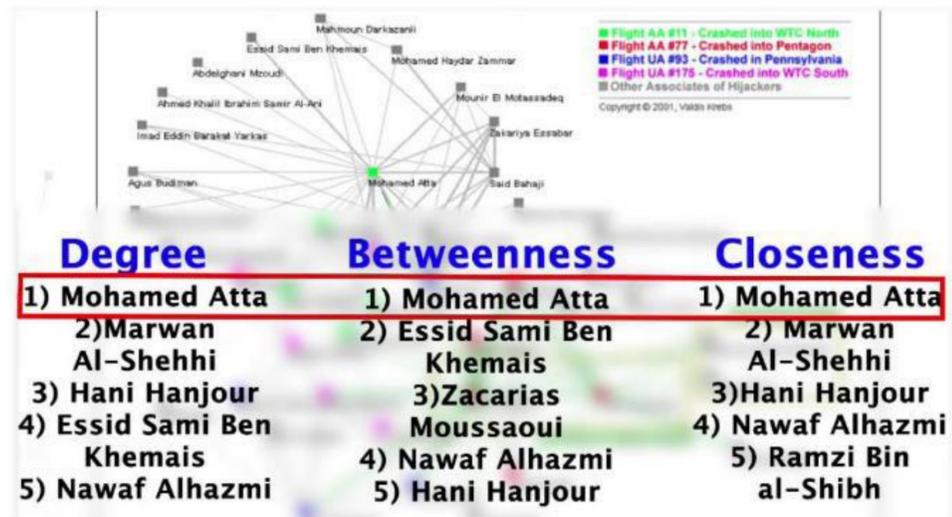
The dot to which the arrow is pointed in the first figure has most connections. But through further analysis we can see that John is the prime connection between the groups, without John no information can be passed. Here comes the importance of highest betweenness score and closeness centrality. Highest betweenness score is the score received to each nodes based on the number of shortest paths that passes through the node. consider the points A and B, the information from A reaches B through John by a number of ways among which two number of ways which is the shortest path with 3 edges. So the betweenness score of John is $\frac{\text{Go through john}}{\text{Shortest path}} = \frac{2}{2} = 1$.



source:<http://brilliant.org>

In a connected graph closeness centrality of a node is a measure of centrality in a network calculated as a reciprocal of the sum of the length of the shortest path

between the node and all other nodes in the graph. Thus more central a node is the closer it is to all other nodes. When considering the given graphs the closeness centrality of John is greater compared to others. So from John we trace back to find out from where a news started spreading. Similarly, During 9/11 attack Mohammed Atta was found to have the highest degree, betweenness score and closeness centrality. So, he was the prime link in the attack.



source: <http://brilliant.org>

3.2 FORENSIC STATISTICS

1) A wounded criminal walks away from the crime scene with blood dripping from a wound on his arms forming elongated, elliptical bloodstain of width $7.2mm$ and length $9mm$. What is the walking speed of the criminal?

SOLUTION:

Angle of Impact θ is given by,

$$\sin \theta = \frac{\text{width}}{\text{length}}$$

$$\theta = \sin^{-1} \left(\frac{\text{width}}{\text{length}} \right)$$

$$\theta = \sin^{-1} \left(\frac{7.2}{9} \right)$$

$$\theta = \sin^{-1}(0.8) = 53^\circ$$

As the wound is on his arms we assume the drop height to be $1m$.

Then the vertical impact speed V_g is given by,

$$V_g = \sqrt{2gh}$$

given, $g = 9.8m/s^2$, $h = 1m$

$$V_g = \sqrt{2 \times 9.8 \times 1} = \sqrt{19.6} = 4.43m/s^2$$

when $\theta = 53^\circ$

$$\tan \theta = \tan 53^\circ = 1.33$$

$$\tan \theta = \left(\frac{V_g}{V_h} \right)$$

The horizontal speed can be calculated by,

$$V_h = \left(\frac{V_g}{\tan \theta} \right)$$

$$V_h = \left(\frac{V_g}{\tan 53^\circ} \right)$$

$$V_h = \frac{4.43}{1.33}$$

$$V_h = 3.33m/s^2$$

2) Magi's DNA profile matched with the DNA evidence discovered at the crime scene. Her identical twin Grace is the defendant. A DNA profile is of a type that is found in only 1 out of a billion people. Check the admissibility of the evidence and evaluate which hypothesis it is in support of.

SOLUTION:

H_p : The DNA source is Grace (Prosecution Hypothesis)

H_d : The DNA source is Magi (Defence Hypothesis)

E: The DNA evidence found matched with Magi's DNA profile.

The chances of a DNA match in siblings is one out of 10,0 whereas identical twins have the same DNA profile. Therefore Grace's DNA profile matches with Magi's DNA profile.

Hence,

$$P\left(\frac{E}{H_p}\right) = P\left(\frac{E}{H_d}\right)$$

Likelihood Ratio

$$LR = \frac{P\left(\frac{E}{H_p}\right)}{P\left(\frac{E}{H_d}\right)}$$

As both the probabilities are equal,

$$LR = 1$$

Since $LR = 1$,

- The evidence is inadmissible concerning the hypotheses.
 - It is neutral and supports neither of the hypotheses.
-

3) Jim is accused of a robbery. The probability that he is innocent is 59%. Timothy, Jim's friend who runs a shop near the crime scene is called on as the prime witness. Timothy's probability of saying the truth is 0.73. The probability of Timothy saying the truth when Jim is guilty is 0.42. What is the probability that Jim is actually guilty if Timothy lied?

SOLUTION:

Defining all the events involved in this problem,

G: Jim is guilty.

G' : Jim is innocent.

T: Timothy says the truth.

T' : Timothy lies.

$\frac{T}{G}$: Timothy says the truth when Jim is guilty.

$\frac{T'}{G}$: Timothy lies when Jim is guilty.

Here, we need to evaluate $P\left(\frac{G}{T'}\right)$,

$P\left(\frac{G}{T'}\right)$ is the probability of Jim being guilty when Timothy gives a false statement.

Given,

$$P(G) = 1 - 0.59 = 0.41$$

$$P(G') = 0.59$$

$$P(T) = 0.73$$

$$P(T') = 1 - 0.73 = 0.27$$

$$P\left(\frac{T}{G}\right) = 0.42$$

$$\begin{aligned} P\left(\frac{T'}{G}\right) &= 1 - P\left(\frac{T}{G}\right) \\ &= 1 - 0.42 = 0.58 \end{aligned}$$

$$P\left(\frac{G}{T'}\right) = \frac{P\left(\frac{T'}{G}\right) \times P(G)}{P(T')}$$

Substituting the values, we get,

$$P\left(\frac{G}{T'}\right) = \frac{0.58 \times 0.41}{0.27}$$

$$P\left(\frac{G}{T'}\right) = \frac{0.23}{0.27}$$

$$P\left(\frac{G}{T'}\right) = 0.88$$

Hence, the probability that Jim is guilty given Timothy says the truth is 0.88.

Chapter 4

CONTROLLING CRIME BY ANALYSING DATA MANIPULATION

In this fast moving world data plays pivotal role in day to day human activities. Data is considered as a resource, crime containing data manipulation must be strictly monitored and stringent actions must be taken against those fraudulent activities. As data is not merely a group of things or numbers but an asset.

4.1 BENFORD'S LAW

4.1.1 INTRODUCTION

In 1981, Canadian-American astronomer Simon Newcomb found out that the former pages of logarithmic table were much worn when compared to other pages. He published a result that the probability of a single number N being the leading digit of a number is $\log(N+1) - \log(N)$ based on his observation.

This was noticed by Frank Benford in 1938, a physicist and he tested the data from 20 different domains including surface area of 335 rivers, 104 physical constants, 1800 molecular weights, 308 numbers contained in an issue of Reader's digest etc. Total 20229 observations were used. Hence this discovery was named after Benford.

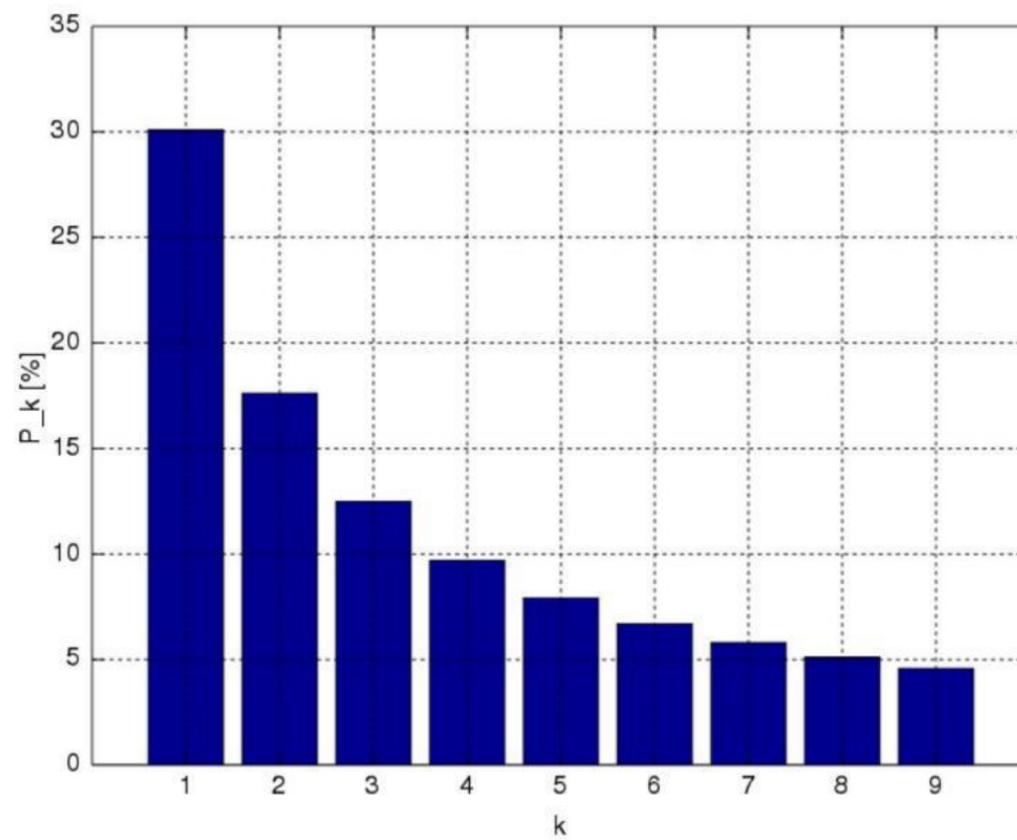
4.1.2 THEORY

Benford's law also known as Newcomb-Benford's law, law of Anomalous number or First digit law. Benford's law states that in a set of natural numbers the leading digit is more likely to be the number 1 the likelihood of leading digit be any other number decreases as it gets closer to number 9. Benford's law predicts the occurrence of digits in large sets of data. This law maintains that we can expect some digits to occur more often than other digits. For sets obeying Benford's law the number 1 appears as leading digit about 30% while the number 9 appears as leading digit less than 5%.

4.1.3 BENFORD'S LAW CONDITIONS

- The numbers in the data set should be the same objects -We cannot apply Benford's law when two different objects data is combined as one data set.
- There should be no built-in maximum or minimum to the numbers -We cannot apply Benford's law when the number is having a maximum or minimum limit.
- The numbers should not be assigned - In case of phone number, invoice number, car numbers Benford's law cannot be applied as they are assigned numbers.
- Does not apply for Uniform Distribution -In uniform distribution the occurrence of each number is about 11%

4.1.4 DISTRIBUTION OF FIRST DIGIT ACCORDING TO BENFORD'S LAW



source:<http://wikipedia.org>

- Each bar represents digits from 1 to 9
- Height of the bar is the percentage of the numbers that start with the digit.

d	$P(d)$	Relative size of $P(d)$
1	30.1%	
2	17.6%	
3	12.5%	
4	9.7%	
5	7.9%	
6	6.7%	
7	5.8%	
8	5.1%	
9	4.6%	

source:<http://wikipedia.org>

A set of numbers is said to satisfy Benford's Law if the leading digit $[d=1,2,\dots,9]$ occurs with probability

$$p(d) = \log_{10}(d+1) - \log_{10}(d) = \log_{10}[(d+1)/d] = \log_{10}(1 + 1/d)$$

4.2 APPLICATIONS

Live case study

From NSE 13th December data is taken. We will look into the total traded value, applied Benford's law then we will see that the real data set is exactly falling into these percentage. And a comparison between what Benford is saying and actual data is done.

	A	J	N	O	P	Q	R	S	T	U
1	SYMBOL	TOTTRDVAL	1st Digit		Digit	Actual	Actual %	Benford	Diff	
2	20MICRONS	40,47,865	4		1	645	29.45%	30.10%	-0.07%	
3	21STCENMGM	5,71,621	5		2	384	17.53%	17.61%	-0.08%	
4	3IINFOLD	38,11,94,542	3		3	279	12.74%	12.49%	0.25%	
5	3MINDIA	2,98,51,519	2		4	207	9.45%	9.69%	-0.24%	
6	3PLAND	4,32,727	4		5	192	8.77%	7.92%	1%	
7	5PAISA	1,21,21,324	1		6	153	6.99%	6.69%	0.30%	
8	610GS2031	50,000	5		7	133	6.07%	5.80%	0.27%	
9	63MOONS	65,71,181	6		8	90	4.11%	5.12%	-1.01%	
10	667GS2050	69,700	6		9	107	4.89%	4.58%	0.31%	
11	676GS2061	1,91,910	1			2,190.0				
12	716GS2050	218	2							
13	762GS2039	10,158	1							
14	772GS2055	44,440	4							
15	795GS2032	110	1							
16	A2ZINFRA	28,48,616	2							
17	AAATECH	1,89,000	1							
18	AAKASH	3,18,06,428	3							
19	AAREYDRUGS	14,22,431	1							
20	AARON	16,72,841	1							
21	AARTIDRUGS	56,89,59,966	5							
22	AARTIIND	1,15,71,37,035	1							
23	AARTISURF	1,24,06,141	1							
24	AARVEEDEN	4,51,112	4							

Inference

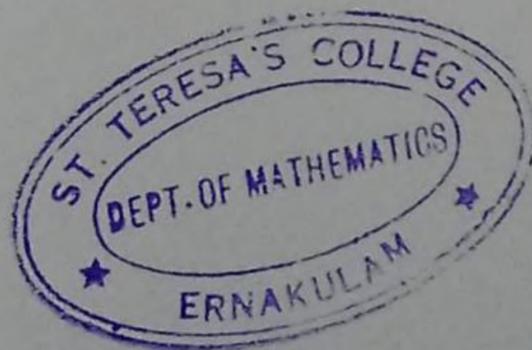
The difference is matching (not even 0.5%). When the difference is more than 1% or 2% .It doesn't mean a fraud ,it could alarm the auditor.

Spreadsheet functions

- Extract the 1st digit out of the database [=LEFT(J2)] click enter
- Column for digits 1 to 9
- Actual frequency column (how many times the digit appearing in the beginning) [COUNTIF] Actual percentage (Actual/Total) \times 100 [Q2/Q11(F4)]
- Benford's percentage [=log10(1/P2+1)]
- Difference (Actual - Benford)

4.2.1 OTHER APPLICATIONS

- Accounting Fraud detection
- Use in Criminal Trails.
- Election Data
- Genome Data
- Scientific Fraud Detection economic Data Digit Analysis



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Project Report

On

**A BRIEF STUDY ON FOURIER SERIES AND
IT'S APPLICATIONS**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

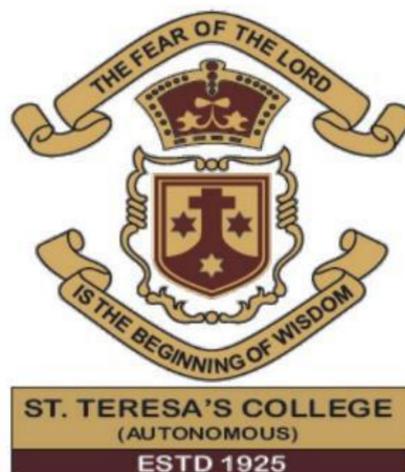
by

RINTA P.J

(Register No. AB19BMAT050)

Under the Supervision of

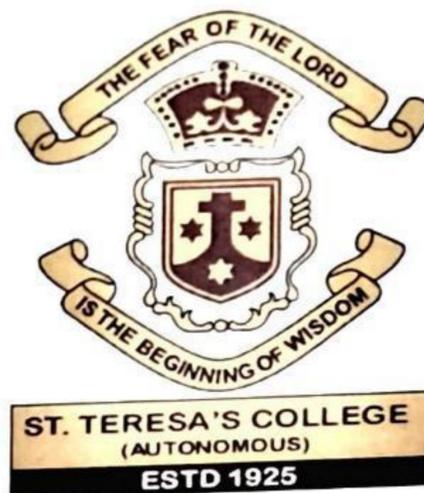
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APRIL 2022



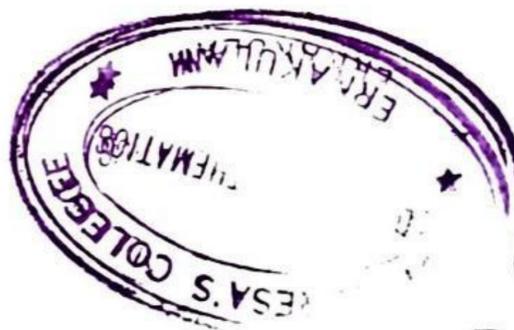
CERTIFICATE

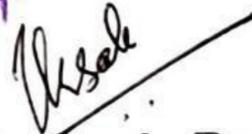
This is to certify that the dissertation entitled, **A BRIEF STUDY ON FOURIER SERIES AND IT'S APPLICATIONS** is a bonafide record of the work done by Ms. **RINTA P.J** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 08-03-2022

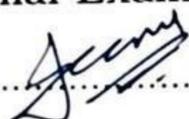
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External Examiners

1: 

2:

DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Mrs. Donna Pinheiro, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Place: Ernakulam.

Date: 08-03-2022


RINTA P.J

AB19BMAT050

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Firstly,I thank God Almighty for giving me his grace to execute this project successfully.I express my deep sense of gratitude to our project guide **Mrs.Donna Pinheiro**,Department of Mathematics,St.Teresa's College(Autonomous), Ernakulam for her valuable guidance and suggestions.I would also like to mention the uncending help and support provided by **Dr.Ursala Paul**, HOD,Department of Mathematics, St. Teresa's College, Ernakulam.

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Place: Ernakulam.

RINTA P.J

Date: 08-03-2022

AB19BMAT050

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INTRODUCTION

JOSEPH FOURIER

Baron Jean Baptiste Joseph Fourier, a French Mathematician (1768-1830), first introduced the idea that any periodic function can be represented by a series of sines and cosines wave in 1828; published in his dissertation *The Analytic Theory Of Heat*, Fourier's work is a result of arriving at the answer to a particular heat equation.



Periodic Functions

A function $f(x)$ is called a periodic function if $f(x)$ is defined for all x , except possibly at some points, and if there is some positive number p called a period of $f(x)$ such that,

$$f(x + p) = f(x) \text{ for all } x$$

Any periodic function can be resolved as a summation of constant value and cosine and sine functions.

Importance of Fourier Series

A Fourier Series is an expansion of a periodic function in terms of an infinite sum of sines and cosines. Fourier series is used to describe a periodic signal in terms of cosine and sin waves. That is it allows us to model any arbitrary periodic signal with a combination of sines and cosines.

A Fourier Series make use of the orthogonality relationships between the sine and cosine functions.

The computation and study of Fourier Series is known as Harmonic Analysis. It establishes a relation between a function in the domain of time and a function in the domain of frequency.

Fourier Series are very vitally used to approximate a periodic waveform in electronics and electrical circuits. It is used in calculators and computers for evaluating values many functions. Especially, in signal processing ,the typical form of Fourier transform is to decompose the signal into amplitude and phase components.

Advanced noise cancellation and cell phone network technology uses Fourier series where digital filtering is used to minimize noise and bandwidth demands respectively.

Fourier Series is broadly used in telecommunication system. Fourier methods are definitely a widely applied tool of analysis. They are used in all signals(audio, image, radar, etc..). One of many application is compression. The compression ratio offered by the use of Fourier transform is dependent on the quality required by the application everyone's favorite MP3 format uses this for audio compression.

Chapter 1

BASIC CONCEPTS OF FOURIER SERIES

1.1 DEFINITION

Suppose that $f(x)$ is a function with period $2L$ and $f(x)$ is differentiable on $[-L, L]$ then $f(x)$ has a Fourier series representation:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

Where,

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, n = 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, n = 1, 2, \dots$$

Here a_0, a_n and b_n are called Fourier coefficients.

If the period of $f(x)$ is 2π that is it ranges from $[-\pi, \pi]$

Then $f(x)$ has a Fourier series representation as follows :

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Where ,

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

1.2 EVEN FUNCTION

If we have an even function and its period is from $[-\pi, \pi]$,

Then $f(x)$ has a Fourier Series representation as follows :

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx)$$

Where,

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \int_0^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

Due to the orthogonality relationship between sine and cosine functions the sine term gets cancelled from the original expansion .

Therefore, the coefficient $b_n = 0$

1.3 ODD FUNCTION

If we have an odd function and its period is from $[-\pi, \pi]$,

Then $f(x)$ has a Fourier series representation as follows :

$$f(x) = \sum_{n=1}^{\infty} (b_n \sin nx)$$

Where,

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx = 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx$$

1.4 COMPLEX FORM OF FOURIER SERIES

The complex form of Fourier Series is algebraically simpler than it and more symmetric. Therefore, it is often used in Physics and other sciences.

If the period of $f(x)$ ranges from $[-\pi, \pi]$,

Then $f(x)$ has a Fourier series representation as follows :

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$$

Where,

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx$$

If the period of $f(x)$ ranges from $[-L, L]$,

Then $f(x)$ has a Fourier Series representation as follows:

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{L}}$$

Where,

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-\frac{in\pi x}{L}} dx$$

1.5 DIRCHLETS CONDITION

Any function $f(x)$ can be formed as a Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Where a_0, a_n and b_n are constants; Provided

- $f(x)$ is periodic, single-valued and finite
- $f(x)$ has atmost a finite number of maxima and minima
- $f(x)$ has a finite number of discontinuities in any one period

Fourier Series are powerful tools for problems involving functions that are periodic or are of interest on a finite interval only.

Chapter 2

FOURIER TRANSFORM

2.1 FOURIER TRANSFORM

The Fourier transform is an extension of the Fourier Series to non-periodic function .

The Fourier transform of $f(t)$ is given by ,

$$\mathbf{F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt}$$

2.1.1 Forward Fourier Transform

It converts the function of time (t) into the function of frequency (ω).

$$\mathbf{f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i\omega t} d\omega}$$

2.1.2 Inverse Fourier Transform

It does the opposite to first equation ie; it converts the function of frequency (ω) into the function of time (t).

2.2 PROPERTIES

1. LINEARITY

If $x(t) \longleftrightarrow^{FT} X(\omega)$ and $y(t) \longleftrightarrow^{FT} Y(\omega)$

Then, $a x(t) + b y(t) \longleftrightarrow^{FT} a X(\omega) + b Y(\omega)$

If $X(\omega)$ is the Fourier transform of function $x(t)$ and $Y(\omega)$ is the Fourier transform of function $y(t)$.

If we add signal to these functions then these added signals will have an effect on Fourier transform also.

2. TIME SHIFTING

If $x(t) \longleftrightarrow^{FT} X(\omega)$

Then, $x(t - t_0) \longleftrightarrow^{FT} e^{-j\omega t_0} X(\omega)$

If $X(\omega)$ is the Fourier transform of the function $x(t)$.

If we change the signal by shifting time, Fourier transform is multiplied by $e^{-j\omega t_0} X(\omega)$.

3. FREQUENCY SHIFTING

If $x(t) \longleftrightarrow^{FT} X(\omega)$

Then, $e^{-j\omega t_0} x(t) \longleftrightarrow^{FT} X(\omega - \omega_0)$

If $x(\omega)$ is the Fourier transform of the function $x(t)$.

If we multiply $x(t)$ by a frequency signal then, it will have an effect on Fourier transform.

4. TIME SCALING

$$\text{If } x(t) \longleftrightarrow^{FT} X(\omega)$$

$$\text{Then, } x(at) \longleftrightarrow^{FT} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

If $X(\omega)$ is the Fourier transform of the function $x(t)$. If the function $x(t)$ is expanded in time by a quantity a .

Fourier transform is compressed in frequency by same amount .

5. DIFFERENTIATION AND INTEGRATION

$$\text{If } x(t) \longleftrightarrow^{FT} X(\omega)$$

Differentiation Property :

$$\text{Then, } \frac{d}{dx}x(t) \longleftrightarrow^{FT} X(\omega)$$

$$\text{Then, } \frac{d^n}{dx^n}x(t) \longleftrightarrow^{FT} (j\omega)^n X(\omega)$$

If $X(\omega)$ is Fourier transform of the function $x(t)$.

If we differentiate the function $x(t)$ one time, Fourier transform will get multiplied by $j\omega$.

If we differentiate $x(t)$ n times, then $X(\omega)$ get multiplied by $(j\omega)$, n times .

Integration Property :

$$\text{Then, } \int x(t)dt \longleftrightarrow^{FT} \frac{1}{j\omega} X(\omega)$$

$$\text{Then, } \int \int \int \dots \int x(t)dt \longleftrightarrow^{FT} \frac{1}{(j\omega)^n} X(\omega)$$

If $X(\omega)$ is the Fourier transform of the function $x(t)$. If we integrate the function $x(t)$ Fourier transform will be divided by $j\omega$.

In a similar way if we integrate $x(t)$ n times, Fourier transform will be having effect in this way as shown .

6. MULTIPLICATION AND CONVOLUTION

If $x(t) \xleftrightarrow{FT} X(\omega)$ and $y(t) \xleftrightarrow{FT} Y(\omega)$

$$x(t).y(t) \xleftrightarrow{FT} X(\omega) * Y(\omega) \rightarrow \text{Multiplication Property}$$

$$x(t) * y(t) \xleftrightarrow{FT} X(\omega).Y(\omega) \rightarrow \text{Convolution Property}$$

If $X(\omega)$ is the Fourier transform of function $x(t)$ and $Y(\omega)$ is the Fourier transform of function $y(t)$.

Fourier transform of product of two signals is the convolution of two Fourier transforms. Similarly , convolution of two signals is the product of two Fourier transforms .

2.3 DISCRETE FOURIER TRANSFORM

The discrete Fourier transform transforms a sequence of N

Complex numbers

$$\{x_n\} = x_0, x_1, \dots, x_{N-1}$$

into another sequence of Complex numbers ,

$$\{X_k\} = X_0, X_1, \dots, X_{N-1}$$

which is defined by,

$$\begin{aligned} X_k &= \sum_{n=0}^{N-1} x_n \cdot e^{-i\frac{2\pi}{N}kn} \\ &= \sum_{n=0}^{N-1} x_n [\cos(\frac{2\pi}{N}kn) - i \cdot \sin(\frac{2\pi}{N}kn)] , \end{aligned}$$

where the last expression follows from the first one by Euler's formula.

2.4 FAST FOURIER TRANSFORM

Fast Fourier Transform (FFT) is an efficient way or algorithm to compute Discrete Fourier Transform (DFT) with reduced computations. DFT is used to digitize signals. The collection of various fast DFT computation techniques is known as the Fast Fourier transform . Fourier analysis converts a signal from its original domain to a representation in the frequency domain and vice versa. The DFT is obtained by decomposing a sequence of values into components of different frequencies.

Basic concepts underlying FFT has to do with an array of time - domain wave form samples processing those to produce a new array of frequency domain spectrum samples. FFT makes the calculation easier as we always deals with various frequency band in communication system .

DFT is useful in many fields, but computing in directly from definition is too slow to be practical. It is also used efficiently solve partial differential equations , and to perform other operations such as convolution or multiplying large integers. It deals with a finite amount of data , it can be implemented in computers by numerical algorithms . FFT manages to reduce the FFT used in digital signal processing to modify, filter and decode digital audio, video and images . It is an important image to decompose an image into its sine and cosine components. It can convert discrete data into a continuous data type available at various frequencies .

The main advantage of an FFT is speed ,which it gets by decreasing the number of calculations needed to analyze a waveform .

Chapter 3

FOURIER SERIES IN DIGITAL SIGNAL PROCESSING

3.1 DIGITAL SIGNAL PROCESSING

Digital Signal Processing is the technique of processing a signal to isolate or obtain the more useful information. Filtering, modulating and sampling, digitizing the analog signal, processing the signal are used to improve the signal performance, resulting in a new ideal signal.

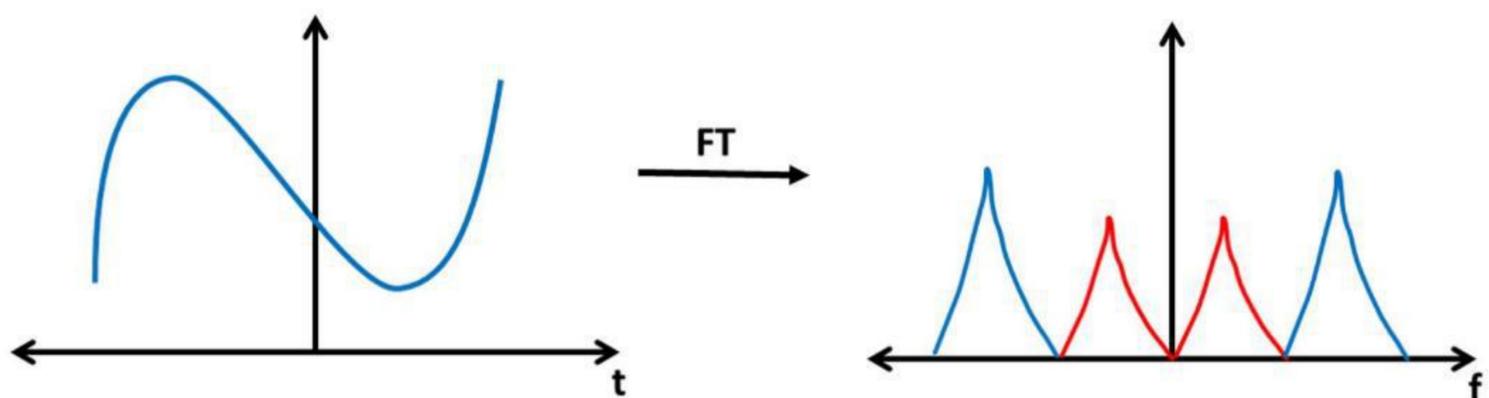
Mostly the voice recordings contain background noise and the images contain specks of light called noise. In the real world professional software is used to minimize these kind of noises. Programmes like Audacity and Lightroom work on the basis of Signal Processing, which uses the mathematical technique called the Fourier Transform.

Since signal is essentially any waveform that is a function of time, signal processing is not limited to photos and voice recordings. Hence

apart from image processing and audio processing it's widely used in wireless communication, array processing, feature extraction etc. Signal filtering works using a system called Linear Shift Invariant (LTI) system. Such a system has two simple properties. Firstly, as the name suggest, the system has to be linear. That is, if we input two waves separately into the system and obtain the corresponding outputs. Similarly if we input the sum of the two waves, result will be the sum of the two corresponding outputs. This is also called the superposition principle. Secondly, it has to be time invariant. Suppose we input a wave and get the corresponding output. Then if we input the same wave anytime later, we should still expect to get the same corresponding output wave.

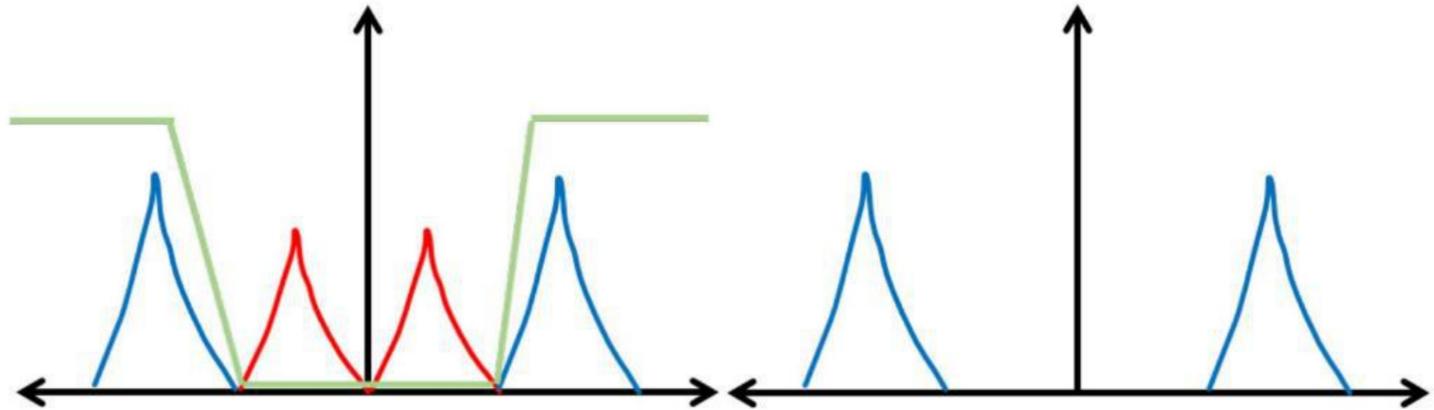
3.2 DENOISING DATA

When we use Fourier Transform to a sound recording, we can decompose one recording into various frequencies that it was originally composed of.



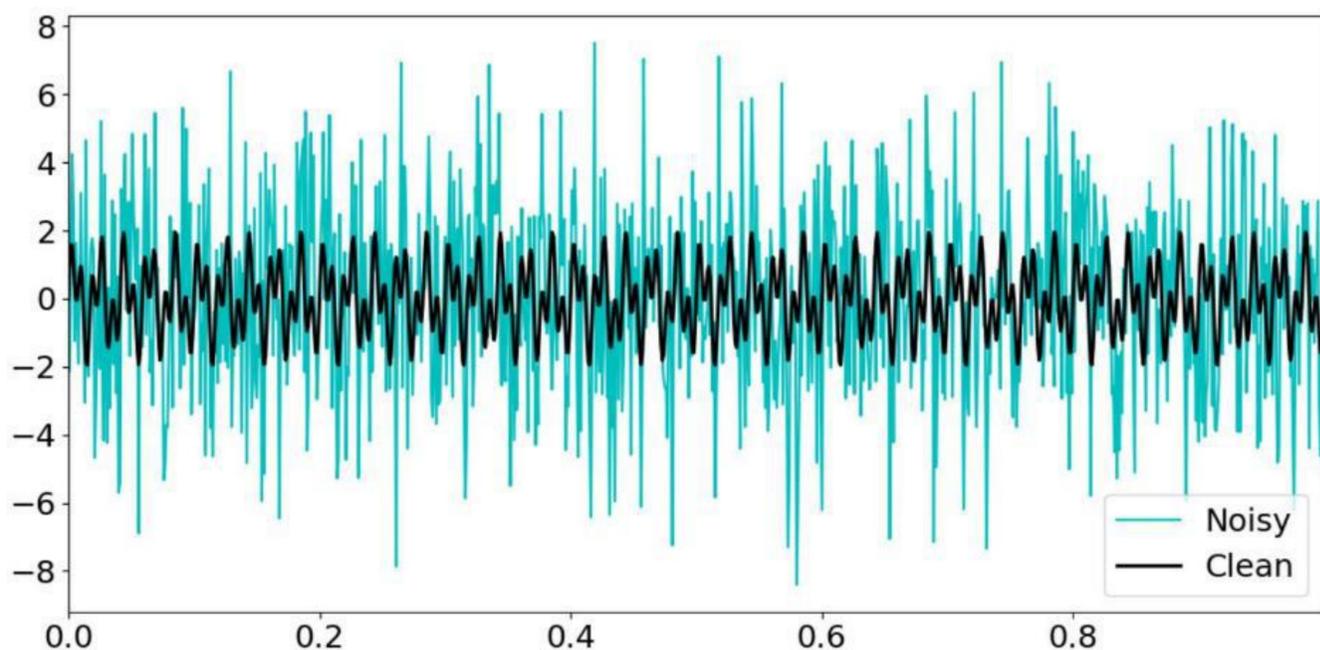
Then it will be easy to isolate the frequencies that are undesirable, and apply our audio filter to remove them. Once the filter is applied,

the Inverse Fourier Transform is applied and hence we get back our voice recording as a function of time.



3.2.1 Denoising Data In Python Using FFT

In the illustrated program, we create a simple dataset which is the sum of two sine waves of different frequencies. Then we add noise to the data set. We use Fast Fourier Transform to pull out the structure of the data and denoise. Basically, the purpose of the program is to denoise and extract the useful information from any signal that is given to us.



On running the given data vector f through the FFT command, we get the vector - \hat{f} Fourier coefficients. These are complex valued

Fourier coefficients which describes what are the magnitudes and phase of sine and cosine components of increasing frequency that is to be added to get the given dataset.

```
import numpy as np
import matplotlib.pyplot as plt
plt.rcParams['figure.figsize'] = [16, 12]
plt.rcParams.update({'font.size': 18})

#Create a simple signal with two frequencies
dt = 0.001
t = np.arange(0, 1, dt)
f = np.sin(2*np.pi*50*t) + np.sin(2*np.pi*120*t) #Sum of 2 frequencies
f_clean = f
f = f + 2.5*np.random.randn(len(t)) #Add some noise

#Compute the Fast Fourier Transform (FFT)
n = len(t)
fhat = np.fft.fft(f,n) #Compute the FFT
PSD = fhat * np.conj(fhat) / n #Power spectrum (power per freq)
freq = (1/(dt*n)) * np.arange(n) #Create x-axis of frequencies
L = np.arange(1, np.floor(n/2), dtype='int') #Only plot first half of

#Use the PSD to filter out noise
indices = PSD > 100 #Find all frequencies with large power
PSDclean = PSD * indices #Zero out all others
fhat = indices * fhat #Zero out smaller Fourier coeffs in Y
ffilt = np.fft.ifft(fhat) #Inverse FFT for filtered time signal

#Plots
fig, axs = plt.subplots(3, 1)

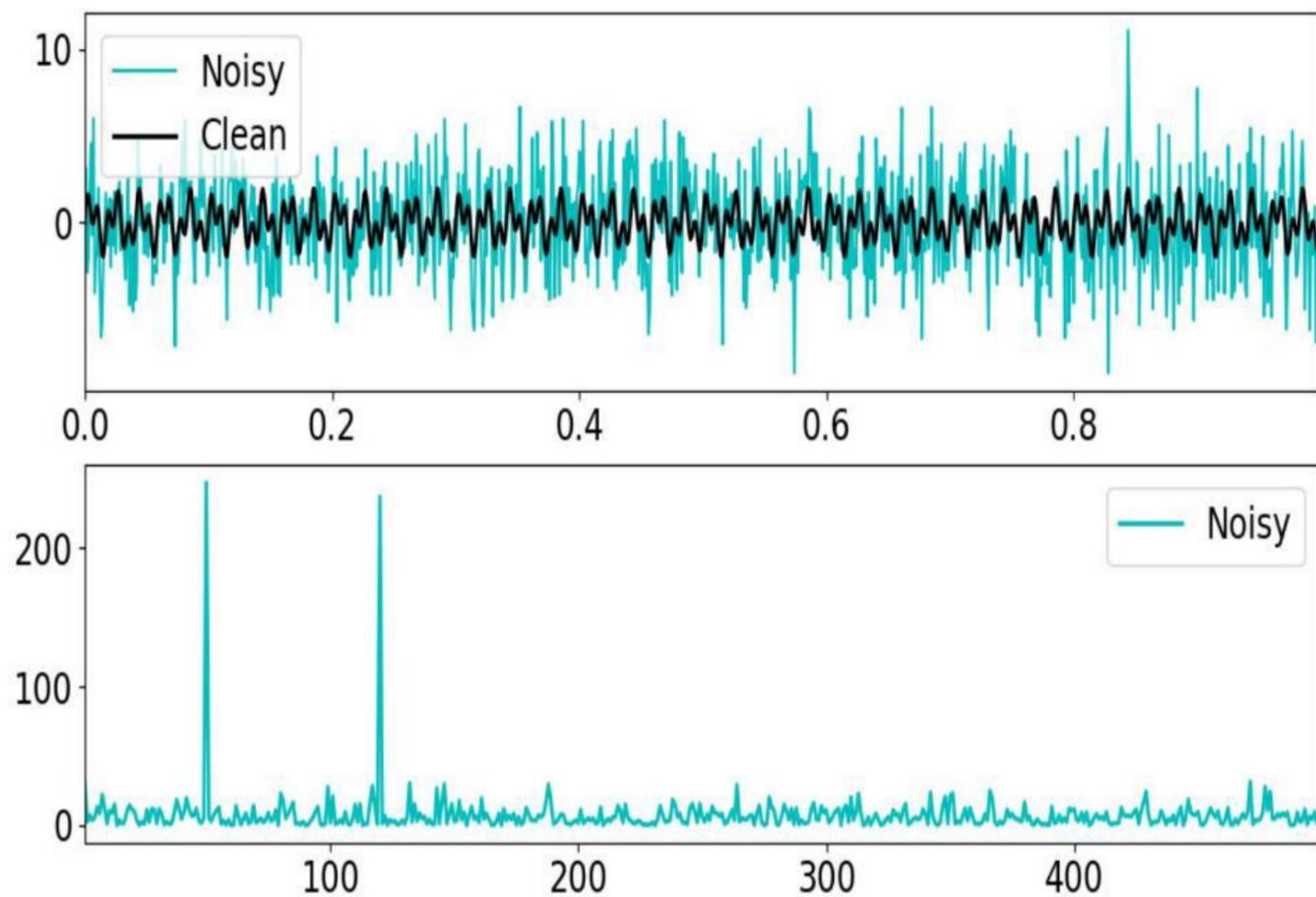
plt.sca(axs[0])
plt.plot(t, f, color='c', linewidth=1.5, label='Noisy')
plt.plot(t, f_clean, color='k', linewidth=2, label='Clean')
plt.xlim(t[0], t[-1])
plt.legend()

plt.sca(axs[1])
plt.plot(t, ffilt, color='k', linewidth=2, label='Filtered')
plt.xlim(t[0], t[-1])
plt.legend()

plt.sca(axs[2])
plt.plot(freq[L], PSD[L], color='c', linewidth=2, label='Noisy')
plt.plot(freq[L], PSDclean[L], color='k', linewidth=1.5, label='Filtered')
plt.xlim(freq[L[0]], freq[L[-1]])
plt.legend()

plt.show()
```

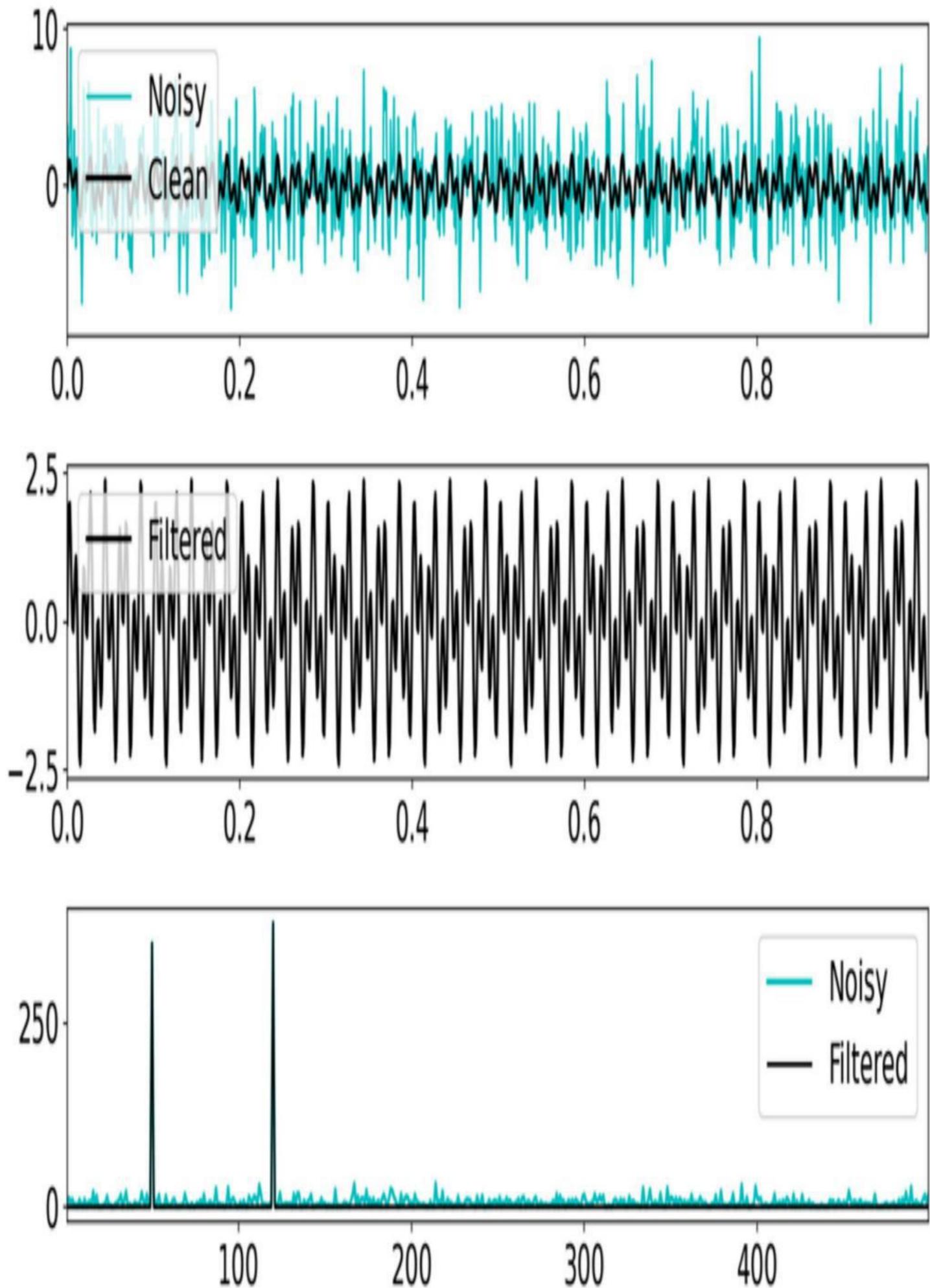
As we have the Fourier Transform, we will compute the Power Spectral Density(PSD), by performing \hat{f} times the conjugate of \hat{f} . That is, we are finding the magnitude of each Fourier Coefficients squared.



As the plot suggest,even though the signal is noisy the power spectrum have two peaks(clean signal).So we can apply a filter function such that any Fourier coefficient less than 100 is phased out and any Fourier coefficient greater than 100 is left out.Finally, we Inverse Fourier Transform and reconstruct our clean signal.

The proposed Fourier transform has a wide range of help in various domains like power distribution system, wireless, signal processing, cell phone manufacturing, mechanical and industrial application.The Fourier transform provides the world an easy and most comfortable method for solution of questions.

The final output is given below. In the final output we have recovered the original signal without the noise.



Fourier transformation is also frequency domain which represent time series analysis proved its application in Quantum mechanics, Signal processing, Image Processing and filters, Transformation, and encoding, Data Processing and Analysis and other fields. The idea behind Fourier transforms is that a function of direct space can be expressed equivalently as a complex valued function of reciprocal space that is frequency sometimes called Fourier space.

Chapter 4

APPLICATIONS OF FOURIER TRANSFORM IN IMAGE ANALYSIS

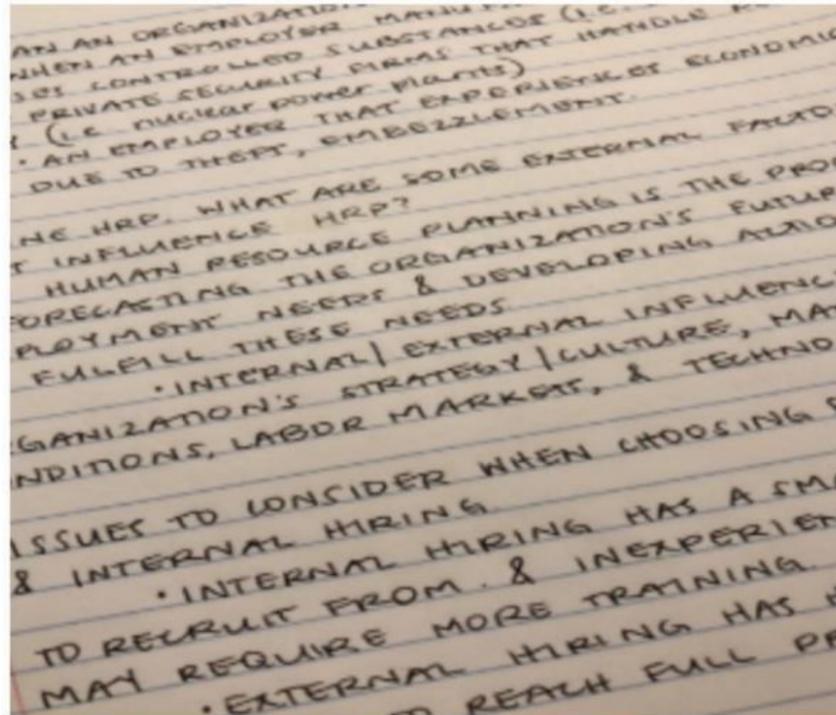
4.1 IMAGE ANALYSIS

Image analysis is used for the extraction of meaningful information from images. The Fourier transform is an important image processing tool which is used to decompose an image into its sine and cosine components. The output of the transformation represents the image in the Fourier or frequency domain while the input image is in the spatial domain equivalent.

In the Fourier domain image, each point represents a particular frequency contained in the spatial domain image. In the modern world we primarily make use of digital images and as we know, these digital images are made of several pixels. So that we make use of

Discrete Fourier Transform (DFT) for Image analysis.

One of the things that we are interested in image analysis is reading text. Currently, we make use of Fourier transform for recognising characters.



The fact is, while humans are very good at reading handwritten and printed texts, computers struggle with it. But if we use Fourier transform to recognize characters, this new representation helps computers to read the texts easily.

Consider tasks such as transferring books to electronic copies, number plate recognition, automating the input of handwritten text and activities in post offices. When human beings do these tasks on a large scale, there are chances for occurrence of errors due to our carelessness. Having a computer read text in these situations is very helpful for completing them in a faster pace. This is where the need for Fourier transform arises.



When we apply Fourier transform to an image, we retain the entire information that is captured in the original data. But we display it in a different way so that it's easier to interpret. We can make use of the lines and circles in the images and their corresponding patterns in the frequency domain to use it to recognize characters.

4.2 LETTER RECOGNITION

We use the Fourier Transform to identify letters and numbers. The representation of letters and numbers in the frequency domain can be used for identifying what letter or number it is, in the time domain. In this way computers can distinguish each letter from another, on observing its Fourier images.

4.2.1 Letter Recognition in Python using FFT

```
import cv2
import numpy as np
import glob

list_images = glob.glob('letters/*') #take all images from the folder

for image_title in list_images: #loop
    img = cv2.imread(image_title, cv2.IMREAD_GRAYSCALE)
    f = np.fft.fft2(img)
    fshift = np.fft.fftshift(f)
    magnitude_spectrum = 20 * np.log(np.abs(fshift))
    magnitude_spectrum = np.asarray(magnitude_spectrum, dtype=np.uint8)
    img_and_magnitude = np.concatenate((img, magnitude_spectrum), axis=1)

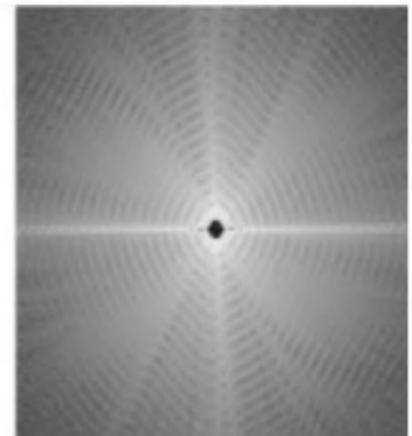
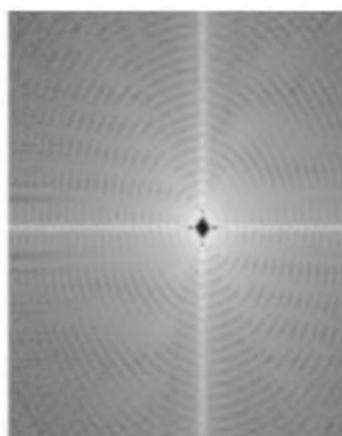
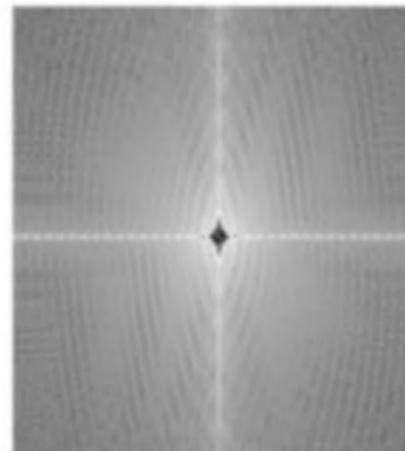
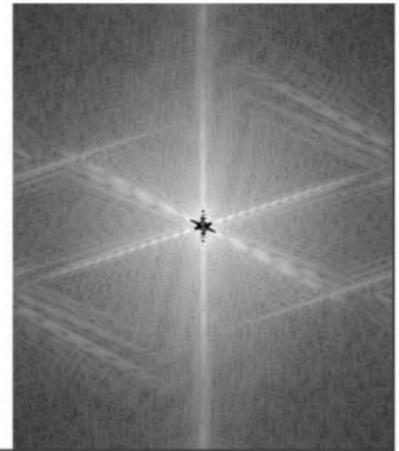
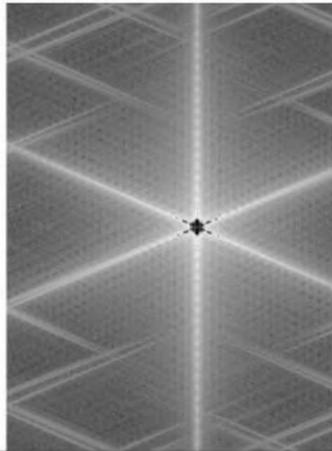
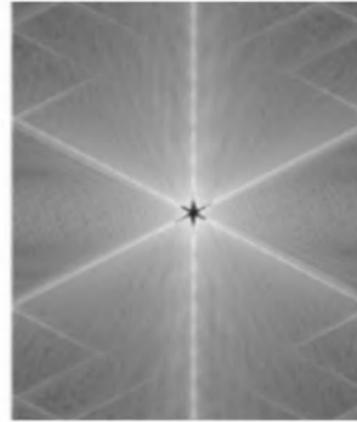
    cv2.imshow(image_title, img_and_magnitude)

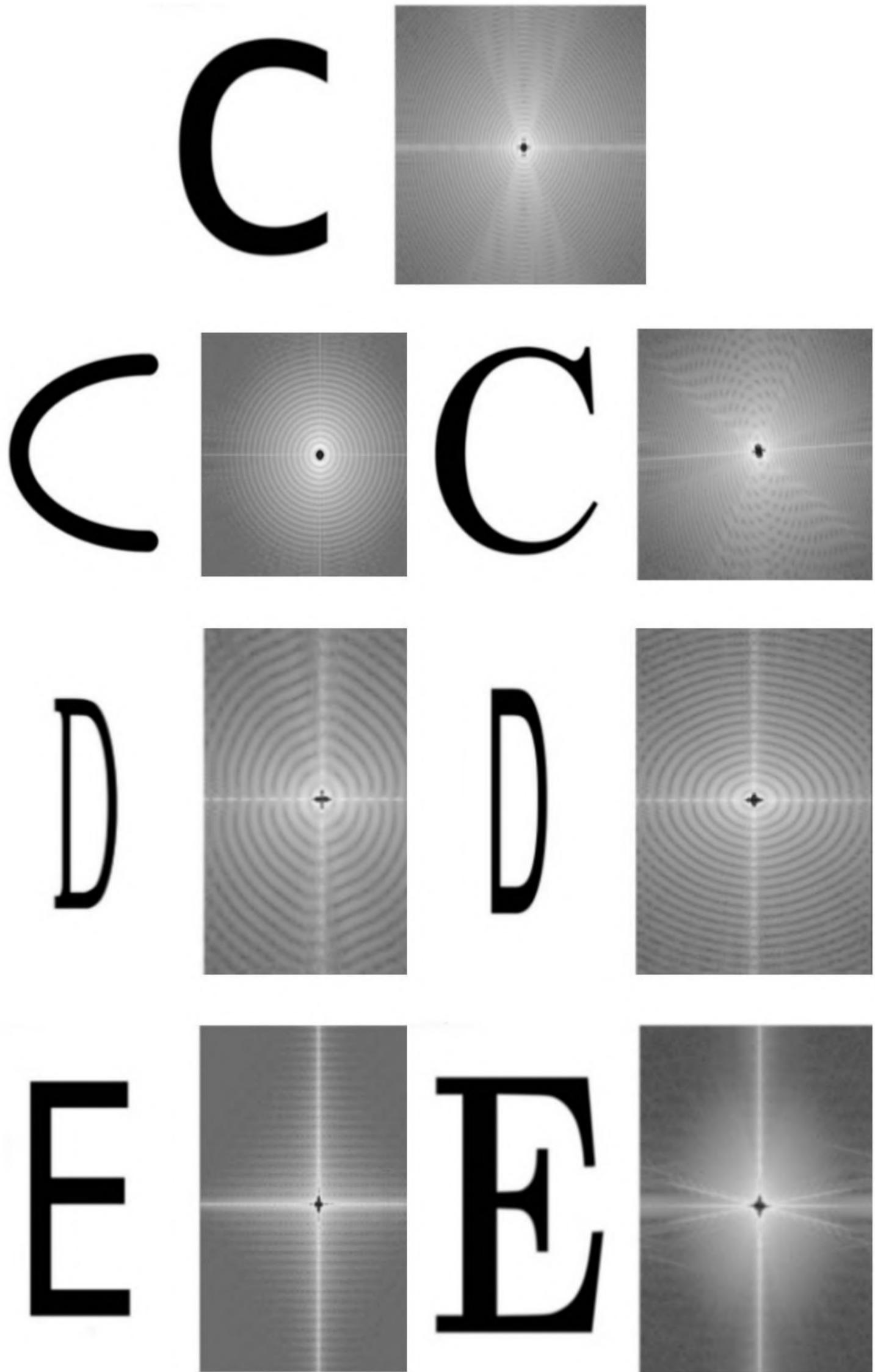
cv2.waitKey()
cv2.destroyAllWindows()
```

In the illustrated program, we created a folder containing different english alphabets written in capital letters in different fonts. We converted them into grayscale and evaluated their Fast Fourier Transform (FFT) and converted images from spatial domain to frequency domain.

Consider the letter A, whatever way you draw it, it has two upright lines and a crossbar at the bottom. When we transfer these to the frequency domain, they all end up looking similar no matter in which style we write them. Also they look definitely different from that of the other letters.

In a similar way, when we analysed different English alphabets we observed that the Fourier images of the same letter written in different fonts gave similar results and they look entirely different from other letters.





CONCLUSION

Fourier series is a way of representing periodic functions as the sum of infinite series of sines and cosines. This representation of periodic functions have wide range of applications in our daily life. It was discovered by a French mathematician Joseph Fourier while he was working on a particular heat equation. In the expansion of this series we make use of the orthogonality relationship of sines and cosines. Fourier series are powerful tools for problems involving functions that are periodic in nature.

As we know many of the functions that we come across in practical situations are not periodic in nature. Hence, Fourier transform has wide applications. It is an extension of Fourier series to non periodic functions. In this project we discussed various properties of Fourier transform and also about Discrete Fourier Transform (DFT) which is widely used in applications.

This project discussed two applications of Fourier series. They are Digital Signal Processing and Image Analysis. We analysed the use of Fourier series in both of these .

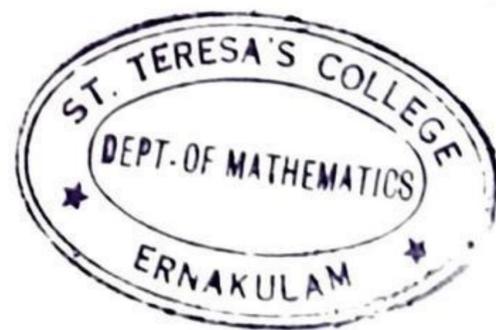
In the case of digital signal processing we used Fast Fourier Transform in Python programming to two sine waves and decomposed these waves into various frequencies that it was originally composed of. In this way we observed that it will be easy to isolate the undesirable frequencies from the original data.

In image analysis we analysed certain English alphabets written in different styles using Fast Fourier Transform (FFT) command in

Python programming and observed their output in frequency domain. Fourier transform images of each alphabets written in different styles showed similarities whereas Fourier image of one alphabet looks entirely different from others.

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**DETERMINATION OF CAUCHY'S CONSTANTS FOR DIFFERENT
LIQUIDS USING HOLLOW PRISM**

PROJECT REPORT

Submitted by

MERLIN DAINU

Register No: AB19PHY011

Under the guidance of

Smt. Susan Mathew, Assistant Professor

**Department of Physics and Centre for Research, St. Teresa's College (Autonomous),
Ernakulam Kochi, 682011**

Submitted to the

Mahatma Gandhi University, Kottayam

In partial fulfilment of the requirement for the award of

BACHELOR'S DEGREE OF SCIENCE IN PHYSICS



ST. TERESA'S COLLEGE (AUTONOMOUS)

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CERTIFICATE

This is to certify that the project report entitled “**DETERMINATION OF CAUCHY'S CONSTANTS FOR DIFFERENT LIQUIDS USING HOLLOW PRISM**” is an authentic work submitted by **MERLIN DAINU** under my supervision at Department of Physics, St. Teresa's College for the partial fulfilment of the award of Degree of Bachelor of Science in Physics during the academic year 2021-2022. The work presented in this dissertation has not been submitted for any other degree in this or any other university.

Supervising Guide

Susan

Smt. Susan Mathew

Assistant Professor

Place : Ernakulam

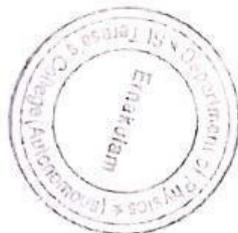
Date : 09/05/2022

Head of the Department

Priya

Dr. Priya Parvathi Ameena Jose

Assistant Professor



ST. TERESA'S COLLEGE (AUTONOMOUS)

ERNAKULAM



B.Sc PHYSICS

PROJECT REPORT

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Registration No. : AB19PHY011

Year of work : 2021-2022

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Supervising Guide

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Smt. Susan Mathew

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Assistant Professor

Submitted for the university examination held at St. Teresa's College, Ernakulam

Date : 09/05/2022

EXAMINERS:

Priya Parvathi Ameena Jose
Susan

DECLARATION

I, **MERLIN DAINU**, Register No: **AB19PHY011** hereby declare that this project work entitled **‘DETERMINATION OF CAUCHY’S CONSTANT FOR DIFFERENT LIQUID USING HOLLOW PRISM’** has been prepared by me during the academic year 2021-22 under the guidance of **Smt. Susan Mathew**, Department of Physics and Centre for Research, St. Teresa’s College, Ernakulam is in the partial fulfilment for the award of degree of bachelor of Physics under Mahatma Gandhi University. I further declare that the work reported in the project has not been submitted and will not be submitted, either in part or in full, for the award of any other degree or diploma in the institute or any other institute or university.

Place: Ernakulam

Date: 09. 05. 2022

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I am grateful to Almighty God for the good health and wellbeing that were necessary to complete this project.

With great pleasure we would like to thank our beloved guide Smt. Susan Mathew, Department of Physics, St. Teresa's College, for her immense support and encouragement.

Also, I wish to thank all the faculty members and supporting staff for helping me to complete the project.

Finally, we would like to thank my parents and friends who have helped me with their valuable suggestions and guidance and have been very helpful in various stages of project completion.

Merlin Dainu

ABSTRACT

The separation of visible light into its different colours is known as dispersion. This experiment is based on the phenomenon of normal dispersion. The angle of the prism A and the angle of minimum deviation D for different wavelength are determined. From this the refractive index n for these colours are calculated. Taking the value of wavelength λ , the Cauchy's constants A and B are calculated for different pairs of spectral colours using the equation. In this project we have determined the refractive indices of different transparent liquids using spectrometer-hollow prism method. Thus Cauchy's constant is determined.

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1. INTRODUCTION

Cauchy's dispersion equation shows an empirical relationship between the refractive index and wavelength of light for a particular transparent material. The equation is named after the mathematician Augustus-Louis Cauchy. He defined the equation in 1836. This equation was actually defined in his work on spectral theory that is 'Cauchy Memoire'.

The validity of the equation is only for the normal dispersion in the visible wavelength region. This equation is useful in some applications due to its mathematical simplicity. The Sellmeier Equation is a later development of Cauchy's work. The curve of the Cauchy's equation represents the visible region with considerable accuracy. A and B are the parameters of Cauchy's constant and depends on refractive index. Here A is a dimensionless parameter, when wavelength, $\lambda \rightarrow \infty$ then refractive index, $n \rightarrow A$. For the medium wavelength in the visible region, the curvature and amplitude of the refractive index is affected by B. So, the refractive index n is inversely proportional to the square of the light's wavelength.

When compared to the red light, the parallel light beams that incident on a lens bend towards the axis more as the wavelength of red is greater than that of violet hence, refractive index of a material for red light would be less than that of the violet light. The dispersion of the red light when compared to the violet light is also less. This is because the inverse proportionality of both wavelength and refractive index which is shown by Cauchy's equation.

2. PRINCIPLE

When an electromagnetic wave is incident on an atom or a molecule, the periodic electric force of the wave sets the bound charges into a vibratory motion. The frequency with which these charges are forced to vibrate is equal to the frequency of the wave. The phase of this motion as compared to the impressed electric force will depend on the impressed frequency. It will vary with the difference between the impressed frequency and the natural frequency of the charges.

Dispersion can be explained with the concept of secondary waves that are produced by the induced oscillations of the bound charges. When a beam of light propagates through a transparent medium (solid or liquid), the amount of lateral scattering is extremely small. The scattered waves travelling in a lateral direction produce destructive interference. However, the secondary waves travelling in the same direction as the incident beam superimpose on one another. The resultant vibration will depend on the phase difference between the primary and the secondary waves. This superimposition, changes the phase of the primary waves and this is equivalent to a change in the wave velocity.

Wave velocity is defined as the speed at which a condition of equal phases is propagated. Hence the variation in phase due to interference, changes the velocity of the wave through the medium. The phase of the oscillations and hence that of the secondary waves depends upon the impressed frequency. It is clear, therefore, that the velocity of light in the medium varies with the frequency of light. Also refractive index depends upon the velocity of light in the medium. Therefore, the refractive index of the medium varies with the frequency (wavelength) of light.

The relation permittivity of the medium in the case of dynamic polarizability is given by

$$\epsilon_r = 1 + X_e = 1 + \frac{Ne^2}{\epsilon_0 m} \sum_i \frac{f_i}{\omega_i^2 - \omega^2} \dots \dots \dots (1)$$

Here, N is the number of electrons per unit volume, e the charge and m is the mass of the electron, ϵ_0 permittivity of free space, f_i oscillator strengths of the substance, ω_i is the angular frequency of the electromagnetic spectrum of the substance, ω is the impressed angular frequency.

Also $\sum f_i = 1$ and relative permeability in majority of substances that transmit electromagnetic waves is equal to 1.

Therefore, $n_2 = \epsilon_r$

Assuming that there is only one atomic frequency ω_o where $\omega \ll \omega_o$

$$n_2 = 1 + \frac{Ne^2}{\epsilon_o m(\omega_i^2 - \omega^2)} \dots \dots \dots (2)$$

Using binomial expansion

$$\begin{aligned} n &= \left[1 + \frac{Ne^2}{\epsilon_o m(\omega_i^2 - \omega^2)} \right]^{\frac{1}{2}} \\ &= 1 + \frac{Ne^2}{2\epsilon_o m(\omega_i^2 - \omega^2)} \\ &= 1 + \left(\frac{Ne^2}{2\epsilon_o m\omega_o^2} \right) \left(1 - \frac{\omega^2}{\omega_o^2} \right)^{-1} \\ &= 1 + \left(\frac{Ne^2}{2\epsilon_o m\omega_o^2} \right) \left(1 + \frac{\omega^2}{\omega_o^2} \right) \end{aligned}$$

As

$$\omega = \frac{2\pi c}{\lambda} \quad \text{and} \quad \omega_o = \frac{2\pi c}{\lambda_o}$$

$$n = 1 + \frac{Ne^2\lambda_o^2}{8\pi^2\epsilon_o mc^2} + \frac{Ne^2\lambda_o^4}{8\pi^2\epsilon_o mc^2\lambda^2} \dots \dots \dots (3)$$

Taking $1 + \frac{Ne^2\lambda_o^2}{8\pi^2\epsilon_o mc^2} = A \dots \dots \dots (4)$

and $\frac{Ne^2\lambda_o^4}{8\pi^2\epsilon_o mc^2} = B \dots \dots \dots (5)$

$$n = A + \frac{B}{\lambda^2} \dots \dots \dots (6)$$

Equation (6) represents Cauchy's dispersion formula. It is evident that the refractive index of the medium decreases with increase in wavelength of light.

If a graph is plotted between μ and $1/\lambda^2$ it will be a straight line. The intercept on the Y-axis gives value A. the slope of the line PC gives value of B.

A and B are called Cauchy's constants. The values of A and B depend on the medium. A is a dimensionless parameter, when $\lambda \rightarrow \infty$ then $n \rightarrow A$.

B has a unit of nm^2 . It affects the curvature of the amplitude of the refractive index for medium of wavelength in the visible region. The wavelength is of the order of a few hundred nanometers so, we can omit the higher powers of the wavelength since the coefficients associated with them are very small.

If the refractive indices n_1 and n_2 for any two known wavelength λ_1 and λ_2 are determined by a spectrometer, the Cauchy's constants A and B can be calculated from the above equation as,

$$n_1 = A + \frac{B}{\lambda_1^2} \dots\dots\dots(1)$$

$$n_2 = A + \frac{B}{\lambda_2^2} \dots\dots\dots(2)$$

On solving, (1) and (2) and rearranging

$$A = n_1 - \frac{B}{\lambda_1^2}$$

And
$$B = \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2} \quad \text{where } n_2 > n_1$$

Refractive indices n_1 and n_2 are calculated using spectrometer.

3. EXPERIMENTAL SETUP

Apparatus and Instrument Used:

Spectrometer

Telescope and collimator are the parts of spectrometer. The three levelling screws on the prism table are engraved with concentric rings and lines. In order to lock the telescope and collimator, clamping devices are provided after the adjustments. For the measurement in spectrometer, a Vernier scale is used.

Adjustments of Spectrometer (Preliminary)

1. Telescope

Telescope is focused on a distant object. In order to get the clear image of the distant object by coinciding with the crosswire without parallax, the length of the telescope is varied by rack and pinion arrangement. So now the collimator is ready to produce parallel rays.

2. Collimator

Bring the collimator in line with the telescope and get the image in the telescope only after the slit is illuminated with the light. It must be noted that the slit is to be sufficiently wide so that the crosswire coincide with image without parallax and the collimator is also ready to produce parallel rays.

3. Prism Table

Here, the hollow prism ABC is placed on the table. It is important that the base must be turned towards the clamp. One reflecting face AC must be arranged in such a way that it is perpendicular to the line joining two screws S_1 and S_2 . Now the table is rotated in a way that the edge A is pointing towards the collimator. From the first face AB, we can see the reflected image of the slit in order to coincide the image with the crosswire; S_1 and S_2 are worked as needed. Now by working the screw S_3 the image from the other face AC is observed through the telescope that is made symmetrical. Now the prism table is levelled.

Hollow Prism

For finding the Cauchy's constant, we use a hollow prism. We use it by filling a particular transparent liquid in it.

Transparent Liquids

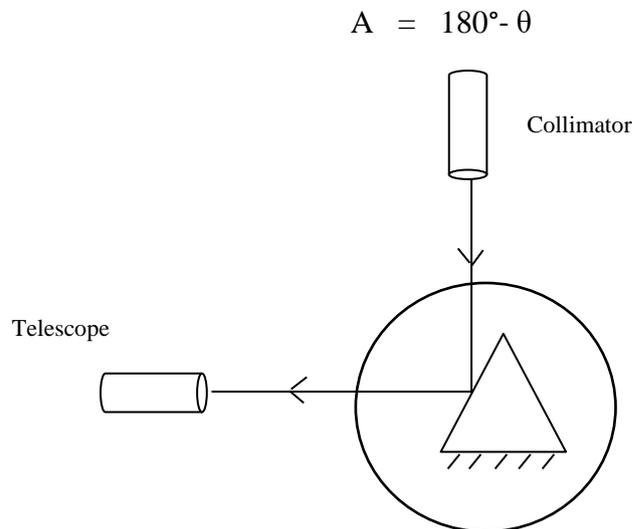
The transparent liquids that we use here for the experiment are

- Water
- Rubbing alcohol
- Vinegar
- Sanitizer
- Glycerine

4. PROCEDURE

Angle of Prism:

After the preliminary adjustments, telescope is brought in perpendicular direction to the collimator and clamped. Now the reflected image of the slit is obtained on the face AB of the prism just by rotating the Vernier table. This is recorded through a telescope and the readings of both verniers are noted. Similarly, the Vernier table is unclamped and the reflected image from the other face AC is obtained through a telescope and its Vernier readings are noted. θ is obtained by taking the mean of the difference between the corresponding vernier readings. θ is the angle of which the table has been rotated. Now the angle of prism A is found out by supplementary method,



Angle of Minimum Deviation (D_m):

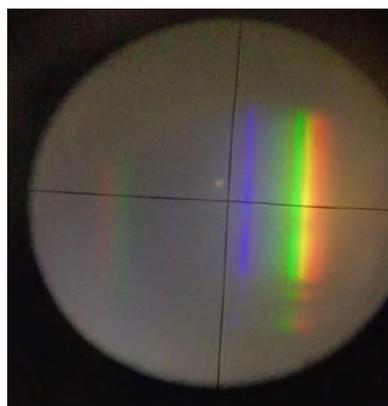
In this case a liquid prism is required to find the angle of minimum deviation. Let say water. Now the Vernier table has to be unclamped and rotated so that the light from the collimator falls obliquely on one face of the prism. The refracted image is seen through it by rotating the telescope. Image should be obtained towards the direct ray position so the Vernier table is slowly rotated in required direction. The telescope is also rotated in the same direction so that the image is always in the field of view. The Vernier table is rotated up to the moment where the refracted image of a colour, let say green is found to be static for a moment then retrace and this is the minimum deviation position for that colour.



Now the Vernier table and telescope are clamped. The tangential screw of the telescope is adjusted in a way that the vertical crosswire coincide with the centre of the image.



Corresponding readings from the main scale and Vernier are obtained. Now to find the direct image of the slit, the prism is removed and the telescope is brought in line with the collimator and the direct reading is taken. The angle of minimum deviation for this particular colour (green) is found out by taking the difference of these readings.



The retraced image is constituted by certain discrete lines of varying colours. To obtain these varying colours, the prism is successfully adjusted to be in the minimum deviation position and in each case the angle of minimum deviation is determined

Repeat the experiment with different liquid for finding out the angle of minimum deviation in each case. The refractive index of each colour is determined by calculating

$$n = \frac{\sin \left(\frac{A + D}{2} \right)}{\sin \frac{A}{2}}$$

where, D is the minimum angle of deviation, A is the angle of prism and n is the refractive index.

Now for finding the Cauchy's constant, we use the equation:

$$n = A + \frac{B}{\lambda^2}$$

where A and B are the Cauchy's constants, λ is the wavelength of the particular colour and n is the refractive index. Consider any two colours of wavelength λ_1 and λ_2 with calculated refractive index n_1 and n_2 (where $n_2 > n_1$) respectively for a particular liquid and apply the rearranged form of the above equation given by,

$$B = \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2}$$

And

$$A = n_1 - \frac{B}{\lambda_1^2}$$

A graph can be plotted by taking refractive index along Y-axis and $1/\lambda^2$ along X axis, straight line is obtained. The slope of straight lines directly gives the Cauchy's coefficient B while the Y intercept gives the other coefficient A.

5. OBSERVATIONS AND CALCULATIONS

Angle of prism

Value of one main scale division = 30'

No. of divisions on vernier scale = 30

READINGS OF	VERNIER 1			VERNIER 2		
	MSR (deg)	VSR (div)	TOTAL READING= MSR +VSR(LC) (deg)	MSR (deg)	VSR (div)	TOTAL READING= MSR +VSR(LC) (deg)
REFLECTED RAYS FROM 1st FACE (a)	291°	5	291°5'	110°30'	28	110°58'
REFLECTED RAYS FROM 2nd FACE (b)	171°30'	15	171°45'	351°	14	351°14'
DIFFERENCE BETWEEN (a) AND (b)	119°20'			119°44'		

$$\text{Least count (LC)} = \frac{\text{Value of one main scale division}}{\text{No. of divisions on vernier scale}} = \frac{30'}{30} = 1'$$

$$\begin{aligned} \text{Total reading} &= \text{MSR} + (\text{VSR} \times \text{LC}) \\ &= 291^\circ + (5 \times 1') \\ &= 291^\circ 5' \end{aligned}$$

$$\text{Mean } \theta = 119^\circ 32'$$

$$\begin{aligned} \text{Angle of prism A} &= 180^\circ - \theta \\ &= 180^\circ - 119^\circ 32' \\ &= 60^\circ 28' \end{aligned}$$

ANGLE OF MINIMUM DEVIATION FOR DIFFERENT COLORS OF WATER

COLOUR	VER-NIER	REFRACTED RAY READING			DIRECT RAY READING			DIFFERENCE D = X-Y (deg)	MEAN D (deg)	$n = \frac{\sin(\frac{A+D}{2})}{\sin \frac{A}{2}}$
		MSR (deg)	VSR (div)	TOTAL READING = MSR +VSR(LC) (deg)	MSR (deg)	VSR (div)	TOTAL READING = MSR +(VSR x LC) (deg)			
VIOLET	1	348°30'	22	348°52'	13°	24	13°24'	24°32'	24°34'	1.3421
	2	169°	11	169°11'	193°30'	17	193°47'	24°36'		
BLUE	1	349°	7	349°7'	13°	24	13°24'	24°17'	24°19'30"	1.339
	2	169°	25	169°25'	193°30'	17	193°47'	24°22'		
GREEN	1	349°30'	1	349°31'	13°	24	13°24'	23°53'	23°55'	1.3338
	2	169°30'	20	169°50'	193°30'	17	193°47'	23°57'		
YELLOW	1	349°30'	6	349°36'	13°	24	13°24'	23°48'	23°50'	1.3327
	2	169°30'	25	169°55'	193°30'	17	193°47'	23°52'		

For violet:

$$\begin{aligned}
 \text{Total reading} &= \text{MSR} + (\text{VSR} \times \text{LC}) \\
 &= 348^\circ 30' + (22 \times 1') \\
 &= 348^\circ 52'
 \end{aligned}$$

$$n = \frac{\sin \frac{(A+D)}{2}}{\sin \frac{A}{2}} = \frac{\sin \sin \frac{(60^\circ 28' + 24^\circ 34')}{2}}{\sin \sin \frac{60^\circ 28'}{2}} = 1.3421$$

CALCULATION OF CAUCHY'S CONSTANT FOR WATER

(i) FROM TABLE:

For green (λ_2) and yellow (λ_1)

$$\lambda_1 = 576.96 \text{ nm}$$

$$\lambda_2 = 546.07 \text{ nm}$$

From the table:

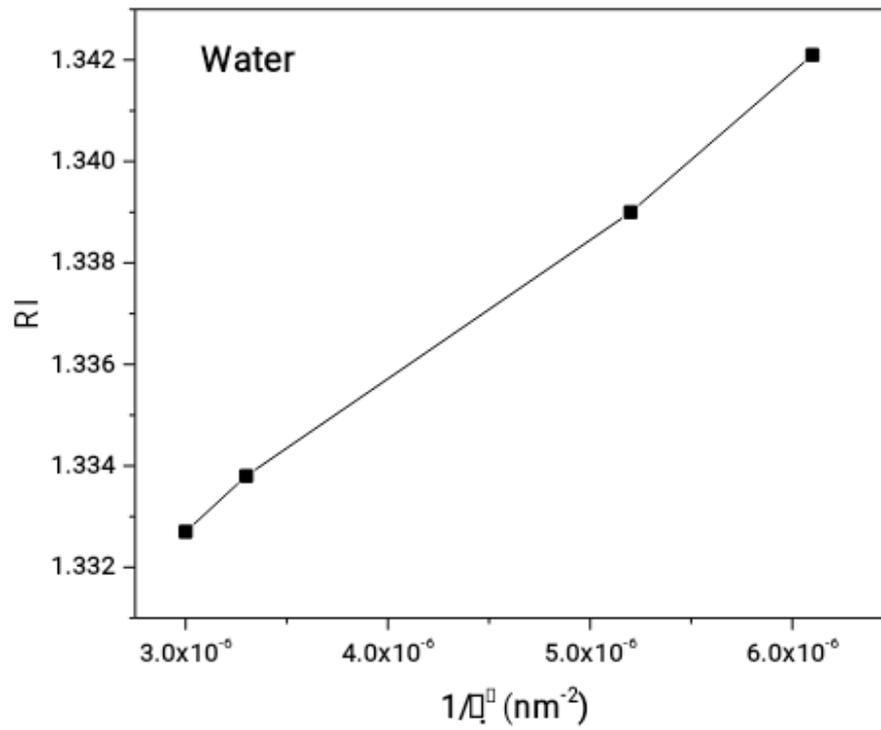
$$n_1 = 1.3327$$

$$n_2 = 1.3338$$

$$\begin{aligned} B &= \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2} \\ &= \frac{(576.96^2)(546.07^2)(1.3338 - 1.3327)}{576.96^2 - 546.07^2} \\ &= \mathbf{3147.541 \text{ nm}^2} \end{aligned}$$

$$\begin{aligned} A &= n_1 - \frac{B}{\lambda_1^2} \\ &= 1.3327 - \frac{0.00314}{576.96^2} \\ &= \mathbf{1.3332} \end{aligned}$$

(ii) FROM GRAPH:



A = 1.32389

B = 2956.716 nm²

ANGLE OF MINIMUM DEVIATION FOR DIFFERENT COLORS OF ALCOHOL

COLOUR	VER-NIER	REFRACTED RAY READING			DIRECT RAY READING			DIFFERENCE D = X-Y (deg)	MEAN D (deg)	$n = \frac{\sin(\frac{A+D}{2})}{\sin \frac{A}{2}}$
		MSR (deg)	VSR (div)	TOTAL READING = MSR +VSR(LC) (deg)	MSR (deg)	VSR (div)	TOTAL READING = MSR +(VSR x LC) (deg)			
VIOLET	1	340°30'	20	340°50'	7°30'	19	7°49'	26°59'	26°57'	1.3723
	2	161°	16	161°16'	188°	11	188°11'	26°55'		
BLUE	1	341°	4	341°4'	7°30'	19	7°49'	26°45'	26°45'30"	1.3699
	2	161°	25	161°25'	188°	11	188°11'	26°46'		
GREEN	1	341°30'	4	341°34'	7°30'	19	7°49'	26°15'	26°15'30"	1.3636
	2	161°30'	25	161°55'	188°	11	188°11'	26°16'		
YELLOW	1	341°30'	9	341°39'	7°30'	19	7°49'	26°10'	26°10'	1.3624
	2	162°	1	162°1'	188°	11	188°11'	26°10'		

For violet:

$$\begin{aligned}
 \text{Total reading} &= \text{MSR} + (\text{VSR} \times \text{LC}) \\
 &= 340^\circ 30' + (20 \times 1') \\
 &= 340^\circ 50'
 \end{aligned}$$

$$n = \frac{\sin \frac{(A+D)}{2}}{\sin \frac{A}{2}} = \frac{\sin \sin \frac{(60^\circ 28' + 26^\circ 57')}{2}}{\sin \sin \frac{60^\circ 28'}{2}} = 1.3723$$

CALCULATION OF CAUCHY'S CONSTANT FOR ALCOHOL

(i) FROM TABLE:

For violet (λ_2) and yellow (λ_1)

$$\lambda_1 = 576.96 \text{ nm}$$

$$\lambda_2 = 404.65 \text{ nm}$$

From the table:

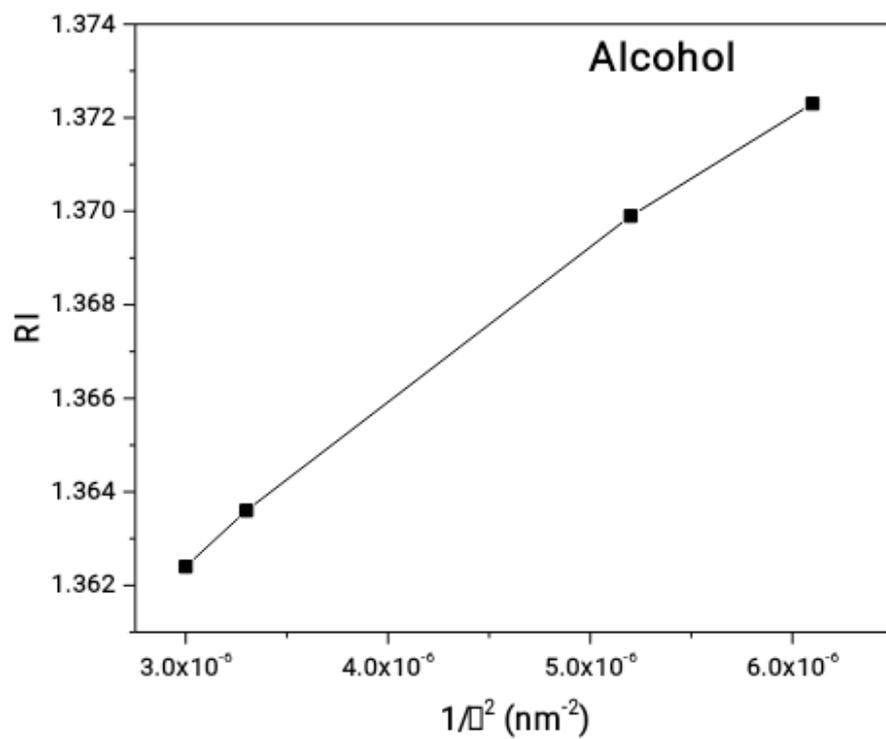
$$n_1 = 1.3624$$

$$n_2 = 1.3723$$

$$\begin{aligned} B &= \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2} \\ &= \frac{(576.96^2)(404.65^2)(1.3723 - 1.3624)}{576.96^2 - 404.65^2} \\ &= \mathbf{3190.334 \text{ nm}^2} \end{aligned}$$

$$\begin{aligned} A &= n_1 - \frac{B}{\lambda_1^2} \\ &= 1.3624 - \frac{0.00319}{576.96^2} \\ &= \mathbf{1.3623} \end{aligned}$$

(ii) FROM GRAPH:



A = 1.35292

B = 3210.448 nm^2

ANGLE OF MINIMUM DEVIATION FOR DIFFERENT COLORS OF VINEGAR

COLOUR	VER-NIER	REFRACTED RAY READING			DIRECT RAY READING			DIFFERENCE D = X-Y (deg)	MEAN D (deg)	$n = \frac{\sin(\frac{A+D}{2})}{\sin \frac{A}{2}}$
		MSR (deg)	VSR (div)	TOTAL READING = MSR +VSR(LC) (deg)	MSR (deg)	VSR (div)	TOTAL READING = MSR +(VSR x LC) (deg)			
VIOLET	1	342°30'	13	342°43'	7°30'	1	7°31'	24°48'	24°48'30"	1.3452
	2	163°	4	163°4'	187°30'	23	187°53'	24°49'		
BLUE	1	342°30'	24	342°54'	7°30'	1	7°31'	24°37'	24°37'30"	1.3429
	2	163°	15	163°15'	187°30'	23	187°53'	24°38'		
GREEN	1	343°	23	343°23'	7°30'	1	7°31'	24°8'	24°10'	1.337
	2	163°30'	11	163°41'	187°30'	23	187°53'	24°12'		
YELLOW	1	343°	28	343°28'	7°30'	1	7°31'	24°3'	24°5'	1.3359
	2	163°30'	16	163°46'	187°30'	23	187°53'	24°7'		

For violet:

$$\begin{aligned}
 \text{Total reading} &= \text{MSR} + (\text{VSR} \times \text{LC}) \\
 &= 342^\circ 30' + (13 \times 1') \\
 &= 342^\circ 43'
 \end{aligned}$$

$$n = \frac{\sin \frac{(A+D)}{2}}{\sin \frac{A}{2}} = \frac{\sin \frac{(60^\circ 28' + 24^\circ 48' 30'')}{2}}{\sin \frac{60^\circ 28'}{2}} = 1.3452$$

CALCULATION OF CAUCHY'S CONSTANT FOR VINEGAR

(i) FROM TABLE:

For blue (λ_2) and green (λ_1)

$$\lambda_1 = 546.07 \text{ nm}$$

$$\lambda_2 = 435.83 \text{ nm}$$

From the table:

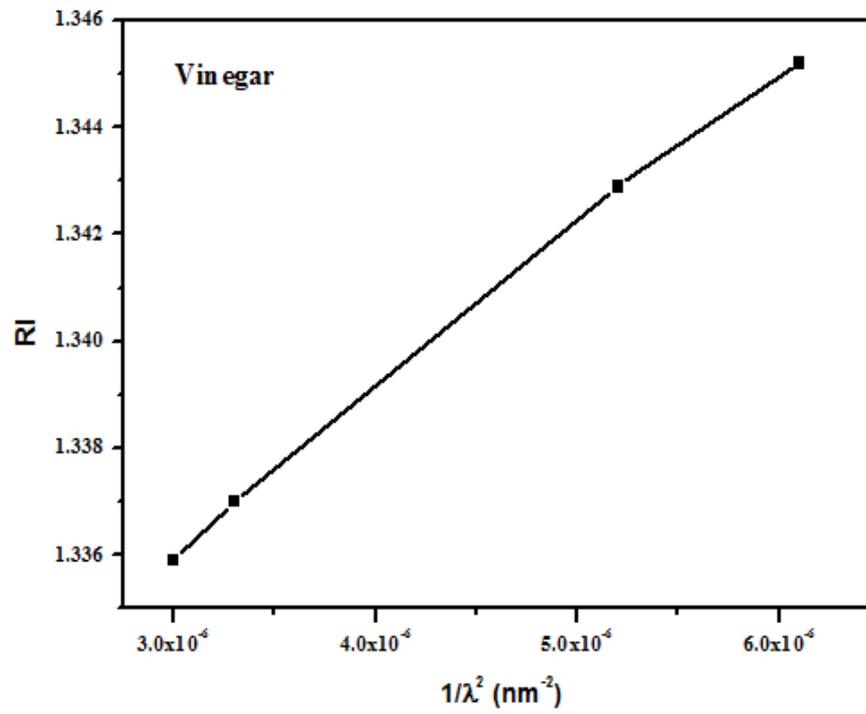
$$n_1 = 1.337$$

$$n_2 = 1.3429$$

$$\begin{aligned} B &= \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2} \\ &= \frac{(546.07^2)(435.83^2)(1.3429 - 1.337)}{546.07^2 - 435.83^2} \\ &= \mathbf{3087.283 \text{ nm}^2} \end{aligned}$$

$$\begin{aligned} A &= n_1 - \frac{B}{\lambda_1^2} \\ &= 1.337 - \frac{0.00308}{546.07^2} \\ &= \mathbf{1.3369} \end{aligned}$$

(ii) FROM GRAPH:



A = 1.32698

B = 3014.925 nm²

ANGLE OF MINIMUM DEVIATION FOR DIFFERENT COLORS OF SANITIZER

COLOUR	VER-NIER	REFRACTED RAY READING			DIRECT RAY READING			DIFFERENCE D = X-Y (deg)	MEAN D (deg)	$n = \frac{\sin(\frac{A+D}{2})}{\sin \frac{A}{2}}$
		MSR (deg)	VSR (div)	TOTAL READING = MSR +VSR(LC) (deg)	MSR (deg)	VSR (div)	TOTAL READING = MSR +(VSR x LC) (deg)			
VIOLET	1	343°	1	343°1'	8°30'	9	8°39'	25°38'	25°40'30"	1.3562
	2	163°	19	163°19'	189°	2	189°2'	25°43'		
BLUE	1	343°	5	343°5'	8°30'	9	8°39'	25°34'	25°32'30"	1.3545
	2	163°30'	1	163°31'	189°	2	189°2'	25°31'		
GREEN	1	343°30'	8	343°38'	8°30'	9	8°39'	25°1'	25°1'	1.3478
	2	164°	1	164°1'	189°	2	189°2'	25°1'		
YELLOW	1	343°30'	15	343°45'	8°30'	9	8°39'	24°54'	24°55'30"	1.3467
	2	164°	5	164°5'	189°	2	189°2'	24°57'		

For violet:

$$\begin{aligned}
 \text{Total reading} &= \text{MSR} + (\text{VSR} \times \text{LC}) \\
 &= 343^\circ + (1 \times 1') \\
 &= 343^\circ 1'
 \end{aligned}$$

$$n = \frac{\sin \frac{(A+D)}{2}}{\sin \frac{A}{2}} = \frac{\sin \sin \frac{(60^\circ 28' + 25^\circ 40' 30'')}{2}}{\sin \sin \frac{60^\circ 28'}{2}} = 1.3562$$

CALCULATION OF CAUCHY'S CONSTANT FOR SANITIZER

(i) FROM TABLE

For blue (λ_2) and yellow (λ_1)

$$\lambda_1 = 576.96 \text{ nm}$$

$$\lambda_2 = 435.83 \text{ nm}$$

From the table:

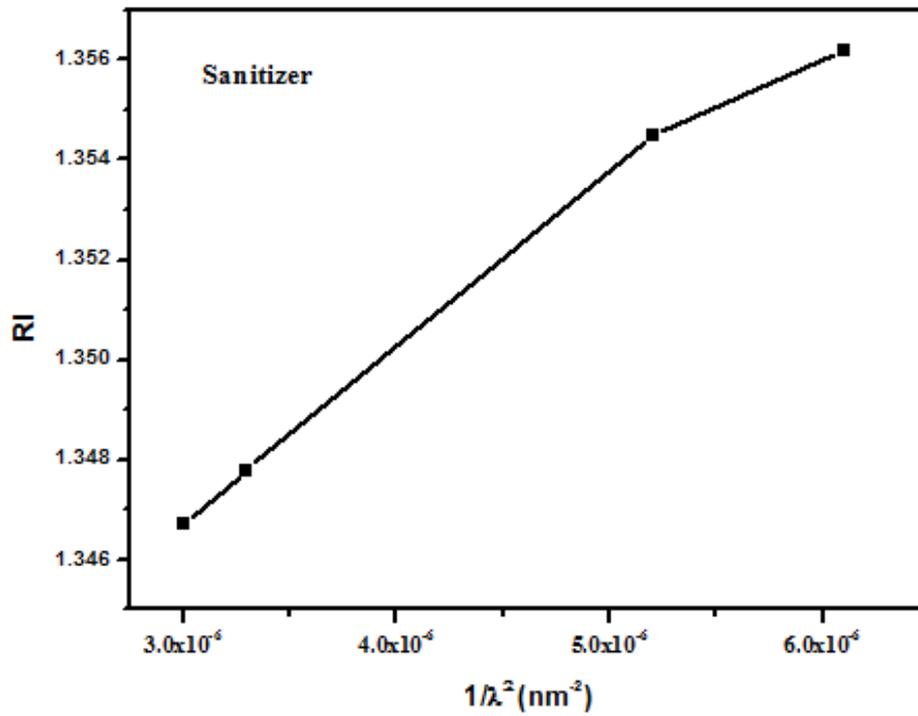
$$n_1 = 1.3467$$

$$n_2 = 1.3545$$

$$\begin{aligned} B &= \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2} \\ &= \frac{(576.96^2)(435.83^2)(1.3545 - 1.3467)}{576.96^2 - 435.83^2} \\ &= \mathbf{3450.415 \text{ nm}^2} \end{aligned}$$

$$\begin{aligned} A &= n_1 - \frac{B}{\lambda_1^2} \\ &= 1.3467 - \frac{0.00345}{576.96^2} \\ &= \mathbf{1.3466} \end{aligned}$$

(ii) FROM GRAPH:



A = 1.33739

B = 3161.194 nm²

**ANGLE OF MINIMUM DEVIATION FOR DIFFERENT COLORS OF
GLYCERINE**

COLOUR	VER-NIER	REFRACTED RAY READING			DIRECT RAY READING			DIFFERENCE D = X-Y (deg)	MEAN D (deg)	$n = \frac{\sin(\frac{A+D}{2})}{\sin\frac{A}{2}}$
		MSR (deg)	VSR (div)	TOTAL READING = MSR +VSR(LC) (deg)	MSR (deg)	VSR (div)	TOTAL READING = MSR +(VSR x LC) (deg)			
BLUE	1	334°	28	334°28'	8°30'	6	8°36'	34°8'	34°12'	1.4603
	2	154°30'	15	169°11'	189°	1	189°1'	34°16'		
GREEN	1	335°	2	335°2'	8°30'	6	8°36'	33°34'	33°38'	1.4536
	2	155°	19	155°19'	189°	1	189°1'	33°43'		
YELLOW	1	335°	10	335°10'	8°30'	6	8°36'	33°26'	33°28'	1.4517
	2	155°30'	1	155°31'	189°	1	189°1'	33°30'		
RED	1	335°	19	335°19'	8°30'	6	8°36'	33°17'	33°20'30"	1.4502
	2	155°30'	7	155°37'	189°	1	189°1'	33°24'		

For blue:

$$\begin{aligned}
 \text{Total reading} &= \text{MSR} + (\text{VSR} \times \text{LC}) \\
 &= 334^\circ + (28 \times 1') \\
 &= 334^\circ 28'
 \end{aligned}$$

$$n = \frac{\sin \frac{(A+D)}{2}}{\sin \frac{A}{2}} = \frac{\sin \sin \frac{(60^\circ 28' + 34^\circ 12')}{2}}{\sin \sin \frac{60^\circ 28'}{2}} = 1.4603$$

CALCULATION OF CAUCHY'S CONSTANT FOR GLYCERINE

(ii) FROM GRAPH

For blue (λ_2) and green (λ_1)

$$\lambda_1 = 546.07 \text{ nm}$$

$$\lambda_2 = 435.83 \text{ nm}$$

From the table:

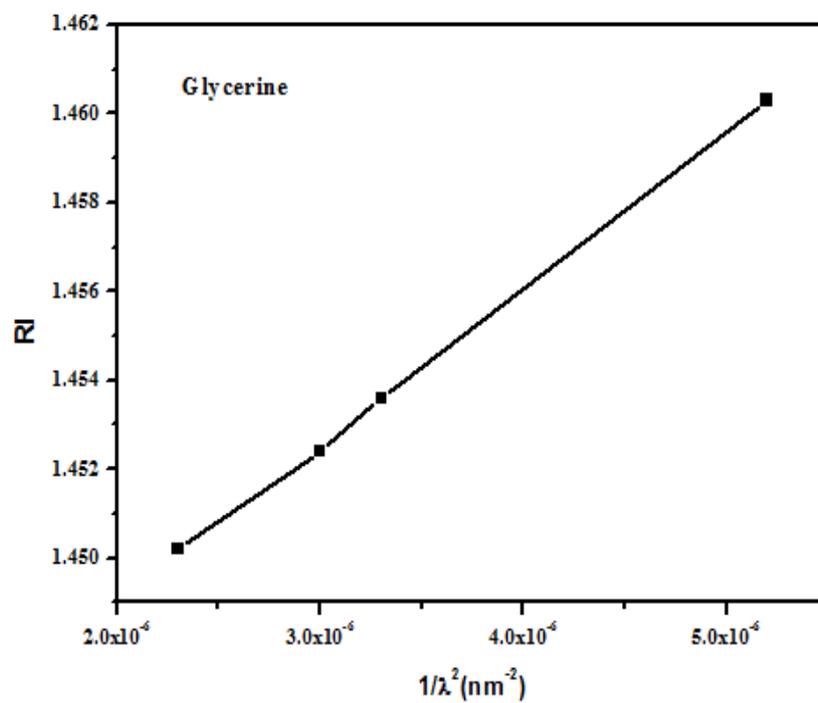
$$n_1 = 1.4536$$

$$n_2 = 1.4603$$

$$\begin{aligned} B &= \frac{\lambda_1^2 \lambda_2^2 (n_2 - n_1)}{\lambda_1^2 - \lambda_2^2} \\ &= \frac{(546.07^2)(435.83^2)(1.4603 - 1.4536)}{546.07^2 - 435.83^2} \\ &= \mathbf{3500.897 \text{ nm}^2} \end{aligned}$$

$$\begin{aligned} A &= n_1 - \frac{B}{\lambda_1^2} \\ &= 1.4536 - \frac{0.0035}{546.07^2} \\ &= \mathbf{1.4535} \end{aligned}$$

(ii) FROM GRAPH:



$$A = 1.44202$$

$$B = 3508.676 \text{ nm}^2$$

6. RESULT AND CONCLUSION

The Cauchy's constant for the given liquids are found to be as follows:

Liquid	From Calculation		From Graph	
	A	B (nm ²)	A	B (nm ²)
Water	1.3332	3147.541	1.3238	2956.716
Alcohol	1.3623	3190.334	1.3529	3210.448
Vinegar	1.3369	3087.283	1.3269	3014.925
Sanitizer	1.3544	3450.415	1.3373	3161.194
Glycerine	1.4535	3500.897	1.4420	3508.676

From Cauchy's equation it is evident that the refractive index of the medium decreases with increase in wavelength.

7. APPLICATIONS

1. Cauchy's equation represents the curve in the visible region with considerable accuracy.
2. Precise determination of refractive index of most popular environmental pollutant gases.

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ROLE OF CONSUMER ATTITUDE TOWARDS SUSTAINABLE FASHION CONSUMPTION

Project Report

Submitted by

NEHA RAJESH

(Reg. No. SB19BMS017)

Under the guidance of

Mrs. Megha Mary Michael

**In partial fulfillment of the requirements for award of the degree of
Bachelor of Management Studies (International Business)**



St. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM

COLLEGE WITH POTENTIAL FOR EXCELLENCE

Nationally Re-Accredited at 'A⁺⁺' Level (NAAC Fourth Cycle)

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Valued by : *[Signature]*
09/05/22
Dr. S. Usha A.A

ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM, KOCHI – 682011

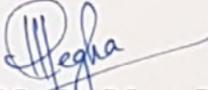


CERTIFICATE

This is to certify that the project entitled "Role of consumer attitude towards sustainable fashion consumption", has been successfully completed by Ms. Neha Rajesh, Reg. No.SB19BMS017, in partial fulfillment of the requirements for the award of degree of Bachelor of Management Studies in International Business, under my guidance during the academic years 2019-2022.

Date: 9/05/2022




Mrs. Megha Mary Michael
INTERNAL FACULTY GUIDE

DECLARATION

I, Neha Rajesh, Reg. No.SB19BMS017, hereby declare that this project work entitled "Role of consumer attitude towards sustainable fashion consumption" is my original work.

I further declare that this report is based on the information collected by me and has not previously been submitted to any other university or academic body.

Date: 9/5/2022



NEHA RAJESH
Reg. No.SB19BMS017

ACKNOWLEDGEMENT

I would like to place on Project Report my debt of gratitude to those who helped me in the preparation of this project.

I thank Dr Lizzy Mathew, Principal and Sr. Emeline, Director, St. Teresa's College Ernakulam for permitting me to take up this opportunity of doing an in-depth study on role of consumer attitude towards sustainable fashion consumption

I take this opportunity to express my deep sense of gratitude and whole hearted thanks to Mrs. Megha Mary Michael who is also the HOD of the department of Management Studies for guiding me in all stages of this project, without whom this project would have been a distant reality.

Last but not the least; I extend my heartfelt thanks to my family and friends for their valuable and proficient guidance and enormous support bestowed during the tenure of this exertion.

NEHA RAJESH

EXECUTIVE SUMMARY

The researcher is presenting this report on role of consumer attitude towards sustainable fashion consumption

Starts with the rationale behind preparing the report, objective, Industry analysis which shows the fashion market in global and national level.

All this research is then followed by a survey which reveals some facts about the consumer attitude, sustainable fashion and customers attitude while purchasing products

At last, the researcher have concluded the report with their interpretation of the whole study and the information collected is presented with the help of charts and diagrams.

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Chapter 1
Introduction to study

1.1 Introduction to study

Susan B. Kaiser, a fashion expert, claims that everyone is "forced to seem" unmediated in front of others. Fashion is a means of self-expression. It refers not only to your wardrobe, but also to how you present yourself to others. Fashion is an expression of one's individual taste and style, as well as an expression of one's individuality.

The Brundtland Commission of the United Nations defined sustainability in 1987 as "filling current demands without jeopardising future generations' ability to satisfy their own needs." The concept of sustainability has been discussed for nearly three decades and has influenced numerous sectors throughout the world. It has had an impact on the way different businesses and industries work as well as their output. The fashion business is one such industry that has recently been affected. Fashion generates nearly 1 trillion dollars in annual income. It is one of the most resource-intensive industries on the planet.

Sustainable fashion is a movement and a process that aims to improve the environmental balance and social justice of fashion goods and the fashion sector. More than merely addressing fashion textiles or goods, sustainable fashion is about meeting today's requirements while also guaranteeing that we can fulfill these needs in the future. One of the biggest causes of environmental degradation is the fashion industry. As a result, sustainable fashion has become a popular trend for both environmental preservation and keeping up with the latest fashion trends.

A trained inclination to behave in a consistently favourable or unfavourable manner with respect to a specific object is referred to as consumer attitude. Thoughts or beliefs, feelings, and behaviours or intentions toward a specific thing, which in this case is usually a good or service, make up attitude. Attitudes are often recognised as one of the most influential factors in purchasing decisions.

This research is to study how different factors determine the attitude of consumers towards sustainability and how that affects their buying behaviour. This research aims to study individuals from different age category and gender. And also, who have an international exposure compared to someone who has not and how these factors affect their attitude.

1.2 Statement of problem

Consumer attitude is a mixture of a consumer's belief, feelings and intentions towards a product or service. It differs with age, gender and occupation. A consumer's attitude is defined from their surroundings. The researcher aims at studying how consumer attitude affects the purchase behaviour towards sustainability within the fashion industry.

1.3 Literature review

1.3.1 Sustainability

WCED, 1987(a: 43), defines Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. Robert Goodland (1995), defined Environment sustainability as natural capital must be maintained both as provider of inputs and as a "sink" for wastes, this means environment sustainability needs sustainable production and sustainable consumption. Bourland (2011); Pookulangara & Shephard (2013), Sustainable fashion as part of the slow fashion movement is often misleadingly described as the opposite of fast fashion. Slow fashion is based on a philosophical ideal that centres on sustainability values, such as good working conditions and reducing environmental destruction. Clark (2008), It challenges the fast fashion paradigm by breaking down existing boundaries between the organisation and its stakeholders, slowing the production process to a more manageable timeframe, moving away from the selfconcept, and focusing on empowering workers by offering a choice that enables change.

1.3.2 Consumer attitude

Attitude is derived from the Latin words for posture or physical position. Wilkie, (1986), put forward that the general notion was that a body's physical attitudes suggested the types of activity or action in which a person would engage. In the words of Allport (1935), an attitude is "a mental and neural state of readiness, organized through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related". Krech et al (1962), define an attitude as a person's enduring favorable or unfavorable evaluations, emotional feelings, and action tendencies toward

some object or idea. Bearden et al (1995) and Kotler (2004) observes that people have attitudes toward almost everything: religion, politics, food, music, clothes, and others. Asiegbu et al (2012), attitudes put them into a frame of mind of liking or disliking an object, moving toward or away from it. Attitudes are an expression of inner feelings that reflect whether a person is favorably or unfavorably predisposed to some 'stimulus' or 'object' (e.g., a restaurant, a brand, a service, a retail establishment).

1.3.3 Antecedents for sustainable fashion consumption

Joergens (2006) notes that consumers have limited choice in sustainable clothing, as the prices are not comparable to the low-cost fashion available to them. Barr, Gilg, and Ford (2001), found out that recycling has received the most attention, primarily because it seems to be the most convenient and most easily accommodated into consumer lifestyles. Lundblad and Davies (2016) mentions that the concept of sustainable fashion covers a variety of terms such as organic, green, fair trade, slow, eco and so forth each attempting to "highlight or correct a variety of perceived wrongs in the fashion industry including animal cruelty, environmental damage and worker exploitation" Ridgway and Price (1994) find out that concepts such as use innovativeness as consumers find new and different ways of using their possessions.

1.4 Significance of study

Consumer attitude is an integral part of consumer behaviour. It's a small factor that has a greater impact on how a product needed to be marketed.

Fashion as an industry is growing with huge market share both globally and nationally. Fashion consumption has increased severely compared to last decade. It has caused a major concern for the environment. The use of synthetic materials and also the maximum exploitation of natural fabrics has caused severe environmental problems. Ethical problems from the fashion industry has also caused major human rights violations.

From this project, the researcher tried to find out how sustainability has a factor has influenced with the customers of Kerala.

1.5 Scope of study

The study will be conducted on the residents and non-residents of Kerala. It aims to reach out to 142 respondents for the study to know the different consumer attitudes towards sustainability in the fashion industry.

1.6 Objective of study

To understand consumer attitude within the fashion industry with the influence of sustainability.

To understand how age, gender, occupation and being an NRI or not affects your consumer behaviours

1.7 Methodology

1.7.1 Data Collection

When it comes to data collection, there are two methods that are generally used by researchers to collect data. These methods are Primary data collection methods and Secondary data collection methods. Primary data collection methods include collection of data through observation, interviews, questionnaire, case studies, projective techniques and schedules. Secondary data is one that already exists and it may be collected through published or unpublished sources. Published sources include publications by the government, public records, records held by banks etc. Unpublished sources include data from letters, diaries unpublished biographies and work etc.

Secondary data in research was used to find out about the industry's profile and the company's profile. It was also used in the introduction of study and literature review. All secondary data related information has been collected from previously done research papers and credible internet websites.

The quantitative approach is used to collect primary data from consumers. The data is collected using survey questionnaire consisting of nominal scale, ordinal scale, interval scale and ratio scale questions.

1.7.1 Sampling

1.7.2 Population

Population is the collection of the elements which has some or the other characteristics in common. The number of elements in the population is the size of the population. In this survey, the population comprises of all fashion consumers around and from Kerala.

1.7.3 Sample Size

The sample for the research is consumers from Kerala. Keeping in view the limitation of time and resources, the sample size taken is 152 respondents. Questionnaires were distributed through social media platforms like WhatsApp and e-mail to the respondents and enough time was given to the respondents to fill the questionnaire to reduce sampling error.

1.7.4 Tools used for Data Collection

The questionnaire is carefully designed to meet the requirements of the research. Most of the questions is constructed using the Likert Scale, ranging between Strongly Disagree; Disagree; Neutral; Agree; Strongly Agree. And Scale of 1 to 5, where 1 being least likely and 5 the most likely

1.7.5 Data Analysis Techniques

The process of evaluating data using analytical and logical reasoning to examine each component of the data provided. This form of analysis is just one of the many steps that must be completed when conducting a research experiment. Data from various sources is gathered, reviewed, and then analyzed to form some sort of finding or conclusion. For the purpose of data analysis the following tools are used:

T-test: The independent samples t-test compares means of one dependent and two independent variables in order to determine whether there's statistical evidence that the associated population means are significantly different.

ANOVA: Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures (such as the "variation" among and between groups) used to analyze the differences among group means in a sample

1.8 Hypothesis

Hypothesis 1 (H1): Sustainable consumption depends on consumer attitude

Hypothesis 2 (H2): International exposure affects consumer attitude

Hypothesis 3 (H3): Influence of age in sustainable consumption

Hypothesis 4 (H4): Influence of gender in sustainable consumption

Hypothesis 5 (H5): Influence of occupation in sustainable consumption

1.9 Limitations

Large and scattered sample size. Time constraints were one of the major limitations as research activities were done alongside academic activities. One of the major constraint - Findings of the survey are based on the assumptions that the respondent has given correct responses. Respondents might not give a proper answer due to nature of the questionnaire.

Chapter 2
Industry Analysis

2.1 Industry Profile

2.1.1 Fashion Industry

The fashion industry is a multibillion-dollar global business dedicated to the creation and sale of clothing. Fashion has no formal definition, although it is best characterised as the style or types of clothing and accessories worn by groups of people at any given moment. There may appear to be distinctions between the high-end designer clothing seen on the runways of Paris or New York and the mass-produced sportswear and streetwear marketed in malls and markets around the world.

Designers, stores, factory workers, seamstresses, tailors, technically skilled embroiderers, the press, publicists, salespersons (or "garmentos"), fit models, runway models, couture models, textile manufacturers, pattern makers, and sketch artists are just a few of the people who make up the fashion industry. The fashion industry may be regarded as the business of creating garments in its most basic form, but it would gloss over the important distinction between fashion and apparel. Fashion incorporates its own prejudices of style, individual taste, and cultural evolution. Apparel is functional clothing, one of humanity's basic needs, but fashion incorporates its own prejudices of style, individual taste, and cultural evolution.

Fashion is no longer only a necessity; it is now much more of a statement. The objective of the modern clothes industry is to create, produce, promote, and market style based on demand. It represents consumers' fluctuating desires to be defined by their clothing, or more generally, to be accepted, which has prompted change throughout fashion history and continues to evolve through an ever-changing haberdashery lexicon. Changing styles necessitate change in industry, particularly in the ever-specialized fields of manufacturing and merchandising, as well as in the promotion of designs and designers, expanding their scope into what were dubbed "lifestyle brands" in the early 2000s, encompassing more than just fashion—including the vernacular of fragrance, accessories, and more.

2.1.2 Fashion Industry in India

India or Indian clothes has one of the oldest techniques and fabrics used. Its one of the ancient fashion industries. Although, the designers from late 80s and 90s did not get much recognition but they saw a change in the industry as the result of increasing exposure to global fashion and the economic boom the economic liberalization of the Indian economy in 1990. The following decades firmly established fashion as an industry across India.

India is a land of diverse cultures and so is the diversity in clothing and fashion. From the Turkish influence in the north to the Hindu influence in the south, the style changes as we move from places to places. Indian industry did not identify clothes as nothing more than a need to cover their body. After the LPG boom in 1991 is where India saw the rise of trends and western influence. Clothing was not a need anymore but a statement, a brand about a person. This influenced the Indian market into designer market, statement outfits and many more. Now, there is a changing trend with styles which has blend of both desi and western styles. Ancient prints are being back in trend with youngsters and also making it casuals and work wear are giving traditional clothing practices importance.

2.1.3 Sustainable fashion

The fashion industry is one of the most rapidly expanding sectors of the economy. It is estimated to be worth over \$2 trillion USD. While the fashion business is flourishing, more and more attention is being drawn to the industry's extensive list of negative environmental repercussions. Fashion production contributes 10% of global carbon emissions, depletes water supplies, and pollutes rivers and streams. Furthermore, according to the United Nations Economic Commission for Europe (UNECE), 85 percent of all textiles are discarded each year, and washing some types of clothing releases a considerable amount of microplastics into the water.

When it comes to the ethical side of fashion or the fast fashion industry, textile workers, especially women in developing nations, are frequently paid low wages and pushed to work long hours in deplorable conditions (UNEP, 2018; WRI, 2019). Human rights are being violated in numerous areas as a result of these circumstances (Human Rights Watch). Chemicals used in clothing production also pose major health risks to both industry workers and consumers. The pollution mentioned above has additional health consequences.

Since 2015, the global ethical fashion market has grown at a compound annual growth rate (CAGR) of 8.7%, reaching a value of roughly \$6.35 billion in 2019. At a compound yearly growth rate (CAGR) of 6.8%, the market is predicted to increase from \$6.35 billion in 2019 to \$8.25 billion in 2023. The growth is mostly due to a growing understanding of the importance of ethical fashion for long-term sustainability. According to estimates from the sustainable fashion sector, the market will expand to \$9.81 billion in 2025 and \$15.17 billion in 2030, with a CAGR of 9.1%.

2.1.4 Sustainable fashion in India

As people grow more aware of the environmental impact of their buying habits, Indian shoppers are flocking to the concept of sustainable fashion. India's retail firms are capitalising on the eco-friendly shopping trend by launching sustainable solutions. Indian Terrain, for example, collaborated with Fairtrade India to establish a sustainable fashion line in 2020, while Madame debuted its Eco Aware collection in April of this year. Over the last several years, a slew of new sustainable fashion firms have emerged. Among them are DaMENSCH, ANI Clothing, Sparsh Organic, and The Pant Project..

The Indian government began the SURE project in August 2019, with the goal of moving towards sustainable fashion that contributes to a clean environment. The programme involves 16 major retail fashion businesses who have committed to sourcing or using sustainable raw materials and techniques for a significant portion of their overall consumption by 2025.

"Sustainability is getting popularity in India," according to a McKinsey analysis, "where the local market is predicted to reach about \$60 billion in revenues in 2022, making it the world's sixth-largest behind the United Kingdom and Germany."

Chapter 3
Data Analysis and interpretation

The data collection and findings of the study are discussed in this chapter of analysis, as well as the descriptive information and statistical analysis created by the obtained survey data. Records are statically analysed with spss software programme.

3.1 Demographic details of respondents

The researcher has tried to study the demographic variables of the respondents.

Table 3.1 - *Demographic Details of Respondents*

	Demographics Characteristics	Number of Respondents	Percentage
Gender	Female	107	74.4%
	Male	35	26.6%
		142	100%
Age	26 and below	98	69%
	41 - 27	32	22.5%
	57 - 42	10	7.0%
	76 - 58	2	1.4%
		142	100%
Occupation	Student	85	59.9%
	Salaried	40	28.2%
	Self Employed	9	5.6%
	Unemployed	8	6.3%
		142	100%
Non-Residential Indians (NRI)	Yes	43	30.3%
	No	99	69.7%
		142	100%

The demographic details of the respondents are shown in table 3.1. It is shown that out of 142 respondents, 74.4% identify as ‘female’ and 26.6% identify as ‘male.’ In the age group, 69% comes under the age group ‘26 and below’, 22.5% come under the age group ‘41 – 27’, 7.0% comes under age group 57 – 42 and 1.4% come under the age group ‘76 – 58.’ Respondents from various occupation groups of 4 categories: Student, salaried, self-employed and unemployed. The data collected depicts that 59,9% are student, 22.5% are salaried, 5.6% are

self-employed, 1.4% are unemployed. The data also shows that out of 142 respondents, 30.3% are Non-Residential Indians and 69.7% are not.

3.2 Relationship of celebrity influence Before Buying a Product with Gender of The Respondent

The independent samples t-test compares means of one dependent and two independent variables in order to determine whether there's statistical evidence that the associated population means are significantly different. The variables considered for the test are, dependent: I would make a sustainable fashion purchase if my favorite celebrity is endorsing it, and independent : gender.

Table 3.2(a) Table showing the variation between celebrity endorsement influence before buying a product on the basis of the gender of respondent - T-test

Dependent Variable	t value	df	Sig(2 – tailed)	Independent variable	N	Mean	Std. Deviation
I would make a sustainable fashion purchase if my favorite celebrity is endorsing it	2.267	140	0.025	Female	107	2.430	1.1984
				Male	35	1.914	1.0675

3.2.1 Difference in The Celebrity Influence Before Buying a Product with Respect to Gender of The Respondent

An independent samples t-test revealed that interpersonal influence before buying a product is comparatively greater for females (F=2.430; SD=1.1984) as compared to males (M=1.914; SD=1.0675), $t(140) = 2.267$. Since $p < 0.05$ (two tailed test), we reject the null hypothesis

Therefore, it can be said that there is a significant variation in celebrity endorsement before buying a product across the two genders.

Table 3.2(b) Summary of Hypothesis Statement

#	Hypothesis statement	Decision
H4	There is a difference between males and females when it comes to purchase products endorsed by a celebrity	Accepted

3.3 Relationship of product knowledge and environment consciousness before buying a Product with Gender of the Respondent

Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures (such as the “variation” among and between groups) used to analyze the differences among group means in a sample

Table 3.3(a) Table showing ANOVA results for Relation between product knowledge and environment consciousness and the gender of the respondent

		ANOVA				
		Sum of Squares	df	Mean Square	F	Sig.
8. Agree or disagree [I research before making a clothing or fashion product purchase]	Between Groups	7.253	1	7.253	6.488	.012
	Within Groups	156.521	140	1.118		
	Total	163.775	141			
10. Agree or disagree [I would describe myself as environmentally responsible]	Between Groups	3.178	1	3.178	4.486	.036
	Within Groups	99.188	140	.708		
	Total	102.366	141			

The one-way anova test compares the means of one dependent variable and one independent variable in order to determine whether there’s statistical evidence that the associated population means are significantly different. The variables considered for the test are –

3.3.1 Dependent Variable – I research before making a clothing or fashion product purchase

Independent Variable - gender of the respondent

In Table 3.3(a), the dependent variable, I would recommend Raymond to a family friend was compared with occupation of the respondent. It was found that the p value was 0.012; $p < 0.05$. Thus, the null (H_0) can be rejected and there is a significant difference between product knowledge and the gender of the respondent.

3.3.2 Dependent Variable – I would describe myself as environmentally responsible

Independent Variable - gender of the respondent

In Table 3.3(a), the dependent variable I would describe myself as environmentally responsible was compared with gender of the respondent. It was found that the p value was 0.036; $p < 0.05$. Thus, the null (H_0) can be rejected and there is a significant difference between environment consciousness and the gender of the respondent.

Table 3.3(b) Summary of Hypothesis Statement

#	Hypothesis statement	Decision
H1	There is a difference between males and females when it comes to product knowledge	Accepted
H2	There is a difference between males and females when being environmentally conscious	Accepted

3.4 Relationship of product pricing, celebrity endorsement and environment consciousness before buying a Product with Age of the Respondent

Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures (such as the “variation” among and between groups) used to analyze the differences among group means in a sample

Table 3.4(a) Table showing ANOVA results for Relation between product pricing, celebrity endorsement and environment consciousness before buying a Product with Age of the Respondent

		ANOVA				
		Sum of Squares	df	Mean Square	F	Sig.
9. On a scale from 1 to 5 [I would spend more for sustainable fashion products]	Between Groups	13.140	3	4.380	3.850	.011
	Within Groups	157.008	138	1.138		
	Total	170.148	141			
9. On a scale from 1 to 5 [I would make a sustainable fashion purchase if my favourite celebrity were endorsing it]	Between Groups	11.877	3	3.959	2.936	.036
	Within Groups	186.101	138	1.349		
	Total	197.979	141			
10. Agree or disagree [My purchase habits are affected by my concern for environment]	Between Groups	12.217	3	4.072	5.143	.002
	Within Groups	109.276	138	.792		
	Total	121.493	141			
10. Agree or disagree [I am concerned about wasting the resources of our planet]	Between Groups	5.089	3	1.696	2.967	.034
	Within Groups	78.883	138	.572		
	Total	83.972	141			
10. Agree or disagree [I would describe myself as environmentally responsible]	Between Groups	10.739	3	3.580	5.392	.002
	Within Groups	91.627	138	.664		
	Total	102.366	141			

The one way anova test compares the means of one dependent variable and one independent variable in order to determine whether there’s statistical evidence that the associated population means are significantly different. The variables considered for the test are –

3.4.1 Dependent Variable – I would spend more for sustainable fashion products

Independent Variable - Age of the respondent

In Table 3.4(a), the dependent variable, I would spend more for sustainable fashion products was compared with age of the respondent. It was found that the p value was 0.011; $p < 0.05$. Thus,

the null (H0) can be rejected and there is a significant difference between product price and the gender of the respondent

3.4.2 Dependent Variable – I would make a sustainable fashion purchase if my favourite celebrity were endorsing it

Independent Variable - Age of the respondent

In Table 3.4(a), the dependent variable, I would spend more for sustainable fashion products was compared with age of the respondent. It was found that the p value was 0.036; $p < 0.05$. Thus, the null (H0) can be rejected and there is a significant difference between celebrity endorsement and the gender of the respondent

3.4.3 Dependent Variable – My purchase habits are affected by my concern for environment

Independent Variable - Age of the respondent

In Table 3.4(a), the dependent variable, my purchase habits are affected by my concern for environment was compared with age of the respondent. It was found that the p value was 0.002; $p < 0.05$. Thus, the null (H0) can be rejected and there is a significant difference between environmentally conscious and the gender of the respondent

3.4.4 Dependent Variable – I am concerned about wasting the resources of our planet

Independent Variable - Age of the respondent

In Table 3.4(a), the dependent variable, I am concerned about wasting the resources of our planet was compared with age of the respondent. It was found that the p value was 0.034; $p < 0.05$. Thus, the null (H0) can be rejected and there is a significant difference between environmentally conscious and the gender of the respondent

3.4.5 Dependent Variable – I would describe myself as environmentally responsible

Independent Variable - Age of the respondent

In Table 3.4(a), the dependent variable, I would describe myself as environmentally responsible was compared with age of the respondent. It was found that the p value was 0.002; $p < 0.05$. Thus, the null (H_0) can be rejected and there is a significant difference between environmentally conscious and the gender of the respondent

Table 3.4(b) Summary of Hypothesis Statement

#	Hypothesis statement	Decision
H1	There is a difference between different age category when it comes to the price of the products	Accepted
H2	There is a difference between different age category when it comes to celebrity endorsements	Accepted
H3 H4 H5	There is a difference between different age category when it comes to being environmentally conscious	Accepted

3.5 Relationship of environment consciousness before buying a Product with occupation of the Respondent

Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures (such as the “variation” among and between groups) used to analyze the differences among group means in a sample

Table 3.5(a) Table showing ANOVA results for Relation between product knowledge and environment consciousness and the occupation of the respondent

ANOVA

10. Agree or disagree [I would describe myself as environmentally responsible]

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	5.830	3	1.943	2.778	.044
Within Groups	96.536	138	.700		
Total	102.366	141			

3.5.1 Dependent Variable – I would describe myself as environmentally responsible

Independent Variable – occupation of the respondent

In Table 3.4(a), the dependent variable, I would describe myself as environmentally responsible was compared with age of the respondent. It was found that the p value was 0.044; $p < 0.05$. Thus, the null (H_0) can be rejected and there is a significant difference between environmentally conscious and the occupation of the respondent

Table 3.5(b) Summary of Hypothesis Statement

#	Hypothesis statement	Decision
H1	There is a difference between occupation category when it comes to being environmentally conscious	Accepted

Chapter 4

Summary of findings, Suggestions and Conclusion

4.1 List of findings

- Table 3.1 – Shows the Demographic Details of Respondents
- Table 3.2(a) Table shows the variation between celebrity endorsement influence before buying a product on the basis of the gender of respondent - T-test
- Table 3.2(b) Shows the Summary of Hypothesis Statement
- Table 3.3(a) Table showing ANOVA results for Relation between product knowledge and environment consciousness and the gender of the respondent
- Table 3.3(b) Shows the Summary of Hypothesis Statement
- Table 3.4(a) Table showing ANOVA results for Relation between product pricing, celebrity endorsement and environment consciousness before buying a Product with Age of the Respondent
- Table 3.4(b) Summary of Hypothesis Statement
- Table 3.5(a) Table showing ANOVA results for Relation between product knowledge and environment consciousness and the occupation of the respondent
- Table 3.5(b) Summary of Hypothesis Statement

4.2 Suggestions

1. Fashion Brands should use popular persons who are related to the sustainable environment or celebrities who has major influence in the minds of the consumers to promote sustainable fashion products. As sustainable fashion or consumption is a new topic and many people are not quite known about it, so a well know person promoting it would attract consumers.
2. Generation” Z” and Millennials are more likely to try new trends and styles. Bringing up styles and trends with sustainable or eco-conscious twist will encourage them to buy those products.
3. Some brands offer sustainable fashion products but with higher price points. So as to encourage consumers to buy these products, promote the product as where it was sourced, many purposes to this product (as in more than one time use) and how the product will degrade or the environmentally conscious packaging.
4. Females tend to be more consumers of fashion products, so fashion brands should bring new innovative fashion products in the female category

4.3 Conclusion

Through this research, the researcher tried to bring light on consumers perspective about sustainability and sustainable consumption. This research was able to identify how different consumers of different age category, income, occupation and international exposure have on the minds of the customers when it came to purchase sustainable.

The research observed that consumers prefer quality, price, comfort and style the top factors before making a purchase and the celebrity endorsements being the least. But that doesn't mean it doesn't have its importance. It has more importance to female consumers than male consumers.

With this research we can conclude that Consumers are new to sustainable fashion. Response from the survey shows how consumers are not aware about what they are wearing or getting their hands on. Most of the answers being neutral to question brings to this conclusion.

Consumers are willing to take a step into sustainable fashion, but the knowledge or awareness regarding a form of fashion is less. Here, the use of celebrity endorsements brings more awareness to the products. Even though, the preference is low for consumers but the impact that can have will be bigger.

The research observed that consumers do know about the environmental damages and problems caused by the fashion industry but is not willing to spend more. Associate groups also don't have much influence on the consumers but rather just their motive and values and ability to buy the products.

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ANNEXURE 1

Questionnaire

1. Gender
 - Female
 - Male
 - Others
2. Age
 - 26 and below (Generation Z)
 - 41 to 27 (Millennials)
 - 57 to 42 (Generation X)
 - 58 to 76 (Baby Boomers)
3. Are you an NRI?
 - Yes
 - No
4. How often do you shop for clothes?
 - Weekly
 - Monthly
 - Yearly Thrice
 - Yearly Twice
 - Yearly once
5. On a scale of 1 to 5 (1 being least likely and 5 the most likely) how likely are you in engaging in the following sustainable practices

S.no		1	2	3	4	5
1	Recycled					
2	Ethically sourced (it's the process of ensuring that the products made are obtained through responsible and sustainable methods)					
3	Reused					

6. Rank the following in the order of your consideration while buying from a fashion brand.
 - Brand name
 - Quality
 - Price
 - Durability
 - Styles
 - Comfort
 - Celebrity endorsed

7. Please indicate to what extent you agree or disagree with the below statements

S.no	Statements	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	I research before making a clothing or fashion product purchase					
2	I care about the working conditions of the factory workers where these products are made					
3	I check if the clothes are sustainably sourced					
4	I check if the clothes are ethically sourced					
5	I buy new fashion products to keep up with the latest trends					

8. On a scale from 1 to 5, 1 being least likely and 5 the most likely, how much would you rate the following statements

S.no	Statements	1	2	3	4	5
1	If I must choose between two equal products, I will choose the one which is less harmful for the environment					
2	I would spend more for sustainable fashion products					
3	I would spend more if my favourite fashion brand went sustainable					
4	I would make a sustainable fashion purchase if my favourite celebrity were endorsing it					
5	I would shift to sustainable fashion if my friend or family also tries to shift					

9. Please indicate to what extent you agree or disagree with the below statements

S.no	Statements	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1	It is important to me that the products I use do not harm the environment					
2	I consider the potential environmental impact of my actions when making many of my decisions					
3	My purchase habits are affected by my concern for environment					
4	I am concerned about wasting the resources of our planet					
5	I would describe myself as environmentally responsible					
6	I am willing to be inconvenienced in order to take actions that are more environmentally friendly					

10. Occupation

- Student
- Salaried
- Self employed
- Unemployed

11. Are you a resident from Kerala?

- Yes
- No

Project Report

On

**AIR POLLUTION VARIATION DUE TO
LOCKDOWN IN KOCHI**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

PARVATHY RAJU

(AB19AMAT059)

Under the Supervision of

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APRIL 2022

ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM



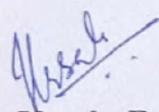
CERTIFICATE

This is to certify that the dissertation entitled, **AIR POLLUTION VARIATION DUE TO LOCKDOWN IN KOCHI** is a bonafide record of the work done by Ms. **PARVATHY RAJU** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

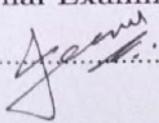
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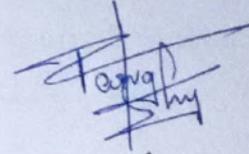
2:

DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Dr Susan Mathew Panakkal, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date: 4/03/2022



PARVATHY RAJU

AB19AMAT059

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Ernakulam.

Date: 4/03/2022

PARVATHY RAJU

AB19AMAT059

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Chapter 1

Introduction

1.1 Air Pollution

Air pollution is one of the major environmental problems all over the world, which affects human health, vegetation, materials, and ecosystems as a whole. Air pollution is caused due to a mixture of solid particles and molecules in the air, called *air pollutants*. It is caused by people, taking the form of emissions from factories, cars, aerosol cans, secondhand cigarette smoke, etc. The combined effects of outdoor and household air pollution cause millions of premature deaths every year, largely because of increased mortality from stroke, heart disease, chronic obstructive pulmonary disease, lung cancer, and acute respiratory infections.

Rapid urbanization, industrialization, vehicular growth, and population growth are responsible for the increase in Air pollution. According to the World Health Organization (WHO), each year air pollution is responsible for the death of seven million around the globe. India is the second most populated country . In order to meet the demands of the Increasing population, industries have increased their production leading to a higher amount of pollution. WHO has suggested India reduce its pollution by 15% every year.

1.2 Air Pollution in Ernakulam District

Kerala's most polluted cities include Kochi and Thiruvananthapuram. Since these cities are highly populated the amount of pollution in these cities is very high. The number of vehicles running on the roads of these cities and various factories contributing majorly to the pollution.

Some of the major factories contributing to air pollution in the Ernakulam district include

1. Eloor FACT
2. Hindalco Alupuram factory & works
3. Hindalco Industries Limited
4. Cochin Petromins factory
5. Hindustan Insecticide Limited
6. Cochin Chemical Industries etc.

1.2.1 Newspaper Article

Our motive for selecting this topic is this [Newspaper article](#) that was published in Hindustan Times, New Delhi by Trisha Sengupta on 4th April 2020. This article pointed out that the residents of Jalandhar, Punjab woke up to the sight of a mountain range peeking from behind the clouds. The only thing is that this mountain range is not located in the land of five rivers but is over 200 km away from the state of Punjab. What people saw from Jalandhar is the mighty Dhauladhar range in Himachal Pradesh. People who shared the images of the incredible sight claimed that the incident took place almost after 30 years. The mighty Dhauladhar range was visible due to the reduction in air pollution during the lockdown period.



1.2.2 Objective

To study the variation in air pollution due to lockdown. The target is to find the difference in air pollution before and during Lockdown. We find declines in the population-weighted concentration of 5 pollutant parameters - Nitrogen dioxide [NO_2], Sulphur dioxide [SO_2] PM 2.5 (Particulate Matter of 2.5 units diameter) , PM 10 (Particulate Matter of 10 units diameter) , Carbon monoxide [CO] . Our goal is to show the difference in the amount of air pollution in Kochi.

1.2.3 Literature Review

There is a lot of research that has been done to evaluate the ambient air quality of various places in Kerala and hence we found out in order to show the intensity of air pollution in those places. A case study of Kerala by using air quality index [2017] on 24 hours basis by Jyothi S.N, Kishan Kartha, Divesh, Adarsh Mohan, Jithin Pai, Geena Prasad reported that the industrial and residential areas of Ernakulam district have the highest air quality index and Kollam has the lowest air quality index among all the six areas studied, [2017], air quality is determined by the particular matter [PM10] concentration.

1.2.4 Prerequisite Knowledge

Data Collection

We have used secondary data in our project. Secondary data is the data collected by another person other than the primary user. Examples of secondary data include government records, census, journals, etc.

The secondary data we have used is a government publication. We have collected the data from the *Central Pollution Control Board*'s [CPCB] official website.

The Central Pollution Control Board has several stations across all states in India. Since our study is concentrated in the Ernakulam district, we have chosen the Central Pollution Control boards station in Kacheripady in Ernakulam. There are two more stations in Ernakulam, one in Vytilla and the other in Eloor.

The concentration of each pollutant is collected on a 24 hour basis in the Central Pollution Control Board's official website.

Statistical Interference

Statistical inference is the process of drawing conclusions about an underlying population based on a sample or subset of the data. Testing of hypotheses basically deals with choosing an appropriate test statistic and dividing its range of variation into acceptance and rejection regions based on the significance level. The significance level is the probability of rejecting the null hypothesis when it is true denoted by alpha (α). We have chosen the significance level to be 5% or 0.05.

Chapter 2

Definitions and Terminologies

2.1 Air Pollutants

Air pollutants are hazardous and toxic substances released from factories, smoke, etc present in the air that causes air pollution.

There are different types of air pollutants, such as gases (including Ammonia, Carbon monoxide, Sulfur dioxide, Nitrous oxides, Methane, Carbon dioxide, and Chlorofluorocarbons), particulates (both organic and inorganic), and biological molecules.

Since it is not practical to study all the pollutants, we have studied the variation in the population-weighted concentration of 5 major pollutants namely:

1. PM 2.5
2. PM 10
3. SO_2
4. NO_2
5. CO

PM 2.5

Particulate matter of 2.5 units diameter contains microscopic particles and liquid droplets which when inhaled causes health issues like shortness of breath as these particles are able to travel directly into the lungs, eye irritation, nose irritation, etc. They are emitted directly from sources like unpaved roads, construction sites, fields, or fires.

PM 10

Particulate matter of 10 units diameter contains solid or liquid particles which are very small and hence PM 10 particles effectively act as gas. Continuous exposure to PM 10 causes serious health issues like asthma, high blood pressure, bronchitis, etc.

SO₂

Sulfur dioxide is an air pollutant in gaseous form that is formed by the reaction between sulfur and oxygen. It is formed when fossil fuels like coal or diesel are burned. Continued exposure at high levels reduces the working capacity of the lungs.

NO₂

Nitrogen dioxide is also a gaseous pollutant that consists of nitrogen and oxygen. It is formed when fossil fuels like oil, coal, or diesel are burned at high temperatures. Continuous exposure to *NO₂* causes serious health issues like asthma attacks.

CO

Carbon monoxide is produced mainly by vehicles and is very toxic. Continuous exposure to CO causes headaches, nausea, vomiting, etc.

2.1.1 Air Quality Index

The air quality index (AQI) is the measure of the amount of air pollutant in the atmosphere. The value of AQI lets us understand how much it can affect human's health. The higher the AQI value, the greater the level of air pollution and the greater the health concerns. Since the AQI value is an index there are six categories which can be broadly classified into Good, Satisfactory, Moderately polluted, Poor, Very Poor, and Severe. Each category corresponds to a different level of health concern. AQI value is calculated for major pollutants namely :

1. Ground-level Ozone
2. Particle Pollution (also known as particulate matter, including PM 2.5 and PM 10)
3. Carbon monoxide
4. Sulfur dioxide
5. Nitrogen dioxide

Chapter 3

Data Analysis

3.1 AQI Calculations

The AQI can be calculated in two ways - manually and by using an online AQI calculator. AQI values are usually measured on a 500-point scale, which consists of 6 divisions.

1. Good(0-50)
2. Satisfactory(51-100)
3. Moderately polluted (101-200)
4. Poor (201-300)
5. Very Poor (301-400)
6. Severe (401-500),

wherein rating between 0 and 50 is considered good. Rating between 301 to 500 range is severe.

The equation for computing AQI :

$$I = \frac{(I_{high}) - (I_{low})}{(C_{high}) - (C_{low})} \times (C - C_{low}) + (I_{low})$$

AQI Category	AQI	Concentration range*							
		PM ₁₀	PM _{2.5}	NO ₂	O ₃	CO	SO ₂	NH ₃	Pb
Good	0 - 50	0 - 50	0 - 30	0 - 40	0 - 50	0 - 1.0	0 - 40	0 - 0.2	0 - 0.5
Satisfactory	51 - 100	51 - 100	31 - 60	41 - 80	51 - 100	1.1 - 2.0	41 - 80	201 - 400	0.5 - 1.0
Moderately polluted	101 - 200	101 - 250	61 - 90	81 - 180	101 - 168	2.1 - 10	81 - 380	401 - 800	1.1 - 2.0
Poor	201 - 300	251 - 350	91 - 120	181 - 280	169 - 208	10 - 17	381 - 800	801 - 1200	2.1 - 3.0
Very poor	301 - 400	351 - 430	121 - 250	281 - 400	209 - 740	17 - 34	801 - 1600	1200 - 1800	3.1 - 3.5
Severe	401 - 500	431 - 500	251 - 500	401 - 500	741 - 500	34 - 500	1600 - 1800	1800 - 500	3.5 - 500

* CO in mg/m³ and other pollutants in µg/m³; 2h-hourly average values for PM₁₀, PM_{2.5}, NO₂, SO₂, NH₃, and Pb, and 8-hourly values for CO and O₃.

Example :

Suppose a value of PM 2.5 of concentration 41.23 micrograms per cubic meter. Then the AQI is calculated using the AQI Formula rounds to 115, and corresponds to the "UNHEALTHY FOR SENSITIVE GROUPS" range.

3.1.1 Z-Test

Z-test is a statistical test to determine whether two population mean are different when the variance is unknown and the sample size is large. It is a hypothesis test that follows *anormal distribution*.

We have done testing of equality of means of two samples. We chose this test because our sample size is large (>30). We have considered two samples, one the period before lockdown and the other sample is the period during the lockdown. The first phase of lockdown was from 23 March 2020 to 17 May 2020 and the second phase was from 8 May 2021 to 9 June 2021 which adds up to a total of 89 days. This period of 89 days has been considered as during the lockdown period (sample 1). We have considered an equivalent number of days before lockdown and this period has been taken as the before lockdown period (sample 2). Here

$$n_1 = 89, n_2 = 89$$

Z -Test: For Two-Sample Mean:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\left(\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}\right)}$$

(\bar{x}_1) -Mean of the sample before lockdown

(\bar{x}_2) -Mean of the sample during lockdown

s_1 - Standard Deviation before lockdown

s_2 - Standard Deviation during lockdown

n_1, n_2 - Sample Size

3.1.2 PM 2.5

Null Hypothesis H_0 : There is no variation in the concentration of PM 2.5 before and during the lockdown.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis H_1 : The concentration of PM 2.5 has reduced during the lockdown.

$$H_1 : \mu_1 > \mu_2$$

	BEFORE LOCKDOWN	DURING LOCKDOWN
Mean	39.53574713	14.84356322
Known Variance	163.4295712	45.31769995
Observations	87	87
Hypothesized Mean Difference	0	
z	15.94074908	
z Critical one tail	1.644853627	
z Critical two tail	1.959963985	

OBSERVATIONS
$\alpha = 0.05$
$z = 15.9407490843669$
$z_{\alpha} = 1.65$

RESULTS
Since $z > z_{\alpha}$, we reject H_0 .
Hence, the concentration of PM 2.5 has reduced during lockdown.

3.1.3 PM 10

Null Hypothesis H_0 : There is no variation in the concentration of PM 10 before and during the lockdown.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis H_1 : The concentration of PM 10 has reduced during the lockdown.

$$H_1 : \mu_1 > \mu_2$$

	BEFORE LOCKDOWN	DURING LOCKDOWN
Mean	69.42574713	33.29045977
Known Variance	295.7989	148.8987
Observations	87	87
Hypothesized Mean Difference	0	
z	15.94074908	
z Critical one-tail	1.644853627	
z Critical two-tail	1.959963985	

OBSERVATIONS
$\alpha = 0.05$
$z = 15.9830164407676$
$z_{\alpha} = 1.65$

RESULTS
Since $z > z_{\alpha}$, we reject H_0 .
Hence, the concentration of PM 10 has reduced during lockdown.

3.1.4 NO₂

Null Hypothesis H_0 : There is no variation in the concentration of NO_2 before and during the lockdown.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis H_1 : The concentration of NO_2 has reduced during the lockdown.

$$H_1 : \mu_1 > \mu_2$$

	BEFORE LOCKDOWN	DURING LOCKDOWN
Mean	5.006190476	2.732222222
Known Variance	23.60916	0.880134
Observations	63	63
Hypothesized Mean Difference	0	
z	3.647258454	
z Critical one-tail	1.644853627	
z Critical two-tail	1.959963985	

OBSERVATIONS
$\alpha = 0.05$
$z = 3.647258454$
$z_{\alpha} = 1.65$

RESULTS
Since $z > z_{\alpha}$, we reject H_0 .
Hence, the concentration of NO_2 has reduced during lockdown.

3.1.5 SO_2

Null Hypothesis H_0 : There is no variation in the concentration of SO_2 before and during the lockdown.

$$H_0 : \mu_1 = \mu_2$$

Alternative Hypothesis H_1 : The concentration of SO_2 has reduced during the lockdown.

$$H_1 : \mu_1 > \mu_2$$

	BEFORE LOCKDOWN	DURING LOCKDOWN
Mean	3.622183908	4.973563218
Known Variance	1.906027	8.46366
Observations	87	87
Hypothesized Mean Difference	0	
z	-3.914299641	
z Critical one-tail	1.644853627	
z Critical two-tail	1.959963985	

OBSERVATIONS
$\alpha = 0.05$
$z = -3.91429964106938$
$z\alpha = 1.65$

RESULTS
Since $z < z\alpha$, we accept H_0 .
Hence, there is no variation in the concentration of SO_2

3.1.6 CO

Null Hypothesis H_0 : There is no variation in the concentration of CO before and during the lockdown.

$H_0 : \mu_1 = \mu_2$ Alternative Hypothesis H_1 : The concentration of CO has reduced during the lockdown.

$H_1 : \mu_1 > \mu_2$

	BEFORE LOCKDOWN	DURING LOCKDOWN
Mean	1.299655172	1.176091954
Known Variance	0.152315	0.231294
Observations	87	87
Hypothesized Mean Difference	0	
z	1.860820338	
z Critical one-tail	1.644853627	
z Critical two-tail	1.959963985	

OBSERVATIONS
$\alpha = 0.05$
$z = 1.86082033794505$
$z\alpha = 1.65$

RESULTS
Since $z > z\alpha$, we reject H_0 .
Hence, the concentration of CO has reduced during lockdown.

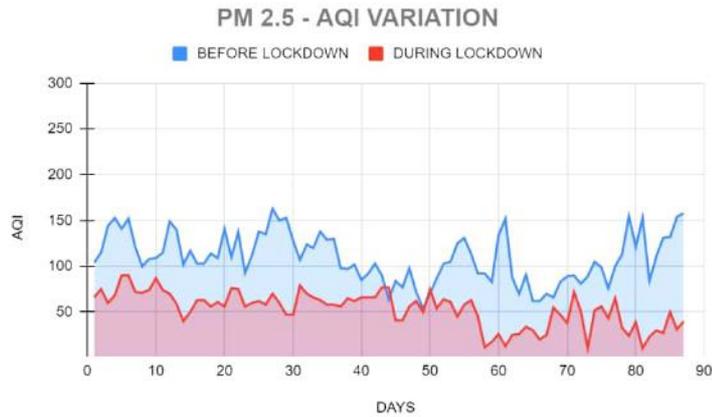
Chapter 4

Graphical Representation

We have not considered AQI values for the z test since AQI is an index, we have represented the variation in AQI values diagrammatically using Area graphs. The AQI values are represented on the y axis and days are represented on the x-axis. We have plotted graphs separately to show the variation in the concentration of the 5 pollutants we have considered before and during the lockdown. Finally, we have also plotted the overall variation in the concentration of all pollutants before and during the lockdown.

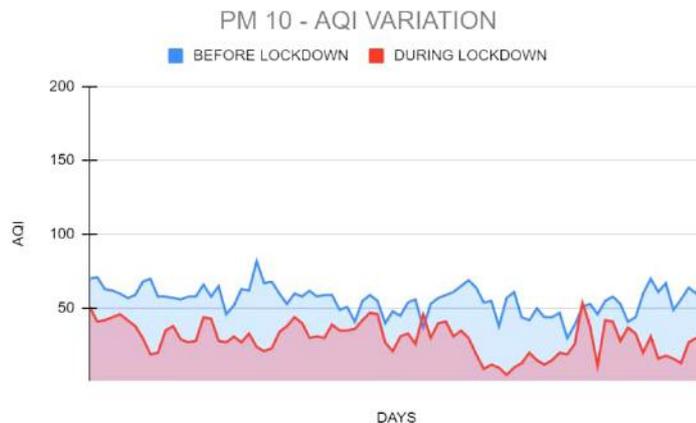
4.0.1 Variations in PM 2.5

Variations in PM 2.5. We took 87 days both before and during lockdown to check whether PM 2.5 is decreased or increased in lockdown. In general, a decrease in PM 2.5 can be observed across during lockdown. The amount of PM 2.5 was increased before lockdown .In which the level of PM 2.5 is varying between $(50 - 100)\frac{g}{m^3}$, $(100 - 150)\frac{g}{m^3}$, $(150 - 200)\frac{g}{m^3}$ in before lockdown. During lockdown load the value of PM 2.5 varying between $(0 - 50)\frac{g}{m^3}$ and $(50 - 100)\frac{g}{m^3}$. However, it has been noted that the value of PM 2.5 has been reduced by 34.75% .



4.0.2 Variations in PM 10

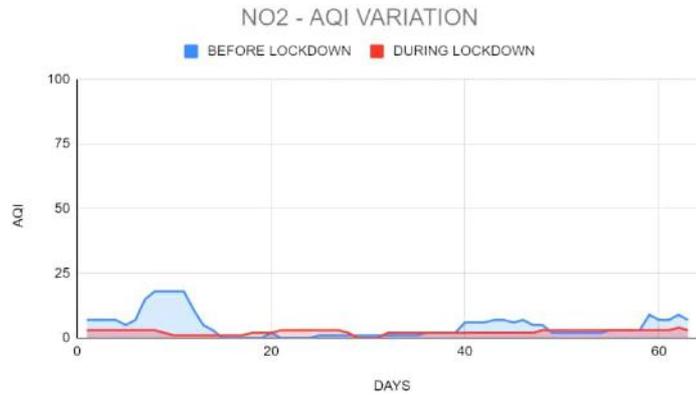
The variation in PM 10. A decrease in PM 10 was observed during the lockdown. This could be due to the decrease in usage of vehicles and the reduction in industrial production .where the level of value of PM 10 between $(0 - 50) \frac{g}{m^3}$, $(50 - 10) \frac{g}{m^3}$, before lockdown. The concentration PM 10 during lockdown is between $(0 - 50) \frac{g}{m^3}$. Hence the amount of PM 10 has been reduced during lockdown by 30.74%.



4.0.3 Variations in NO_2

NO_2 variation before and during lockdown is depicted in figure 3. NO_2 is emitted from fossil fuels like petroleum and diesel and petrol and biogenic sources like soil and lightening, pyrogenic sources like natural fires. NO_2 controls the formation of O_3 where the level of value of NO_2

is between $(0 - 50) \frac{g}{m^3}$ before lockdown. During the lockdown, the level of value of NO_2 is between $(0 - 25) \frac{g}{m^3}$. However, the value of NO_2 has been reduced by 40% in during the lockdown.



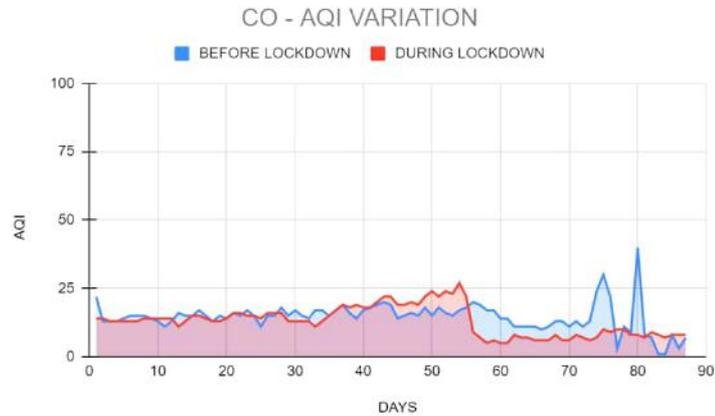
4.0.4 Variations in SO_2

The variation in SO_2 before and during lockdown is shown in figure 4. The primary source of SO_2 is the combustion of sulfur-containing fuels, ie, coals, and diesel used in thermal power plants, industries, and transport. SO_2 can also be sourced from volcanic eruptions and wildfires. The level of value of SO_2 is between $(0 - 25) \frac{g}{m^3}$. During the lockdown, the level of value of SO_2 is also between the range of $(0 - 25) \frac{g}{m^3}$. There is an increase in the value of SO_2 during lockdown by 0.27%.

4.0.5 Variations in CO

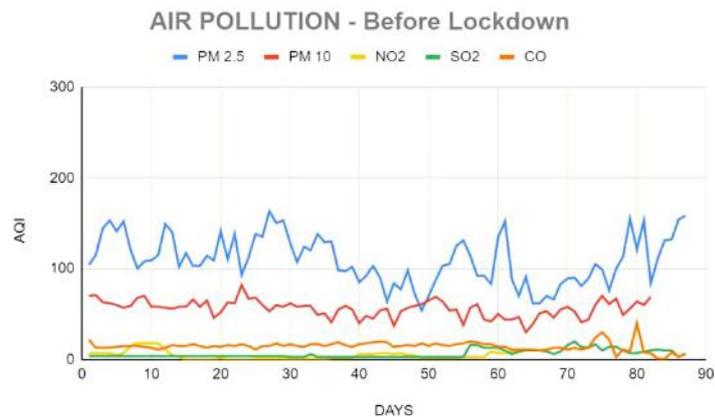
Variation of carbon monoxide is shown in figure 5. CO is mostly emitted from vehicular sources. However, other sources include forest fires agricultural waste burning, bio fuel burning, oxidation of hydrocarbons, and combustion of Fossil fuels. A higher concentration of CO is observed before lockdown. $(25-50) \text{ mg}/\text{m}^3$. The level of value of CO in before lockdown is $(0-25) \text{ mg}/\text{m}^3$ and $(25-50) \text{ mg}/\text{m}^3$. And the level of CO during lockdown $(25-50) \text{ mg}/\text{m}^3$. Both before and during lockdown

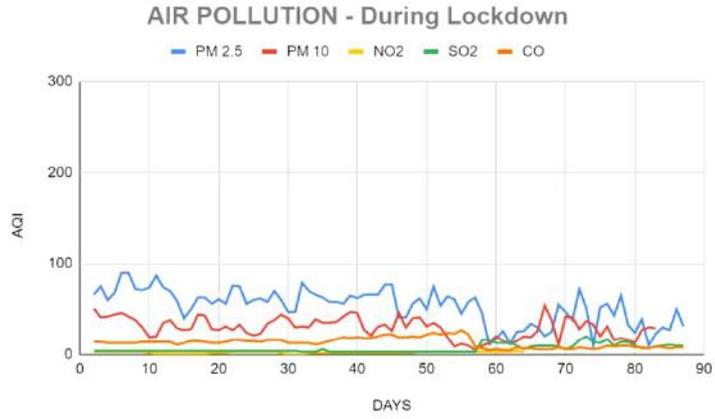
value of CO is poor. Hence the value of CO has been reduced by 5.72 %.



4.0.6 Overall Variations- Before and During lockdown

The variation of all pollutants is shown in figure 6,7. we took 87 days both before and during the lockdown. However, the value of the concentration of all pollutants is reduced during lockdown except SO_2 . SO_2 had an increase during the lockdown. The value of the concentration of all pollutants is greater than that during the lockdown. Hence air pollution was reduced during the lockdown.





Chapter 5

Conclusion

Data was collected from the Central Pollution Control Board's official website for before and during lockdown periods to estimate if there is variation in the concentrations of 5 major pollutants.

We arrived at conclusions based on the z test for two-sample mean and graphical representation of AQI values for each pollutant.

5.0.1 Z-Test

Based on the z test we concluded that the concentration of PM 2.5, PM 10, NO_2 , and CO has decreased during the lockdown. There has been no variation in the concentration of SO_2 during the lockdown.

5.0.2 AQI Graph

Based on the graphs we plotted we concluded that the concentration of PM 2.5, PM 10, NO_2 , and CO has decreased during the lockdown period. There has been a slight increase in the concentration of SO_2 during the lockdown.

Lockdown has resulted in improvements in air quality in Kochi to some

extend. The results obtained from this study gives an insight into the betterment of air quality.

The government of Kerala has introduced the e-mobility project to reduce air pollution. According to this scheme subsidies and incentives are provided for purchase and manufacture of electric vehicles to reduce pollution and switch to green transportation.

Chapter 6

Case-study Comparison

Diurnal and temporal changes in air pollution during COVID-19 strict lockdown over different regions of India” study looked at the temporal and diurnal changes of the six criteria air pollutants, including PM 2.5, PM 10, NO_2 , O_3 , CO, and SO_2 during lockdown (25th March - 3rd May 2020) over various regions of India using the observations from 134 real-time monitoring sites of Central Pollution Control Board (CPCB).

The study selected four regions namely, Indo Gangetic Plain, Central India, South India, northwest India representing a significant reduction has been found in PM 2.5, PM 10, NO_2 , and CO , all the regions during the lockdown.

The identification of commonalities across a range of AQI in the Southern region as Kochi is a part of South India was the purpose of this comparison. The AQI values of the study were similar to our study. We tabulated the results to clearly depict the similarities.

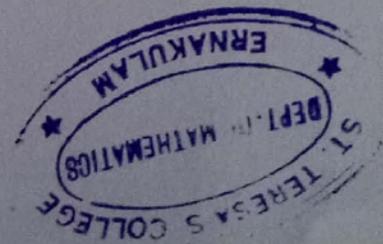
POLLUTANTS	CASE STUDY RESULTS	OUR RESULTS
PM 2.5	40-60% reduction in concentration	34 .75% reduction in concentration
PM 10	40-60 %reduction in concentration	30 .47 % reduction in concentration
NO_2	30-70% reduction in concentration	40% reduction in concentration
SO_2	mixed behavior with slight increase at some sites	0.027% increase in concentration
CO	Same levels during the lockdown	5.72 %reduction in concentration

From the table, we analyze that the reduction in Particulate Matter 2.5, 10 of

Kochi does not lie between the range of South India. Whereas, NO_2 lies in the range of South India. SO_2 has been increased as it has been mentioned. When the South India range of CO hasn't changed there was a minimum reduction in Kochi.

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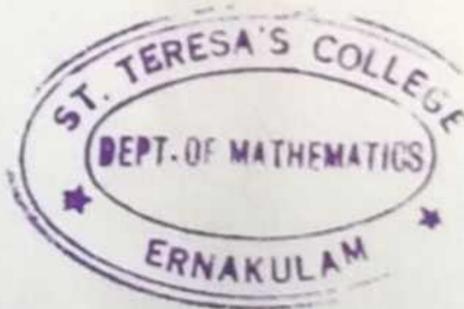


CERTIFICATE

This is to certify that the dissertation entitled, **STATISTICAL STUDY ON YOUTUBE USAGE DURING COVID-19** is a bonafide record of the work done by Ms. **MEGHA GOPALAKRISHNAN** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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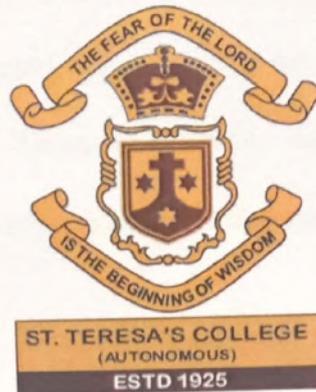
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Project Report

On

**THE EFFECT OF COVID-19 ON ONLINE
FOOD DELIVERY SERVICES.**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

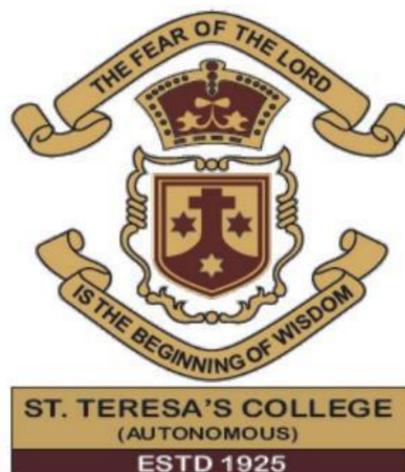
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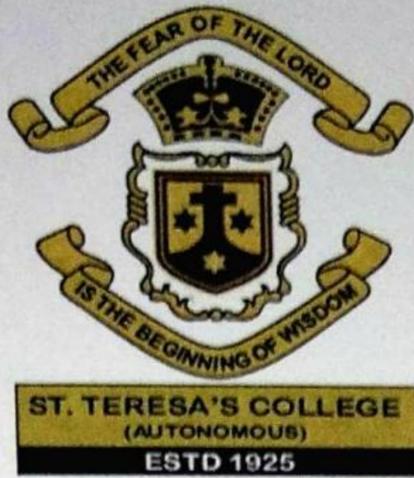


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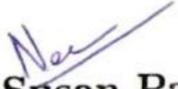
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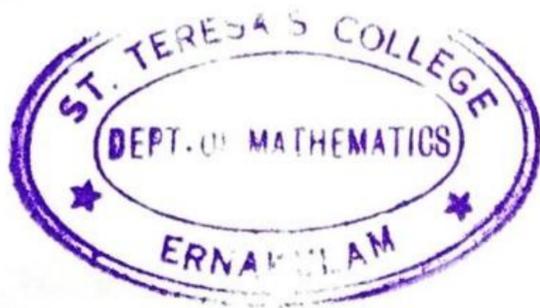


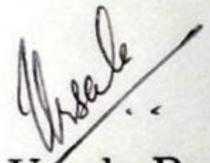
CERTIFICATE

This is to certify that the dissertation entitled, **THE EFFECT OF COVID-19 ON ONLINE FOOD DELIVERY SERVICES**. is a bonafide record of the work done by Ms. **ANGEL GRACE SABU** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

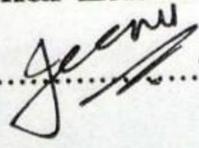
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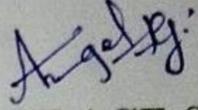
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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Neenu Susan Paul, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date: 4/03/22


ANGEL GRACE SABU

AB19AMAT005

ACKNOWLEDGEMENT

When we set goals for ourselves, there are always obstacles in the way that may deter us from accomplishing the goals. There are also people in our lives that are aware of those goals, and encourage us and also support us to continue regardless of the obstacles, it is now that I can formally thank those people for doing just that for me, before thanking anyone on this earth, I must first thank God for being at my side during this challenging time of my life. The spiritual support has helped to keep me focussed. Thanks to my Guide Neenu Susan Paul, St. Teresa's College for the support and encouragement through this process.

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Chapter 1

Introduction

The COVID-19 pandemic has disrupted nearly every area of people's lives, including their capacity to buy products. Consumers have been confined at home as a result of government-imposed lockdowns, inhibiting regular purchasing habits, and many brick-and-mortar companies have closed. Pharmacies and supermarkets, for example, have stayed open albeit with shortened hours. Many eateries have shuttered or resorted to takeout in order to stay afloat. For many clients, home delivery has given a solution to some of COVID-19's issues. Employees that are required to work remotely, as well as a range of other groups, such as parents who must combine work and parental responsibilities, or individuals who are at risk of serious COVID-19 health concerns, may find e-commerce and home delivery to be a useful option. Despite the difficulties people have had in ordering food online as a result of the pandemic, such as a lack of food inventories, a lack of public transportation, and fewer hours to work at convenience stores and supermarkets, a large percentage of people have been able to obtain sufficient amounts of food stocks. All of this can be attributed to the rapid increase in the number of online food apps, which encourage people to order their meals online and have them delivered to their homes or picked up by themselves without having to enter the restaurant. Programs that encourage grocery shopping and food ordering online can also help to reduce the spread of the Covid-19 virus by reducing interaction between customers and sellers, thereby creating a safe environment in and

of itself. Considering all of the facts, the fact that customers are shifting to ordering food and shopping for groceries online as a result of the pandemic has implications for the retail market as well.

1.1 Background of the study

The early stages of the covid pandemic, as well as the strict lockdown, harmed India's online food delivery companies, which were dominated by Zomato and Swiggy. Consumers had resisted online food ordering due to safety concerns and a general preference for home cooked food. However, things improved later on, and recovery was quicker. According to analysts, the pandemic has assisted companies in improving unit economics and adding more customers, as the frequency of dining out has decreased.

As we stay at home to mitigate the impact of virus many of us have turned to delivery services for meals and groceries for the first time. Before the advent of the Covid nineteen, young people were increasingly buying food online. But now people of all ages are buying food online equally. In this way the spread of the covid is greatly reduced. In the past, large restaurants were mostly using online food delivery services but now it has changed and even smaller hotels started to use online food delivery platforms. The hospitality industry has adapted to make it easier for everyone to support a favourite local restaurant, avoid going out, and simply find a relaxing respite during these stressful times. Because of the nature of how Covid-19 spreads, close contact with others may pose the greatest risk of infection. Many restaurants now allow customers to pay ahead of time, either by phone or online, eliminating the need for physical cash transfers or credit card handling, both of which can potentially harbor the virus.

During the pandemic, diners became accustomed to ordering delivery, and the habit may persist long after dining rooms reopen. However, restaurants and delivery companies continue to be uneasy part-

ners, haggling over fees and struggling to make the service profitable for both parties. Companies such as Zomato and Uber Eats assisted many restaurants in remaining open during lockdowns by allowing diners to stay in and still order out. However, the convenience came at a cost delivery companies can charge commission fees of 30% or more per order, reducing restaurants already meager profits. Delivery was already increasing prior to the pandemic, but it sky rocketed during lockdowns around the world.

Prior to the outbreak of Covid-19, many young people preferred online food delivery. People belonging to other age categories mostly prefer home-cooked meals because they are safe and healthy. During the Covid-19 scenario, everyone was confined to their homes. There were no stores or hotels open, and all local businesses began delivering products through online platforms. And because it was convenient and time-saving, everyone began ordering food online. Online delivery was also safe and reduced the spread of the virus.

1.2 Literature Review

The impact of COVID-19 on restaurant meal ordering via apps 2020: This article was published on April 22, 2021 by Statista's Research Department. According to a Local Circles poll conducted in May 2020, approximately 65 percent of respondents stated that they would not order restaurant meals for delivery within 30 days of the corona virus lockdown being lifted. Approximately 3% of those polled said they would order more than four times during this time period.

According to Priyadharshini (2017), India has more people between the ages of 10 and 24, making it the world's largest young population. With more young people joining the workforce every day, economic growth, increased female labour power, and increased consumer mobility, the traditionally difficult Indian market has evolved and is in need of a more diverse menu.

Samsudin et al (2011) points out that alongside client feedback for an eatery, a plan and execution of wireless food ordering framework was completed. It empowers cafes proprietors to setup the framework in wireless environment and update menu presentations effectively. Advanced mobile has been coordinated within the adaptable wireless food ordering system requesting framework with continuous client criticism execution to encourage ongoing correspondence between eatery proprietors and clients.

Rathore et al (2018) states that 50.8% of consumers use a food delivery service because they don't want to cook because it allows them to have food delivered to their home or office in under 60 minutes.

According to Pathan et al (2017), an online food ordering system can be used to build up a restaurant and mess menu, and customers can quickly place orders. Also, with an online food menu, orders can be readily traced, the client database can be maintained, and the meal delivery business may grow. Restaurants and mess may quickly update their online restaurant menus and upload photographs. Potential consumers can quickly examine a restaurant menu on the internet and place orders at their leisure. As a result, an automated food ordering system with feedback and wireless communication is shown.

1.3 E-Commerce

According to Garret (1996), electronic commerce or e-commerce is the exchange of goods and services via the internet or other computer networks. Buyers and sellers conduct business via networked computers in e-commerce. Electronic commerce also includes the exchange of business information, the maintenance of business relationships, and the conduct of business transactions via communication networks. It contains the relationship between companies (business-to-business), customers (customer-to-customer), and companies and cus-

tomers (business- to-customer). Currently, the business to business sector dominates e-commerce, while the customer-oriented segment lags far behind, accounting for fewer than 10 even though they are all experiencing an exponential growth (Vladimir, 1998).

Buyers like the convenience that e-commerce provides. They can compare prices and make purchases without leaving the house by visiting the World Wide Web (www) sites of numerous suppliers 24 hours a day, seven days a week. For sellers, e-commerce offers a way to cut costs and expand their markets. They do not need to hire staff or maintain a store or distribute mail order catalogs. For retailers, e-commerce allows them to save money while also expanding their customer base. They don't have to hire people, keep a store running, or print and distribute catalogues. Sellers have the ability to market their products or services globally because they sell over the global internet and are not limited by the physical location of a store.

There are several drawbacks to e-commerce. Some customers are hesitant to make online purchases. Customers want to test the comfort of an expensive item, thus online furniture enterprises, for example, have failed for the most part. Many people consider shopping to be a social experience; for example, they may enjoy going to a shop or a shopping mall with family and friends, which they cannot get online. Customers Furthermore, must be guaranteed that credit card transactions are safe and secure. Their personal information is kept confidential. E-commerce not only expands the range of products available to customers, but it also makes it easier for them to find what they want, not only in terms of products and services, but also in terms of attracting new customers and retaining existing ones.

1.4 Objective

- To interpret and to find if there is any relation between place of residence and the mode of preference of food delivery services.

- To interpret and to find if there is any relation between age group and variation in the usage of online food delivery services.
- To interpret and to find if there is any relation between age group and amount of money spend on online food delivery services.
- To interpret and to find if there is any relation between age group and reasons for choosing on online food delivery services.

Chapter 2

METHODOLOGY

2.1 Data Collection

In order to meet the research objectives, it is critical that the data collected is accurate. All data sources can be divided into two categories: Primary data is gained by direct observation or data collected by the researcher. It refers to data that has been acquired for a specific purpose from a field of inquiry and is of a unique type. Primary data for the project was acquired primarily using the survey approach, utilising the tool questionnaire. Secondary data are ones that have already been acquired by others for a specific reason and are then used in a variety of situations. It is second hand information on an incident that the researchers have not personally witnessed.

Customers who order food online are considered the study's target group. The data was taken between November 23rd and January 1st, 2022. India's statewide lockdown began on March 25, 2020, in order to restrict population movement. The government, on the other hand, allowed e-commerce businesses to continue operating during this time. A well-structured online questionnaire was created using the Google forms and distributed to the responders. During the lockdown, an online-based survey is a viable option for data collection to safeguard the safety of respondents and researchers. Students, employed, and unemployed citizens were among those who responded. We distribute the survey via WhatsApp and social media sites. This study was carried out

with the permission of all participants, and no personal information was gathered. The samples were collected from Ernakulam district, Kerala. Numerous Indian state governments did not allow the operation of on-line food delivery during the statewide lockdown, although many well-established online food delivery businesses like Zomato and Swiggy were fully active in Ernakulam during the nationwide shutdown.

2.2 Questionnaire

The Questionnaire is used to collect data from responders, and it is created using Google Forms. It consists of a sequence of questions that the investigators are expected to ask and the respondents are supposed to choose an alternative for each individual enquiry. Questions are in the form of multiple choice questions. There is no personal information collected. Customers that order food online and live in the Ernakulam city area were chosen at random for primary data. Data was collected using a standardised questionnaire. The sample size was determined by taking 500 respondents from the total population of respondents in Ernakulam.

2.3 Chi-Square Test

A chi-square statistic is a measure of the difference between the observed and expected frequencies of the outcomes of a set of events or variables. Chi-Square depends on the size of the difference between actual and observed values, the degrees of freedom, and the sample size. Chi-Square can be used to test whether two variables are related or independent from one another. It can also be used to test the goodness-of-fit between an observed distribution and a theoretical distribution of frequencies.

When the chi-square test is used as a test of independence, it allows the researcher to test whether the two attributes being tested are associated or not. For this test, a null and alternative hypothesis is for-

mulated where the *null hypothesis* that the two attributes are not associated, and the *alternative hypothesis* is that the attributes are associated.

From the given data, the expected frequencies are then calculated i.e.,

$$\text{ExpectedFrequency} = \text{RowTotal} \times \text{ColoumnTotal} \div \text{GrandTotal} \quad (2.1)$$

followed by the calculation of chi-square value. The null or alternative hypothesis is accepted based on the calculated chi-square value. If the calculated chi-square value is less than the value in the table at the given level of significance, the null hypothesis is accepted, indicating that no relationship exists between the two attributes. If the calculated chi-square value is greater than the value in the table, the alternative hypothesis is accepted, indicating that there is a relationship between the two attributes.

The *p-value* or the calculated probability is the best probability to provide the smallest level of significance at which the null hypothesis is not true. If the *p-value is small (less than 0.05)*, it indicates a piece of strong evidence against the null hypothesis. As a result, the null hypothesis is rejected and the alternative hypothesis is accepted. This means that the results of the research study are statistically significant. If the *p-value is large (greater than 0.05)*, it indicates weak evidence against the null hypothesis. As a result, the null hypothesis is not rejected and the alternative hypothesis is not accepted. This means that the results of the research study are not statistically significant.

In order to find the p-value from the chi-square test, at first, the chi-square test is to be performed to obtain the chi-square value. While performing the test, the *degree of freedom* is also calculated by the formula, $d.f = (c-1)(r-1)$ where c is the number of columns and r is the number of rows. Now the chi-square distribution table is entered, with the obtained degree of freedom, and the value of the chi-square is found in the table.

Chapter 3

DATA ANALYSIS

3.1 Survey Analysis

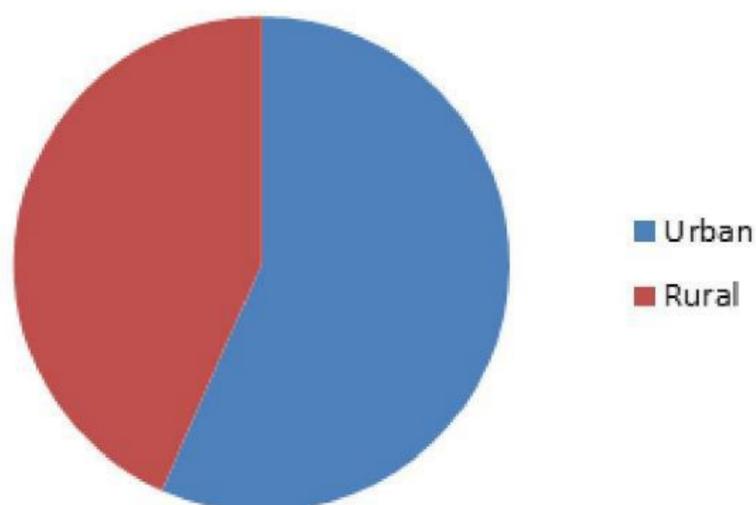
1.PLACE OF RESIDENCE

About 55.2% of responses were from urban area while 44.8% were from rural area.

Area	No Of Responses
Urban	276
Rural	224

Table 3.1: Frequency

Place of residence.

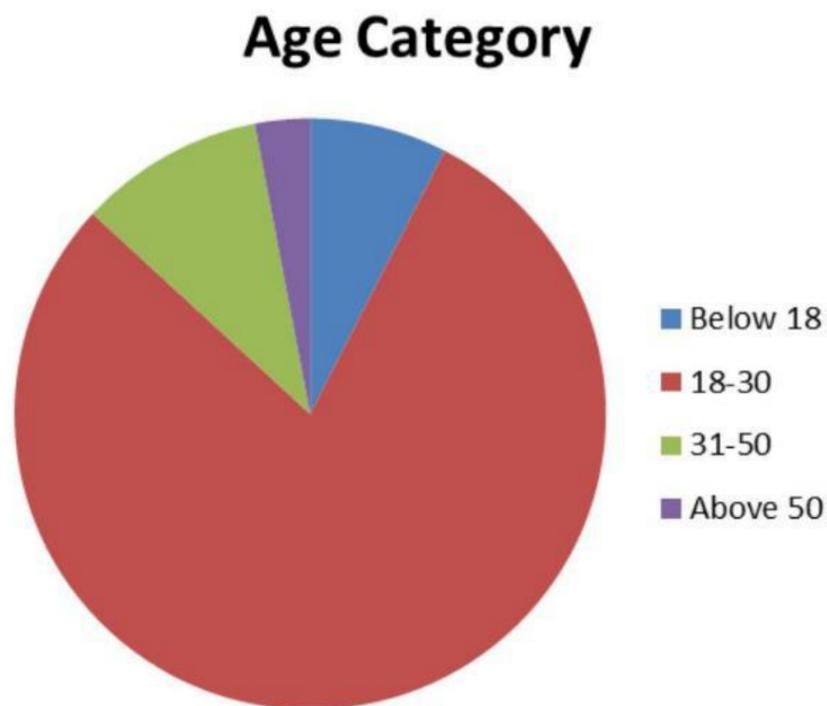


2) AGE CATEGORY

About 8.8% of the respondents belong to below 18 category, 74.2% belong to 18- 30 category, 12.6% belongs to 30-50 category and 4.4% belongs to Above 50 category.

Area Group	No Of Responses
Below18	44
18-30	371
30-50	63
Above50	22

Table 3.2: Frequency

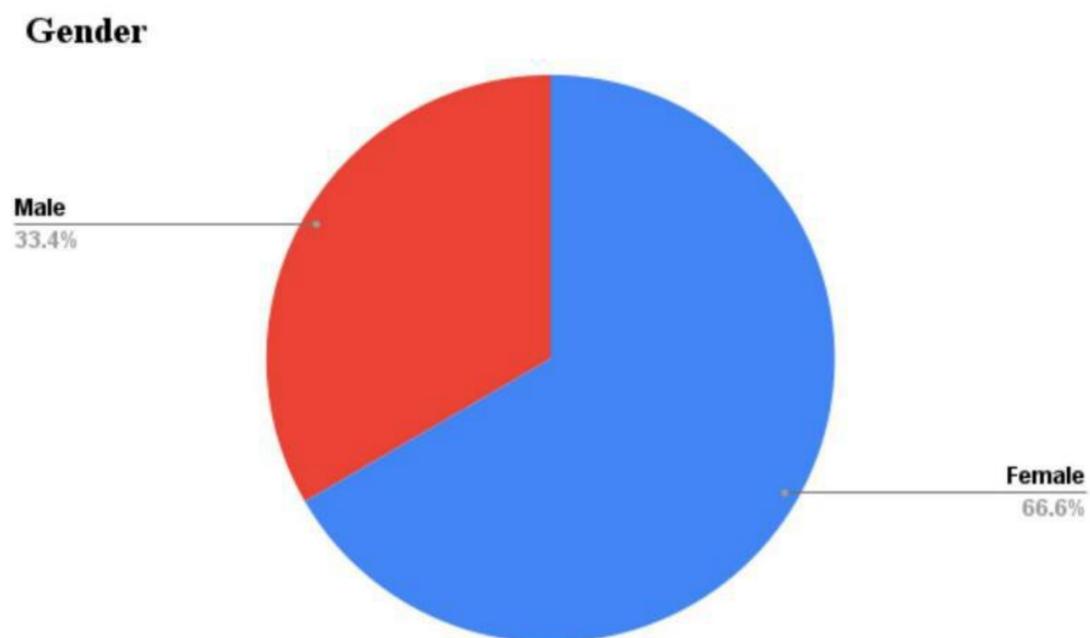


3) GENDER

About 66.6% responses were received from female category, about 33.4% responses were received from male category.

Gender	No Of Responses
Male	167
Female	333

Table 3.3: Frequency

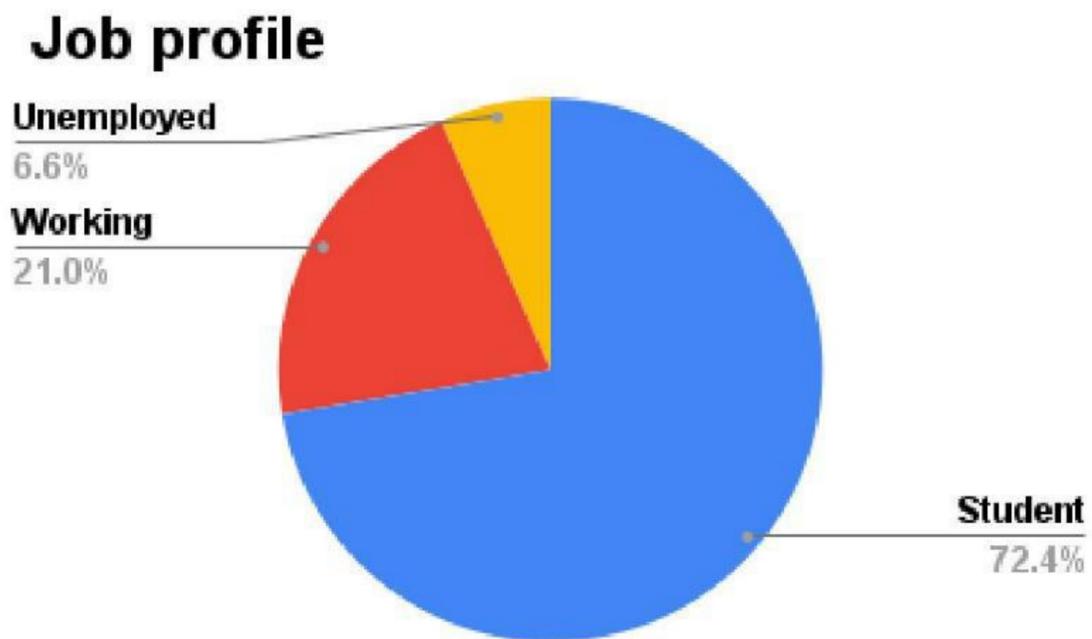


4) JOB PROFILE

About 72.4 % of respondents are students, 21 % of respondents are working and about 6.6 % of respondents are unemployed.

Job Profile	No Of Respondents
Students	362
Working	105
Unemployed	33

Table 3.4: Frequency

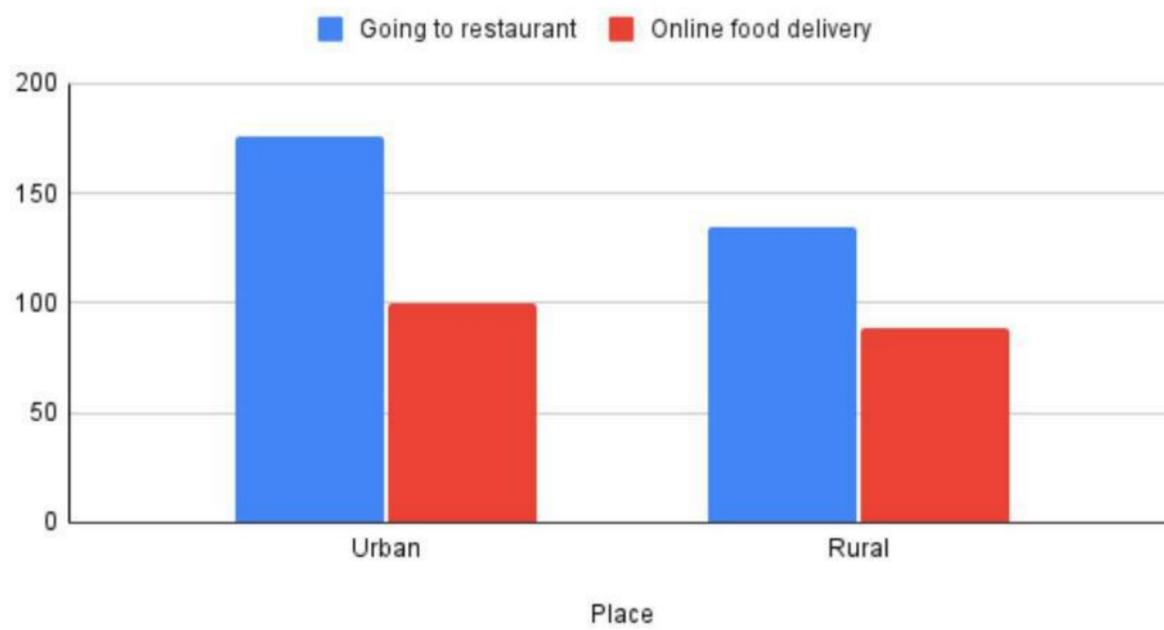


5) PLACE OF RESIDENCE v/s FOOD SERVICE PREFERENCES.

Place	Going To Restaurants	Online Food Delivery
Urban	176	100
Rural	135	89

Table 3.5: Frequency

Place	Going to restaurant	Online food delivery
Urban	56.6%	52.91%
Rural	43.4%	47.09%

PLACE OF RESIDENCE v/s FOOD SERVICE PREFERENCES.

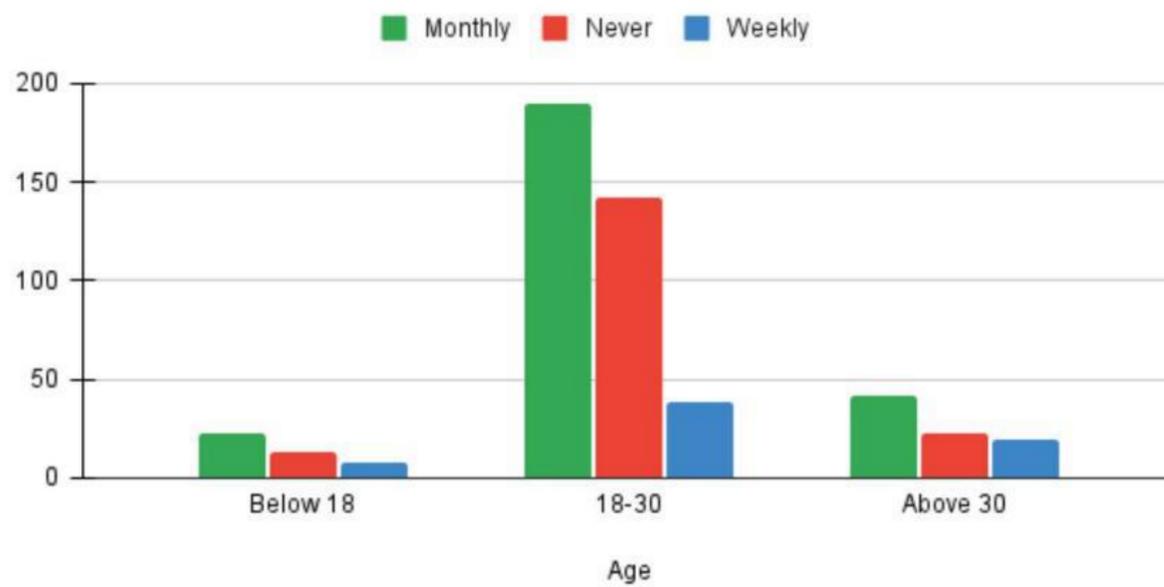
6. AGE CATEGORY V/S USAGE OF ONLINE FOOD DELIVERY SERVICES BEFORE COVID

Age	Weekly	Monthly	Never
Below 18	8	23	13
18-30	39	190	142
Above 30	20	42	23

Table 3.6: Frequency

Age	Weekly	Monthly	Never
Below 18	11.94%	9.01%	7.3%
18-30	58.21%	74.5%	79.78%
Above 30	29.85%	16.47%	12.92%

AGE CATEGORY V/S USAGE OF ONLINE FOOD DELIVERY SERVICES BEFORE COVID.



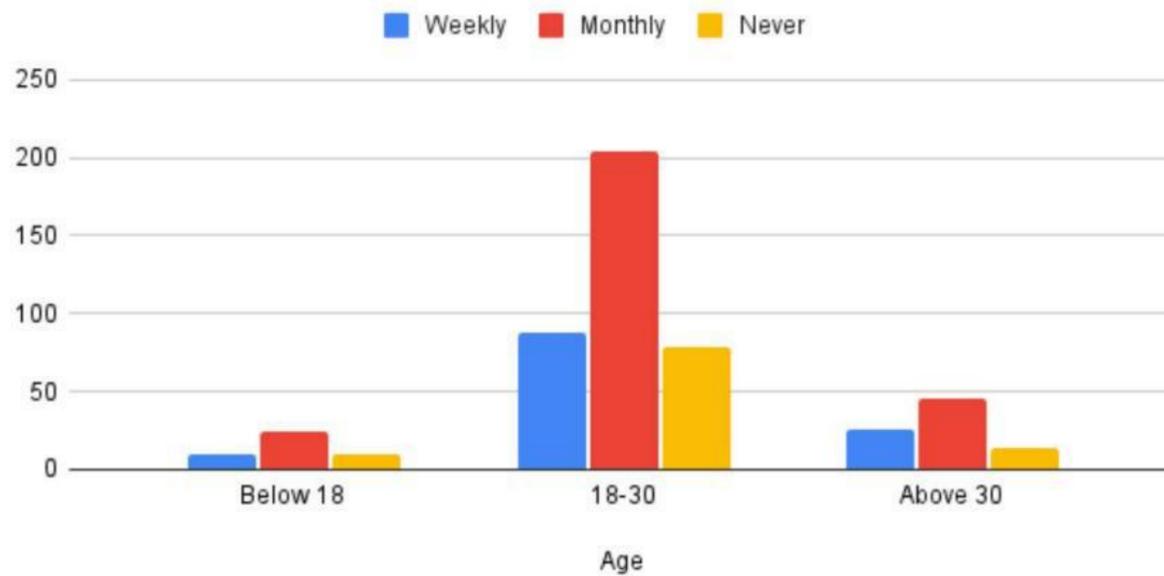
7. AGE CATEGORY V/S USAGE OF FOOD DELIVERY SERVICES DURING COVID

Age	Weekly	Monthly	Never
Below 18	10	24	10
18-30	88	204	79
Above 30	26	45	14

Table 3.7: Frequency

Age	Weekly	Monthly	Never
Below 18	8%	8.8%	9.7%
18-30	71%	75.72%	76.7%
Above 30	21%	16.48%	13.6%

AGE CATEGORY V/S USAGE OF ONLINE FOOD DELIVERY SERVICES DURING COVID.



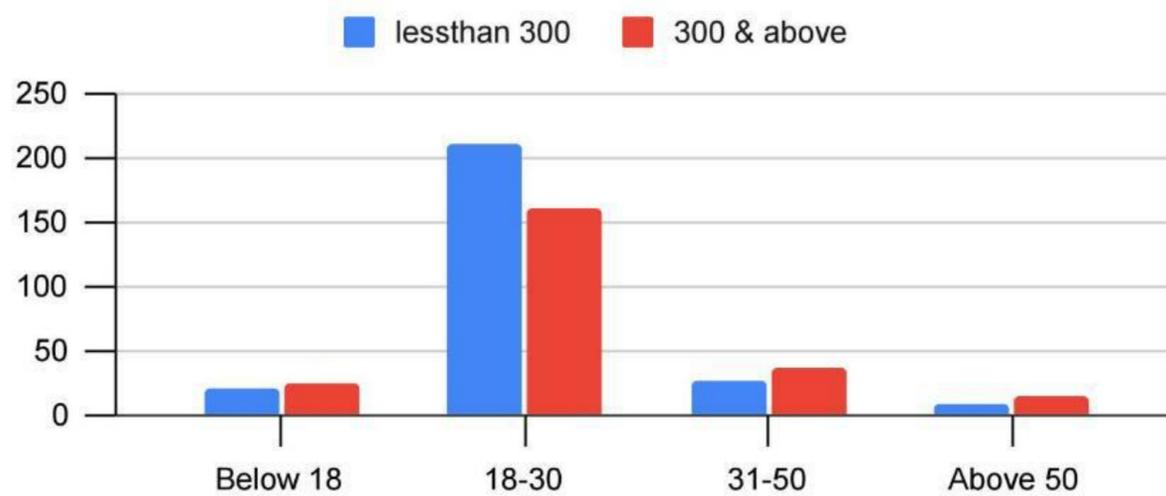
8. AGE CATEGORY V/S AMOUNT SPEND ON ONLINE FOOD DELIVERY BEFORE COVID

Age	Less than 300	300 and above
Below 18	20	24
18-30	211	160
31-50	27	35
Above 50	8	14

Table 3.8: Frequency

Age	Less than 300	300 and above
Below 18	7.51%	10.25%
18-30	79.32%	68.37%
31-50	10.15%	15.38%
Above 50	3%	6%

AGE CATEGORY VS AMOUNT SPEND ON ONLINE FOOD DELIVERY BEFORE COVID



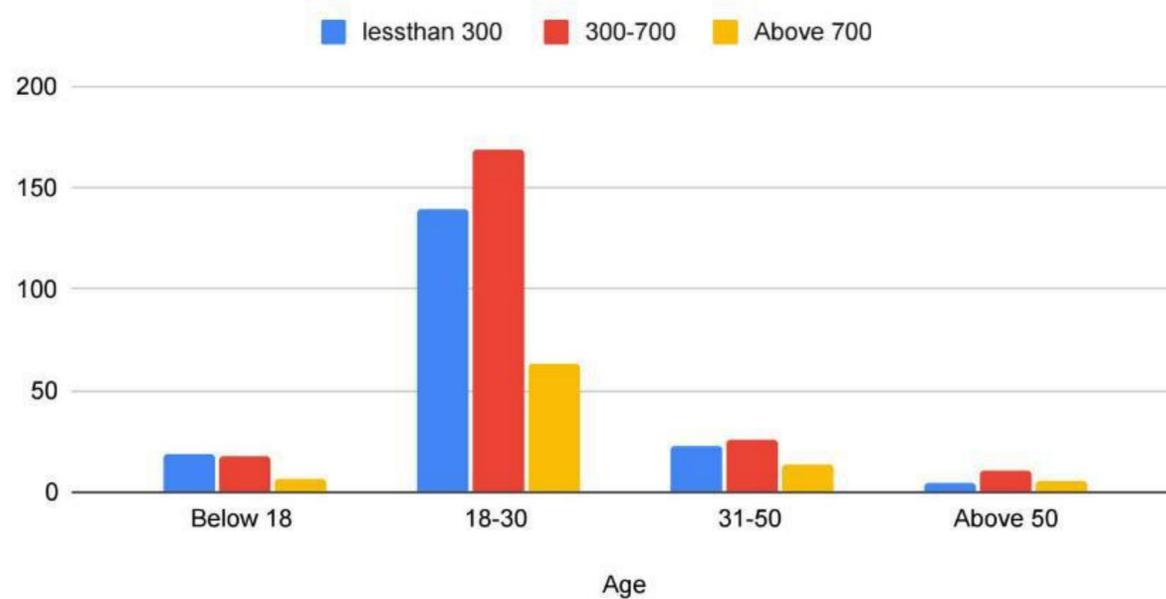
9. AGE CATEGORY V/S AMOUNT SPEND ON ONLINE FOOD DELIVERY DURING COVID

Age	Less than 300	300-700	Above 700
Below 18	19	18	7
18-30	139	169	63
31-50	23	26	14
Above 50	5	11	6

Table 3.9: Frequency

Age	Less than 300	300-700	Above 700
Below 18	10.21%	8.03%	7.78%
18-30	74.73%	75.44%	70%
31-50	12.36%	11.6%	15.55%
Above 50	2.68%	4.91%	6.67%

AGE CATEGORY V/S AMOUNT SPEND ON ONLINE FOOD DELIVERY DURING COVID



10. AGE CATEGORY v/s REASONS FOR CHOOSING ONLINE FOOD DELIVERY SERVICES BEFORE COVID.

Age	Convenient	Fast delivery	Time saving	All of the above
Below 18	8	8	6	22
18-30	114	22	55	180
Above 30	25	7	19	34

Table 3.10: Frequency

Age	Convenient	Fast delivery	Time saving	All of the above
Below 18	5.44%	21.62%	7.5%	9.32%
18-30	77.55%	59.45%	68.75%	76.27%
Above 30	17%	18.91%	23.75%	14.4%

AGE CATEGORY VS REASONS OR CHOOSING ONLINE FOOD DELIVERY BEFORE OVID



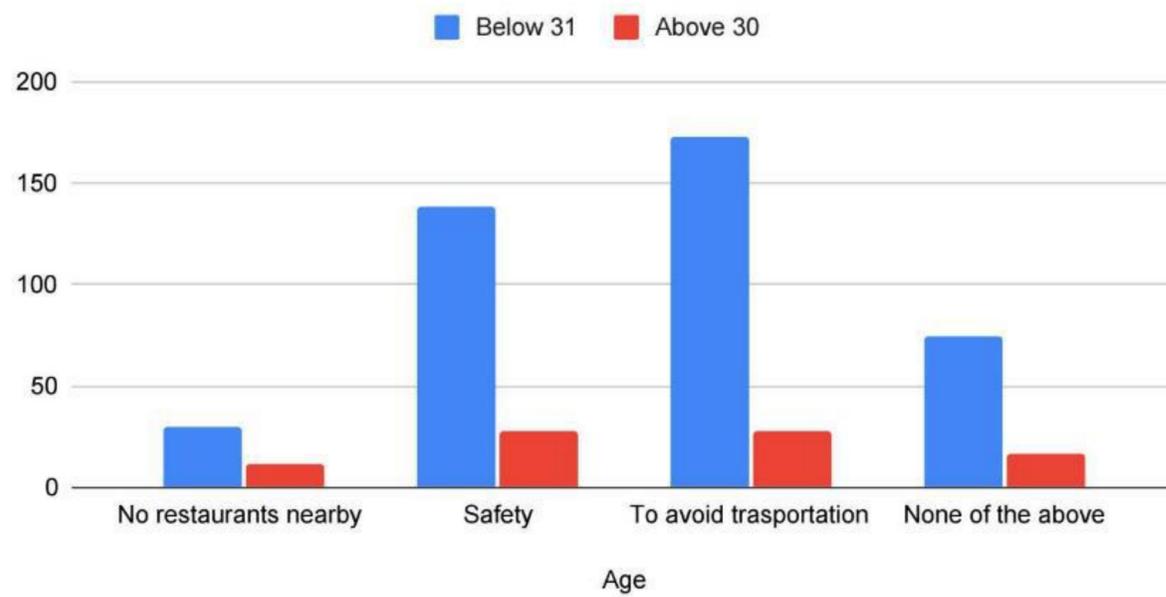
11. AGE CATEGORY v/s REASONS FOR CHOOSING ONLINE FOOD DELIVERY SERVICES AFTER COVID

Age	No restaurants nearby	Safety	To avoid transportation	None of the above
Below 31	30	138	173	74
Above 30	12	28	28	17

Table 3.11: Frequency

Age	No restaurants nearby	Safety	To avoid transportation	None of the above
Below 31	71.42%	83.13%	86.06%	81.31%
Above 30	28.57%	16.86%	13.93%	18.68%

AGE CATEGORY v/s REASONS FOR CHOOSING ONLINE FOOD DELIVERY DURING COVID



3.2 Data Analysis

1. MODE OF PREFERENCE V/S PLACE OF RESIDENCE

Place	Going to restaurant	Online food delivery	Total
Urban	176	100	276
Rural	135	89	224
Total	311	189	500

Table 3.12: Observed frequency

H_0 : There is no relation between place of residence and mode of preference.

H_1 : There is relation between place of residence and mode of preference.

Significance Value: 0.05

Place	Going to restaurant	Online food delivery	Total
Urban	171.672	104.328	276
Rural	139.328	84.672	224
Total	311	189	500

Table 3.13: Expected frequency

χ^2 value : $P = 0.422 > 0.05$

H_0 is accepted

Hence there is no relation.

2.USAGE OF FOOD DELIVERY BEFORE COVID V/S AGE CATEGORY

Age	Weekly	Monthly	Never	TOTAL
Below 18	8	23	13	44
18-30	39	190	142	371
Above 30	20	42	23	85
Total	67	255	178	500

Table 3.14: Observed frequency

H_0 There is no relation between age category and usage of food delivery services before covid

H_1 There is relation between age category and usage of food delivery services before covid

Significance Value: 0.05

Age	Weekly	Monthly	Never	Total
Below 18	5.896	22.44	15.664	44
18-30	49.714	189.21	132.076	371
Above 30	11.39	43.35	30.26	85
Total	67	225	178	500

Table 3.15: Expected frequency

χ^2 value : $P = 0.00714 < 0.05$

H_0 is rejected

Hence there is relation

3.USAGE OF FOOD DELIVERY DURING COVID V/S AGE CATEGORY

Age	Weekly	Monthly	Never	Total
Below 18	10	24	10	44
18-30	88	204	79	371
Above 30	26	45	14	85
Total	124	273	103	500

Table 3.16: Observed frequency

H_0 : There is no relation between age category and usage of food delivery services after covid

H_1 : There is relation between age category and usage of food delivery services after covid

Significance Value: 0.05

Age	Weekly	Monthly	Never	Total
Below 18	10.912	24.024	9.064	44
18-30	92.008	202.566	76.426	371
Above 30	21.08	46.41	17.51	85
Total	124	273	103	500

Table 3.17: Expected frequency

χ^2 value : $P = 0.24 > 0.05$

H_0 is accepted

Hence there is no relation

4. AMOUNT SPEND ON FOOD DELIVERY BEFORE COVID V/S AGE CATEGORY

Age	Less than 300	300 and Above	Total
Below 18	20	24	44
18-30	211	160	371
31-50	27	36	63
Above 50	8	14	22
Total	266	234	500

Table 3.18: Observed frequency

H_0 : There is no relation between age category and the amount spend on food delivery before covid.

H_1 : There is relation between age category and the amount spend on food delivery before covid

Significance Value: 0.05

Age	Less than 300	300 Above	Total
Below 18	23.408	20.592	44
18-30	197.372	173.628	371
31-50	33.516	29.484	63
Above 50	11.704	10.296	22
Total	266	234	500

Table 3.19: Expected frequency

χ^2 value : $P = 0.0405 < 0.05$

H_0 is rejected

Hence there is a relation.

5.AMOUNT SPEND ON FOOD DELIVERY DURING COVID V/S AGE CATEGORY

Age	Less than 300	300-700	Above 700	Total
Below 18	19	18	7	44
18-30	139	169	63	371
31-50	23	26	14	63
Above 50	5	11	6	22
Total	186	224	90	500

Table 3.20: Observed frequency

H_0 : There is no relation between age category and the amount spend on food delivery during covid

H_1 : There is relation between age category and the amount spend on food delivery during covid

Significance Value: 0.05

Age	Less than 300	300-700	Above 700	Total
Below 18	16.368	19.712	7.92	44
18-30	138.012	166.208	66.78	371
31-50	23.436	28.224	11.34	63
Above 50	8.184	9.856	3.96	22
Total	186	224	90	500

Table 3.21: Expected frequency

χ^2 value : $P = 0.652 > 0.05$

H_0 is accepted

Hence there is no relation

6. AGE CATEGORY V/S REASONS FOR CHOOSING ONLINE FOOD DELIVERY SERVICES BEFORE COVID

Age	Convenient	Fast delivery	Time saving	All of the above	TOTAL
Below 18	8	8	6	22	44
18-30	114	22	55	180	371
Above 30	25	7	19	34	85
TOTAL	147	37	80	236	500

Table 3.22: observed frequency

H_0 : There is no relation between age category and the reasons for choosing online food delivery before covid

H_1 : There is relation between age category and the reasons for choosing online food delivery before covid

Significance Value: 0.05

Age	Convenient	Fast delivery	Time saving	All of the above	TOTAL
Below 18	12.936	3.256	7.04	20.768	44
18-30	109.074	27.454	59.36	175.112	371
Above 30	24.99	6.29	13.6	40.12	85
TOTAL	147	37	80	236	500

Table 3.23: Expected frequency

χ^2 value : $P = 0.0302 < 0.05$

H_0 is rejected

Hence there is relation

7. AGE CATEGORY V/S REASONS FOR CHOOSING ONLINE FOOD DELIVERY SERVICES DURING COVID.

Age	No restaurants nearby	Safety	To avoid transportation	None of the above	TOTAL
Below 31	30	138	173	74	415
Above 30	12	28	28	17	85
TOTAL	42	166	201	91	500

Table 3.24: observed frequency

H_0 : There is no relation between age category and reason for choosing online food delivery during Covid.

H_1 : There is a relation between age category and reason for choosing online food delivery during Covid.

Significance level: 0.05

Age	No restaurants nearby	Safety	To avoid transportation	None of the above	TOTAL
Below 30	34.86	137.86	166.83	75.53	415
Above 30	7.14	28.22	34.17	15.47	85
TOTAL	42	166	201	91	500

Table 3.25: Expected Frequency

χ^2 value : $P = 0.137 > 0.05$

H_0 is accepted.

Hence there is no relation.

Chapter 4

RESULT AND CONCLUSION

4.1 Conclusion

1.PLACE OF RESIDENCE V/S FOOD SERVICE PREFERENCES.

In the case of online food delivery, people living in both rural and urban areas have equal accessibility and availability to online food services. Restaurants are growing at a fast rate in both rural and urban areas. People can directly go to the restaurants according to their needs. Hence there is no relation between place of residence and food service preferences.

2.USAGE OF FOOD DELIVERY SERVICES V/S AGE CATEGORY.

There is a relation between age categories and usage of food delivery services before covid. According to our survey, people belonging to the age category between 18-30 are most likely to order food online as most of them are working class and they do not have enough time to cook food. So it would be convenient for them to order food online. These are the ones who use smartphones more frequently and they are more familiar with food ordering applications. As we can see from the above conclusion, people belonging to the category above 50 are less likely to order food online because they are less aware of modern technologies. So they prefer to have healthy homemade foods. Hence here we can

conclude that age affect the usage of food delivery.

There is no relation between age categories and usage of food delivery services during covid. People have been held at home due to the govt imposed lockdowns, preventing regular shopping habits. Many restaurants have closed, thus forcing the consumers to buy food online irrespective of their preferences. Covid-19 hit everyone so badly that the working industry had to change to work from home. The people belonging to this sector didn't have enough time to cook, forcing them to order food online. All categories started showing a slight increase in the usage of online food delivery, because of the less accessibility of transportation services and safety concerns. All age groups faced equal difficulty during Quarantine period. Online food delivery platforms played a vital role in the lives of all people. Thus irrespective to their gender and age all people started using online food delivery services. Hence here we can conclude that age does not affect the usage of online food delivery services.

3. AMOUNT SPEND ON ONLINE FOOD DELIVERY SERVICES V/S AGE CATEGORY.

There is a relation between age categories and the amount spent on online food delivery services before covid-19. The people belonging to the age categories between 18-30 are usually the ones that spend more money on ordering food online. They love to live their lives to the fullest and also they love to try more new dishes. Before corona most of them were financially stable and it was affordable for them to order food. And also because they are hesitant to cook and they prefer to order food instead. On the other hand people of other age groups are very much self aware of their income and preferably would like to save money rather than spending more on eating junk food. Hence here we can conclude that age affects the amount spend on food delivery before covid.

There is no relation between age categories and the amount spent on online food delivery services during covid-19. Food is essential to human life since it supplies energy for all activities, growth, and development. People of all ages require food. During Covid-19, more individuals, particularly those in quarantine, had no choice but to purchase food online. People are willing to order food online even if it means spending more money for delivery because of food safety, quick delivery, and the availability of a wide variety of food items. As a result, when the pandemic began, people were not interested in purchasing food through online platforms. Because the population's fear and anxiety levels have increased, the perception of risk can also include the fear of contagion via food, packaging, and contact with the delivery person at the time of delivery. However, people's perspectives have gradually improved. All categories have begun to buy meals through these online platforms because they restrict customer contact with restaurant employees and let customers to enjoy their favourite restaurant food at home during a pandemic like COVID-19. Hence here we can say that age does not affects the amount spend on online food delivery services during covid.

4.REASONS FOR CHOOSING ONLINE FOOD DELIVERY SERVICES V/S AGE CATEGORY.

There is a relation between age categories and the reasons for choosing online food delivery services before covid. Because even before covid 19 pandemic people belonging to different age groups had different opinions. Nowadays people don't have enough time to go out and eat, so they started taking more advantage of the online food delivery platforms. Some might order food online as to reduce transportation, hesitation to cook (especially for those who find it difficult to cook well) or to save time. People are generally more satisfied with online delivery because it is done at their convenience and avoids the stress of waiting, standing in line, takeout, and so on. The waiting time is well spent by

doing something else at home or at work. The people belonging to the age category 18-30 are more likely to order food online and older adults are also making use of this online food delivery platforms. Therefore each of them have their own reasons. Hence here we can conclude that age affects the reasons for choosing online food delivery services.

There is no relation between age categories and the reasons for choosing online food delivery services during covid. Because majority of the respondents across all age groups started ordering food online as they practiced social distancing and safety as to minimise the spread of covid. Home delivery of food items helped people to avoid transportation. Hence, there was no any shortage or unavailability of food items occurred during the pandemic. Because of how covid-19 spreads, the greatest risk of infection could occur through intimate contact with others. Many restaurants now allow customers to pay in advance, either via phone or online, reducing the need for physical cash transfers or credit card processing, both of which might carry the virus. Hence here we can conclude that age doesnot affect the reasons for choosing online food delivery services.

4.2 Result

The sudden onset of the Covid-19 crisis has surprised consumers, companies and government agencies. This study focuses on the effect of Covid-19 pandemic on online food delivery services. Based on the data, we analysed online food delivery platform usage, how it has helped during the Covid-19 pandemic and whether current changes could be long lasting.

Overall, Covid-19 has had a net positive impact on frequency and spending on online food delivery. For those not cooking at home, online delivery service from restaurants is one way to limit the number of trips outside the home. From the survey, we can conclude that usage of food delivery services varies with age category before Covid-19. People belonging to age category 18-30 are likely to order food more often. And

also these people spend more money on food delivery. Despite the fact that a large section of the population in Ernakulam uses online food delivery apps, there are still people who do not use food apps owing to health and quality concerns.

In a nutshell, the majority of users are students and working people, indicating that the online food ordering system is gaining popularity among young people. The changing lifestyles of consumers, as well as the growth of online activity in India, have unquestionably changed the trends in online food ordering.

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Annexure

1. What age category do you belong to?

- Below 18
- 18-30
- 31-50
- Above 50

2. Gender

- Male
- Female
- Others

3. Place of residence

- Urban
- Rural

4. Job profile

- Working
- Unemployed
- Student

5. What do you prefer?

- Going to restaurants
- Online food delivery

6. How often do you order food online before covid-19?

- Daily
- Weekly
- Monthly

- Never

7. How often do you order food online during covid-19?

- Daily
- Weekly
- Monthly
- Never

8. How much do you spend on online food delivery before covid-19?

- Less than 300
- 300-700
- Above 700

9. How much do you spend on online food delivery during covid-19?

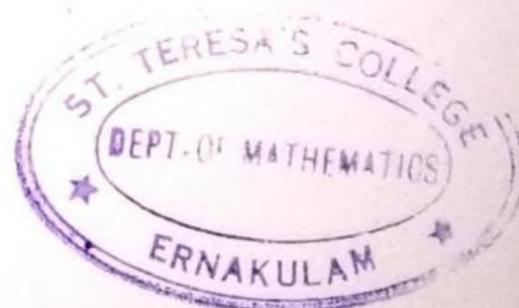
- Less than 300
- 300-700
- Above 700

10. Why do you prefer online food delivery before covid-19?

- Fast delivery
- Convenient
- Time saving
- All of the above

11. Why do you prefer online food delivery during covid-19?

- Safety
- To avoid transportation
- No restaurant nearby
- None of the above



Project Report

On

**STUDY ON APPLICATIONS OF CHAOS
THEORY**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

Shanolia Joseph

(Register No. AB19AMAT028)

Under the Supervision of

MRS. DONNA PINHERIO



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APRIL 2022

ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM



CERTIFICATE

This is to certify that the dissertation entitled, **STUDY ON APPLICATIONS OF CHAOS THEORY** is a bonafide record of the work done by Ms. **SHANOLIA JOSEPH** under my guidance as partial fulfilment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 09.03.2022
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External Examiners

1:.....*Juma*.....*Sex* 

2:

DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Mrs. Donna Pinherio, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date: 09.03.2022



SHANOLIA JOSEPH

AB19AMAT028

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Ernakulam.

Date: 09.03.2022

SHANOLIA JOSEPH

AB19AMAT028

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Chapter 1

INTRODUCTION TO CHAOS THEORY

1.1 CHAOS THEORY

We can see chaos around us from changes in the weather, arrhythmia in the heart, Patterns of traffic movement, Development and decay in cities, Epidemics and can be found even in People's collective behaviour.

Mathematicians say it is tough to define chaos, but is easy to "recognize it when you see it". Chaos and order are two sides of the coin as they are interrelated.

Chaos Theory is the study of non linear, complex, dynamic system. It's a branch of mathematics that deals with the systems that appear to be ordered (deterministic) but actually exhibit chaotic behaviour. It also deals with systems that appear to be chaotic but are actually ordered beneath the surface. But this deterministic nature does not make it predicable, so this behaviour can be called deterministic chaos or simple chaos. we understands that nature is highly complex and we can make predict that it is unpredictable but Chaos theory has succeeded in capturing and displaying the beauty of the unpredictable in the most amazing patterns.

Chaos theory studies the behavior of dynamic system that are highly

sensitive to initial condition an effect which is perfectly connected as an butterfly effect. A butterfly flapping it's wings can give rise to a chain of events that might end up with creating a thunderstorm in some distinct place. This is only an example and it's an idea that applies to everything in our universe .Tiny changes in initial condition can results in that are very different from each other and thus unpredictable.

1.2 HISTORY OF CHAOS THEORY

EDWARD LORENZ: THE FATHER OF CHAOS THEORY

Lorenz, the great mathematician is the father and author of Chaos theory. Through his great effort and discovery, the concept of Chaos theory has grown to one of the branches of mathematics. Lorenz's discovery has achieved great attention now, but it was not much appreciated in the initial stages by some Meteorologists and Climatologists and fails to realize the broader mathematical implications. Later in history, Lorenz's publications gained all the possibilities to the principle of Sensitive Dependence on Initial Conditions (SDIC).

The principle of SDIC was agreed upon by most mathematicians including Maxwell, Hadamard, and Poincare in 1890. Renaissance was one of the mathematicians who played a great role to for show the modern Chaos Theory. Polymath and Leonard da Vinci were able to depict spectacular turbulence with the concept of Chaos Theory.

Another mathematician named Richardson writes poetically on Chaos Theory: "Big whorls have little whorls, This feed on their velocity: And so on to Viscosity: And little whorls have lesser whorls."

While reading the pages on the origin of Chaos Theory might have a notion that Lorenz did much less compared to all other mathematicians. But the important thing is that he discovered the base for all the above theories. He can be claimed to have discovered "Chaos" amidst smooth runways in the mathematical field. Therefore Lorenz is

the one who derived the mathematical understanding by integrating all the elements together with Sensitive Dependence on Initial Conditions (SDIC), Fractals, Attractors, and resulting erratic dynamics. Today the concept of Chaos Theory has become a part of our real life and it's a new way of looking at things that already exist amidst us.

1.3 CHAOS THEORY IN MATHEMATICAL SENSE

We will consider the following systems of two linear equations:

$$X_{n+1} = 2x_n + y_n + 1$$

$$Y_{n+1} = x_n + 2y_n + 2$$

The equation can be written in matrix form:

$$\begin{Bmatrix} x_{n+1} \\ y_{n+1} \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} x_n \\ y_n \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

Let repeat the equations several times to see if chaos occurs in this example. Let us start with the initial condition and make the first repeatedly as follows:

$$\begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

The second iteration is then obtained as follows:

$$\begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

Substituting the values of the first iteration in the above equation, we obtain:

$$\begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} \right) + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

Simplifying the above equation, we obtain:

$$\begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^2 \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

Continuing with the third iteration upon the above equation and simplifying the results, we obtain:

$$\begin{Bmatrix} x_3 \\ y_3 \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^3 \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^2 \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

In general, we obtain the following formula after performing the n th iteration:

$$\begin{Bmatrix} x_n \\ y_n \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{n-1} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{n-2} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \dots + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} + \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

Next, we will re-write the above equation in the following more simplified form:

$$\begin{Bmatrix} x_n \\ y_n \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} + \left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{n-1} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{n-2} + \dots + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

The expression inside the parenthesis is a geometric series of the following form:

$$[A]^{n-1} + [A]^{n-2} + \dots + [I]$$

Let us now consider what happens when we change the initial condition. In order to simplify the above expression, we need to use the following mathematical identity:

$$\left([A]^{n-1} + [A]^{n-2} + \dots + [I] \right) \left([I] - [A] \right) = [I] - [A]^n$$

Therefore, the n th iteration of the linear system of equations becomes:

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \right) \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right)^{-1} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

The above equation is an explicit equation for the n-th iterate of $\begin{pmatrix} X \\ Y \end{pmatrix}$

Let us now consider what happens when we change the initial condition

$\begin{pmatrix} X(0) \\ Y(0) \end{pmatrix}$ slightly. Now consider the new initial condition $\begin{pmatrix} x(0) + \varepsilon \\ y(0) + \delta \end{pmatrix}$

where ε and δ are very small numbers. Let us call the new iterate in this

case $\begin{pmatrix} X_m \\ Y_m \end{pmatrix}$ m- new initial value

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^m \begin{pmatrix} x_0 + \varepsilon \\ y_0 + \delta \end{pmatrix} + \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^m \right) \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right)^{-1} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

subtracting two equation, we get,

$$\begin{pmatrix} x_m \\ y_m \end{pmatrix} - \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^m \begin{pmatrix} x_0 + \varepsilon \\ y_0 + \delta \end{pmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

+

$$\left\langle \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^m \right) \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right)^{-1} - \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \right) \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right)^{-1} \right\rangle$$

if this linear equation is chaotic, then we let $n = m$, we get

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} - \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n \begin{pmatrix} \varepsilon \\ \delta \end{pmatrix}$$

we can say that ε and δ are very small numbers. When the value of n increases the above equation will not get zero. This is because of the matrix that we got in equation. The expression goes to infinity as n increases to infinity. May be the values of matrix be equal to one or more than one the expression goes to infinity. If the eigenvalues of the matrix are less than one, then the expression goes to zero. If the value of one or more of the eigenvalues is greater than 1, then the system

is chaotic. On the other hand, if all the eigenvalues of the coefficient matrix have values less than 1, then the system is not chaotic.

Chapter 2

CHAOTIC SYSTEM AND FRACTALS

2.1 Chaotic System

A key concept of Chaos Theory is that chaos and order are not completely in opposition. Chaotic systems are a mix of these two i.e. when observed from the outside they show unpredictable and chaotic behaviour but on the inside they are a mixture of perfectly deterministic set of equations. Due to sudden, unstable and chaotic causes some systems show order. Therefore, chaotic systems are predictable for a while and then 'appear' to become random. The amount of time that the behaviour of a chaotic system can be effectively predicted depends on three factors: accuracy in measuring its current state, a time scale associated with the dynamics of the system, called the Lyapunov time and the amount of uncertainty in the prediction. Some examples of Lyapunov times are: weather systems-a few days (unproven); the inner solar system-4 to 5 million years. In chaotic systems, the uncertainty in a prediction increases rapidly as the time increases rapidly. Hence, mathematically, when the prediction time is doubled the uncertainty in prediction is squared. Therefore, in practice a meaningful prediction cannot be made over an interval of more than two or three times the Lyapunov time. When meaningful predictions cannot be made, the system appears random.

Chaotic systems are deterministic but it's said to be chaotic when its evolution depends on the initial conditions. Chaotic systems are immensely found in nature. Many natural phenomena can be marked as chaotic. They can be found in the solar system, in heart, in meteorology and so on. Even a small change in the initial condition can lead to a wide difference in the outcome. Hence, chaotic systems are very much sensitive to the initial condition and this makes them fairly unpredictable.

Characteristics of a chaotic system:

- No periodic behaviour
- Sensitivity to initial conditions
- Chaotic motion is difficult to forecast
- The motion looks random
- Non-linear

The structure of a chaotic system is its preferred set of behaviours and that is determined by a key which is called mathematically as attractors. The mathematician Ian Stewart used the following example to illustrate an attractor. Consider a ping pong ball which is released into an ocean. When it is released above into the water it will fall and when it is released from below it will float. No matter where it is released from, the ball will move to its attractor i.e the ocean surface and is predictable. Though it is harder to predict the behaviour of a chaotic system, knowing its attractor minimizes this feature of unpredictability of a chaotic system. It also allows us to accurately predict how the system will respond if it is jolted off its attractor.



A strange attractor; <http://en.wikipedia.org/wiki/Attractor>

Chaotic systems can be found anywhere around us. Whatever the origin of the chaotic system maybe, it will have the same characteristics.

2.2 Fractals and its Applications

”Fractal” comes from the Latin word *fractus*, which means ”to break into irregular fragments.” The word ”fractal” was first used in 1975 by father of fractals, Benoit B. Mandelbrot, New York. A fractal is a pattern that repeats the same design and detail over a wide range of scale. Hence it is self similar and also chaotic. Even if we repeatedly magnify any piece of a fractal, it looks the same. For instance, a twig and its appendages from the edge of some species of trees form a pattern that repeats the design of the trunk and main branches of the tree. Such repetition of detail or recurrence of statistically identical geometrical patterns as we look at smaller upon smaller parts of the original object, is the unifying theme of fractals. Driven by repetition, fractals are images of dynamic systems – the pictures of Chaos. Fractal patterns don’t have any characteristic size. The definition just given is general. Experts don’t agree on a more explicit or mathematical definition. Fractals are all around us. Examples are:

- branching configurations (blood vessels, river networks, trees, ferns)
- rough surfaces in general (landscapes, mountains)
- objects that undergo fragmentation (coal and rock, soil, asteroids and meteorites)
- earthquake features (frequency distribution of aftershocks)
- flow patterns (jets, clouds, smoke, mists)
- galaxy distributions, size distribution of craters, rings of Saturn, fluctuations in interplanetary magnetic fields
- lightning
- changes in stock prices
- incomes of rich people.

Fractals and Chaos

Fractals deal with geometric patterns and quantitative ways of characterizing those patterns. Chaos, in contrast, deals with time evolution and its underlying or distinguishing characteristics. Even though fractals show chaotic behaviour, one can get a sight of self similarity in fractals. Fractals are a class of geometric forms; chaos is a class of dynamical behaviour. Fractals and chaos are closely connected and they occur together. For instance, fractals with narrow textures can be seen when we look closely into the chaotic attractors. Points on such chaotic attractors when plotted, gives a set of layers which looks the same over a broad range. In general, the chaotic attractors of invertible maps are fractals whereas the chaotic attractors of non-invertible maps may or may not be fractals. Fractals can be used to detect chaos. Examples of fractals showing chaotic behaviour in nature:



(i) clouds



(ii) earth surface topography



(iii) asteroid impact craters in the moon

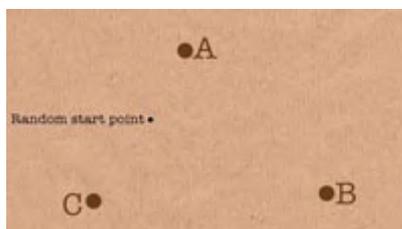


(iv) gullies

2.3 Applications of Fractals

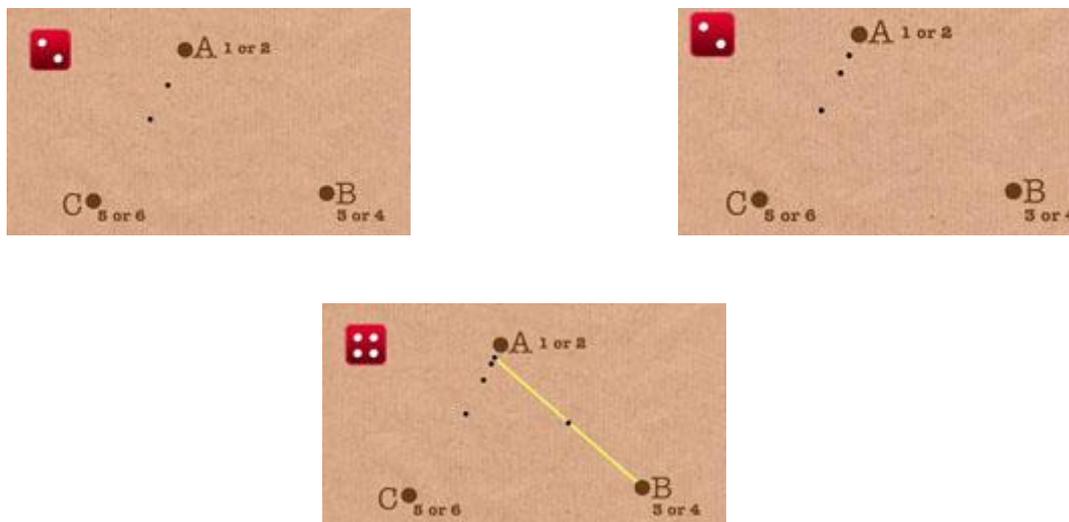
2.3.1 The Sierpiński Triangle

An interesting way in which fractals can be generated is the chaos game. Consider three points; A,B and C forming an equilateral triangle. Then a starting point; M, is chosen randomly on or inside the triangle.

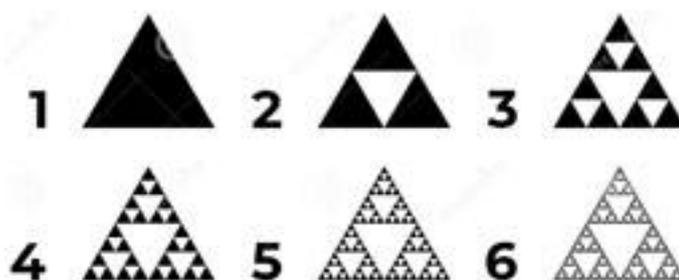


A dice is rolled to decide which point to go towards such that whatever the point we choose we are going halfway towards that point. When the outcome is 1 or 2, we go halfway towards M. When the outcome is 3 or 4, we go halfway towards N and when it's 5 or 6, we go

halfway towards O. if we get the same outcome then, we go halfway to the point (based on the number obtained on the dice) from the last obtained point.



This can be repeated for a long time. We can also run a computer simulation using the same rules. After a lot of iterations, a fractal emerges from total chaos. This is a beautiful fractal called Sierpiński Triangle. Its perimeter tends to infinity since the fractal continues infinitely and its area tends to zero since the black regions are excluded from the fractal and hence its dimension lie between 1 and 2. Therefore, it has a fractal dimension. It can be measured only roughly.



If a one dimensional straight line is scaled by one-half then its mass is also scaled by one half.

$$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^1$$

The power i.e 1 is the dimension of a straight line. Similarly if a two dimensional square is scaled by one-half then its mass is scaled by one-fourth.

$$\left(\frac{1}{4}\right) = \left(\frac{1}{2}\right)^2$$

The power i.e 2 is the dimension of a square. If a three dimensional cube is scaled by one-half then its mass is scaled by one-eighth.

$$\left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^3$$

The power i.e 3 is the dimension of a cube. If a Sierpiskiński triangle is scaled by one-half then we get similar and same pattern, three of which, when arranged in the right pattern, gives back the original triangle. Thus, the mass has been scaled by one-thirds. Following the above line of reasoning, this means that one-half raised to the power of (say) \mathbf{X} , should equal one-third. And \mathbf{X} is the fractal dimension of the Sierpiskiński triangle. Thus,

$$\left(\frac{1}{2}\right)^x = \frac{1}{3}$$

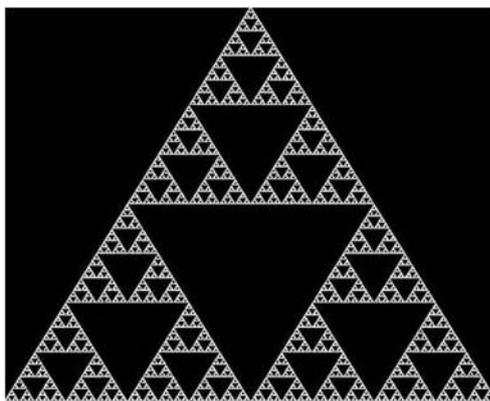
which implies

$$2^x = 3$$

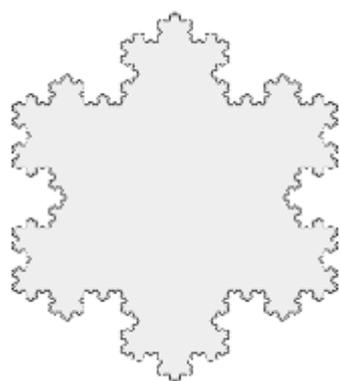
By taking logarithm on both the sides, to the base 2, we get

$$x = \log_2 3 \approx 1.585$$

which is the fractal dimension of a Sierpinski triangle.

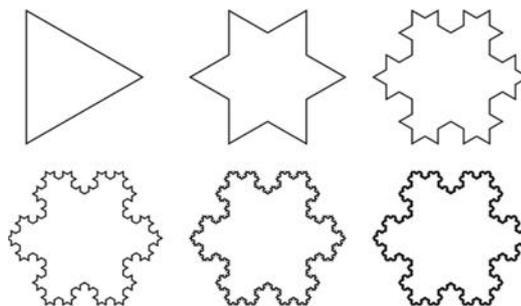


2.3.2 The Koch Snowflake



Helge von Koch, a Swedish mathematician came up with a fractal whose shape is similar to that of a snowflake. It was called the Koch's Snowflake. It can be constructed using an equilateral triangle. The steps to obtain a Koch snowflake from an equilateral triangle are as follows:

1. Draw an equilateral triangle.
2. Another equilateral triangle should be drawn on the middle of each side.
3. Pull out the middle and repeat the process with 1,2,3,4 times 3 which is 12 sides in total.
4. Repeat this process over and over. At the end a fractal which is similar to that of a snowflake is formed.



No matter how much one zoom's in, the same pattern is seen over and over. Never ending patterns like this drawn on any scale or on any maximization look roughly the same called fractals. Each time we add a triangle, one side of the Koch snowflake will turn into four. After the first repetition, we'll get

$$3 \times 4^1 = 12 \text{ sides}$$

After the second repetition, we'll get

$$3 \times 4^2 = 48 \text{ sides}$$

When it's repeated n times, we'll get

$$(3 \times 4^n) \text{ sides}$$

If we do this an infinite number of times, we'll get infinitely many sides. So the perimeter of the Koch snowflake will be infinite. But the area of the Koch snowflake wouldn't be infinite. If a circle is drawn with a finite area around the snowflake, it will completely fit inside the

circle no matter how many times we increase the number of sides. So the Koch fractal has an infinite perimeter, but a finite area. The fractal dimension of the Koch snowflake lies between 1 and 2.

2.3.3 The Barnsley Fern

The Barnsley fern is an example of natural fractals. It was named after the British mathematician, Michael Barnsley.



Like the Sierpinski triangle, the Barnsley fern also produces a beautiful pattern when a particular mathematical formula is iterated over and over again. This can also be done graphically using computer.

Chapter 3

APPLICATIONS OF CHAOS THEORY

3.1 Chaos Theory in Weather Prediction



Figure 3.1: Chaos and Weather

If we look into our surroundings today, we would understand the uncertainty that exists everywhere. The weather is one of them. It's a chaotic system. People can no more predict it as earlier days. Amidst all the uncertainties' let us try to make use of the applications of Chaos Theory to predict the weather. As we know small errors in the initial conditions grow rapidly, and affect predictability is called Chaos. Similarly, small changes in the initial stages of forecast cause chaos in the weather predictability. Furthermore, predictability is limited by errors due to the approximate atmospheric processes of the state-of-the-art numerical models which is a combination of a large number of mathematical equations and depends upon computers to find an accurate solution to the underlying physical problem. . These numerical model sources of uncertainties limit the skill of single, deterministic forecasts

in an unpredictable way, with days of high/ poor quality forecasts which are also randomly followed by days of high/poor quality forecasts. Some dynamical system which is ‘Sensitive Dependence on Initial Conditions’ (SDIC) experience a chaotic behavior when most orbits are sensitive to initial conditions (Lorenz 1993). An orbit is characterized by sensitive dependence if most other orbits that pass close to it at some point do not remain close to the same because the time is taken to reach that particular point advances.

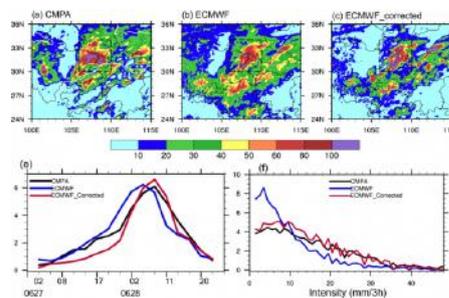


Figure 3.2: Weather Prediction using Chaos Theory

Scientists involved in chaos theory is attempting to examine, describe, and quantify complex and unpredictable dynamics of chaotic systems that are sensitive to their initial conditions but follow mathematics laws—even when their outward appearance appears random. For example, Meteorology, the prediction of weather and climate, is a classic example of such an unpredictable (chaotic) system.

3.1.1 Scientific Foundations of Chaos Theory

Chaotic system was discovered by the Father of Chaos Theory, an American Mathematician and Meteorologist Edward Lorenz (1917-) while he was performing research at Massachusetts Institute of Technology in the United States. In the late 1950s and early 1960s, Lorenz was able to model the weather using twelve differential equations. In order to save time on one occasion and he started the program in the middle, rather than at its initial conditions, and stored computer data to three decimals rather than the usual six. As a result, instead of getting an expected close approximation to his result, Lorenz got a very different

answer. Lorenz rationalized that a small change in the initial conditions can drastically change the long-term behavior of a meteorological system. He called this phenomenon the “butterfly effect.” His 1962 paper “Deterministic Non periodic Flow” is considered the beginning of chaos theory also explained the same. In its extreme case, Lorenz contended that it was possible for the flapping of butterfly wings to cause a massive storm a half-world away. His 1972 paper “Predictability: Does the Flap of a Butterfly’s Wings in Brazil Set off a Tornado in Texas?” also gave him a strong reason to agree to the butterfly effect. Based on these results, Lorenz stated that it is impossible to predict the weather accurately. Therefore meteorological processes and forecasting of weather and climate, along with various other natural systems, like Lorenz’s Attractors are subject to the second law of thermodynamics, which states that “Entropy (disorder) of an isolated system not in equilibrium will increase in Entropy over time.” Ultimately, the ability to predict meteorological events is tied with chaos theory.

3.1.2 Impacts and Issues

Even though Lorenz had the idea that it was impossible to accurately predict meteorological events, but luckily when computers were invented and their ability to handle massive amounts of variables changed that impossibility to possibility. Meteorologists in the twenty-first century were made an attempt to predict weather and climate using complicated mathematical equations that model the behavior of Earth’s atmosphere.

For instance, as we learned that the atmosphere is chaotic, a weather forecast for a 10-day period has little validity by the tenth day and sizeable errors can be noticed after only a few days. However, advanced weather systems can be characterized from historical records by knowing initial conditions in the atmosphere. To determine these initial conditions, of course, a large number of initial, but not identical, states are to be collected.

In the beginning, the flapping of wings in the butterfly effect was actu-

ally just the rounding of numbers. Lorenz, a meteorology professor at MIT, had been constantly entering atmospheric readings by hand into a computer to run some forecasting programs. He was using a printout of the data on which numbers had been shortened from their original accuracy values (.506127, and, had been rounded to .506).

When Lorenz ran the program after entering the rounded numbers in the computer, he could find dramatic differences from the forecast using the full six-digit data. Therefore he would discover the key concept behind chaos i.e. Sensitive Dependence on Initial Conditions (SDIC). As the years rolled down the century, Lorenz worked out vigorously to get the implications for the weather. In the 1963 paper, he had published a research paper but not been able to calculate just how far the limit to accurate long-range forecasting would be. “Probably it could take a few days or a few centuries,” he wrote. Lorenz thought a few days or centuries may not come soon so by 1969 he had pinned down the limit to something like three weeks. Of course, reaching even that theoretical limit would require readings from different stations placed much too close together to be feasible. At last in 1963, American mathematician Edward Lorenz simplified the problem and proposed the atmospheric model which can be described as “only” by three differential equations:

$$\frac{dx}{dt}(t) = \sigma(y - x) \quad (3.1)$$

$$\frac{dy}{dt}(t) = x(\rho - z) - y \quad (3.2)$$

$$\frac{dz}{dt}(t) = xy - \beta z \quad (3.3)$$

Where numbers (x, y, z) specified a state of the atmosphere, which motion follows a vector field. Constants: σ, ρ, β are specific for certain system. Lorenz used (also used for following simulations):

For example,

$$\sigma = 10, \rho = 28, \beta = 8/3$$

x can represent a temperature, the second y displays the humidity

and the last z is a pressure.

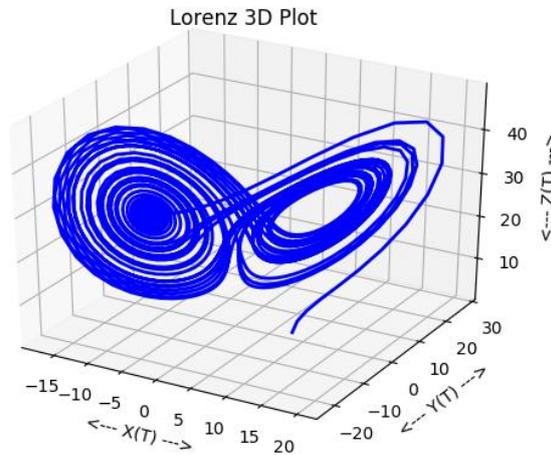


Figure 3.3: Lorenz Attractor

When we follow a trajectory by solving the differential equation proposed by E. Lorenz we can observe the vast evolution (motion) of the weather.

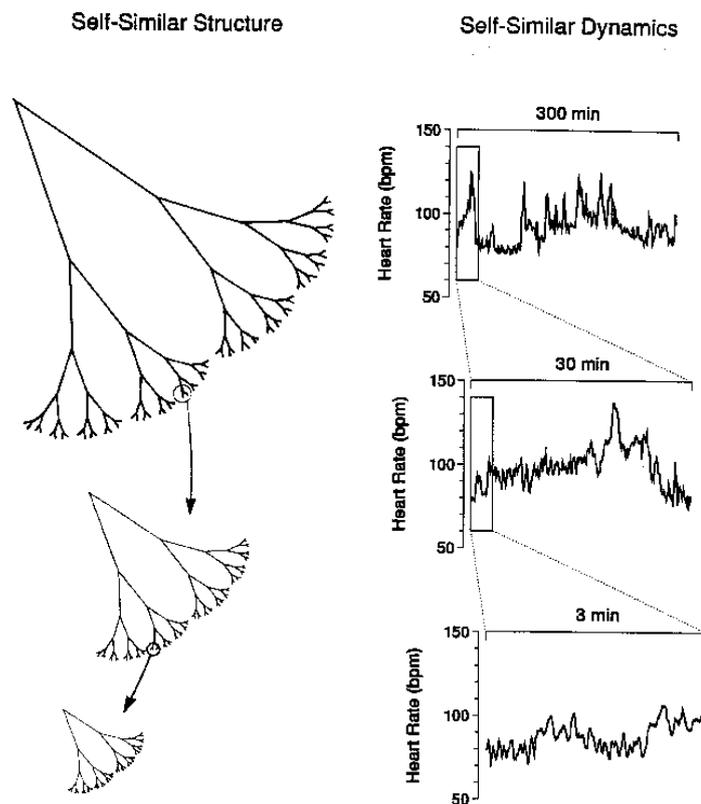
However, these equations were many extraordinary useful phenomena called again the butterfly effect. Which states that even a small change of the initial condition (for example the temperature in one certain point increased for about 0.001 degrees, etc) will surely have a tremendous impact on the shape of trajectory that the atmosphere follows.

We can populate this butterfly effect on different fields like the stock market, macroeconomy processes, traffic, people life, and many others. There is always a butterfly effect and the other awesome feature of these differential equations and solutions we compute. Though the trajectories are different and unpredictable, possibly they accumulate on the same butterfly-shaped object, which is created completely and independently from the initial position. Of- course all trajectories seem to be attracted to this butterfly shape. Therefore we call this effect the

Lorenz attractor.

3.2 Chaos Theory in Human Body

Another real-life application of Chaos theory can be applied to human biological rhythms. Because the Human Body is governed and controlled by rhythmical movements of many dynamical systems both knowingly and some unknowingly: like The rhythm of the heartbeat, the regular cycle of inhaling and exhaling air that makes up breathing, the cardiac rhythm of waking and sleeping, and the regularities and irregularities in the brain waves of mental health. Sadly these are not necessarily regular always. However, even healthy hearts often exhibits brief chaotic fluctuations and sick hearts can have regular rhythms of the same. Using Chaos theory we are able to get information about how to reduce sleep disorders, heart disease, and mental disease.



Of course in recent years, medical has drawn more attention to chaos theory in diagnosing cardiac dynamics, psychological disorders and etc. Chaos theory is much advanced now and allows researchers to predict and maybe even counteract, certain outcomes. An important example is the chaotic behavior of Ventricular fibrillation, which gives detailed information about a severely abnormal heart rhythm that is often life-threatening.

3.3 Chaotic Cryptography

Chaotic Cryptology is the application of chaotic theory to the practice of cryptography, the study of the methods used to securely and privately transmit information in the presence of an adversary or third party. The use of chaos or randomness in cryptography has long been sought after by entities seeking to develop new encryption techniques. Although chaotic cryptography is a promising technique, its shortcomings include inadequate security properties and low performance.

Chaotic cryptology is made up of two opposing processes: chaotic cryptography and chaotic cryptanalysis. Cryptography refers to encrypting information for secure transmission, whereas cryptanalysis refers to decrypting and deciphering encoded encrypted messages.

This Chapter discusses about chaotic maps , the bifurcation diagram of the logistic map and then finally on how to generate the key for a cryptosystem with various values of the control parameter. In order to use chaos theory efficiently in cryptography, the chaotic maps should be implemented such that the entropy generated by the map can produce required Confusion and diffusion. Properties in chaotic systems and Cryptographic primitives share unique characteristics that allow for the chaotic systems to be applied to cryptography.

If chaotic parameters, as well as Cryptographic keys, can be mapped

Logistic map

$$x_{n+1} = rx_n(1 - x_n)$$

- Bifurcation diagram - r vs x
- Bifurcation points

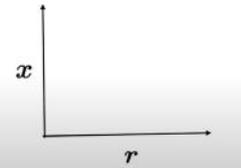


Figure 3.4: Logistic Map

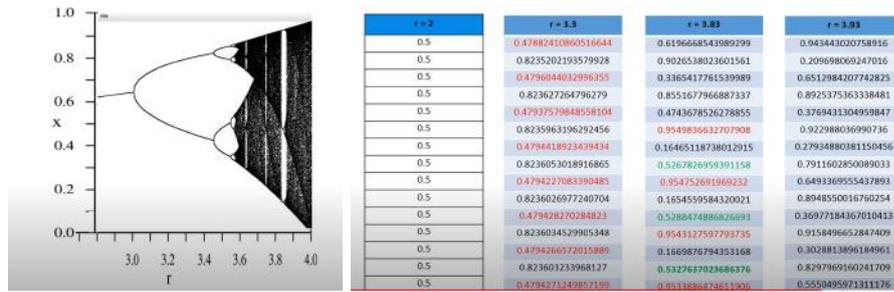
symmetrically or mapped to produce acceptable and functional outputs, it will make it next to impossible for an adversary to find the outputs without any knowledge of the initial values.

Chaotic maps are broadly classified as One dimensional map and multi dimensional map. Logistic map, sine map, tent map are popular examples of One Dimensional map and for the multi dimensional map the lenon map and the 2D logistic map are the common example Any of these map can be used for the key generation of cryptosystem but before using them it is very crucial to understand the dynamic behaviour of the map and the control parameter range where they behave chaotically.

Here we are going to visualize the dynamic behaviour of the logistic map which is expressed as-

Where r is the control parameter ,this visualization is done with the help of the diagram called Bifurcation diagram where the plot is done between r and x ,ie, as we vary the value of r the different values of x is being plotted. Here we can also identify the bifurcation points(these are the points where the chaotic map changes topologically), It is really interesting to see how the logistic map behaves under different conditions of r .

For r less than 3 ,the logistic map is stable and at $r=3$ it undergoes period doubling and oscillates between 2 points. At $r = 3.5$,the region



again is periodic and it bifurcates and oscillates between 4 points and after a cascade of period doubling at approximately 3.567 we can visualize the onset of chaos. It is also interesting to see periodic changes and pattern The darker region is a chaotic regime.

Now to experiment with the logistic map, when we select the values of r for key generation, the different values of x will be generated. It can be seen that x holds the value 0.5 for all values of r =2, ie at r=2 the logistic map is stable. The second table displays the value of x for r=3.3, here the x value oscillates between the 2 points which are approximately 0.47 0.82, Now consider r=3.83, the x values are shown in the third column, it is interesting to find a pattern because of the presence of the periodic window near 3.83 Now for r=3.93, the values of x are displayed in the last column, here the nature of x is of no pattern as seen in the previous columns, no repeated values are observed, ie it is not periodic and such values are called the chaotic values.

Similarly to generate the key for encryption, we consider the previous values of r, First considering r=2, as it generated only periodic value, the key also reflects periodicity and pattern. similarly, for r=3.3 we observe a pattern in the key, the third key is generated with r=3.83 though it seems to be a good choice we can still observe a pattern marked in green because r = 3.83 is in the periodic window.

The last is $r=3.93$, the key is perfect without any pattern, this key is therefore chaotic and nonlinear, So $r= 3.93$ is a good choice for generating a key for encryption. Therefore selecting the r -value from the chaotic regime is very crucial for generating a strong key for encryption. It is not only fascinating to use a logistic map for encryption but it is also highly recommended to understand the dynamic behavior of the chaotic map for selecting the right control parameter.

Chapter 4

LIMITATIONS OF CHAOS THEORY

One of the major limitations of chaos theory is its sensitive dependence on initial conditions in which a small change in one state of a deterministic nonlinear system can result in large differences in a later state. Many factors influence chaos theory, including actions and reactions, which makes it nearly impossible to determine the exact source of the phenomenon. According to Levy, “The size of fluctuations from one period to the next in chaotic systems vary in size” Predictability is one of the big negative aspects of chaos theory. Since other factors beyond our control can play a role in results, it can be difficult to determine how they affected the outcome. Chaos theory doesn’t reveal much about the distribution and impact of external variables on our outcomes. We are more likely to see a chaotic outcome when there are more factors at play. Some systems do not seem to benefit from the effects of chaos theory as a whole. Chaos does not occur in slow systems, such as those with infrequent events or where there is a significant amount of friction that dissipates energy and dampens disturbances.

Since Chaos theory is still developing and there are still many things to be discovered and hence concepts and ideas are redefined or complemented continuously. Many scientists are trying to link chaos theory with other theories and ideas to create a more general theory.

Although chaos theory can help us make better decisions and de-

sign new strategies for the future, we should not rely on its outcomes, otherwise, this habit of short-term observe-and-manage will impair an organization's long-term success. Chaos theory is not as simple to find an immediate and direct application in the business environment, but mapping the business environment using the knowledge of chaos theory is worthwhile. To withstand the ever-growing chaos and detect subtle changes in it soon, a new statistical hypothesis test needs to be designed.

Chapter 5

CONCLUSION

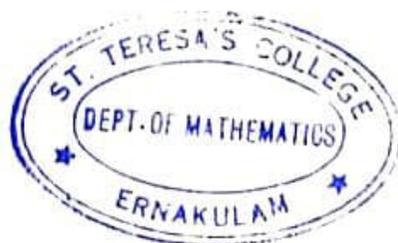
Everything around the world is subjected to chaos theory. We can find chaos from the currents of the oceans, stock market, cryptography, weather predictions, Psychology to chaos in the human body.

Chaos theory is a field of study in Mathematics, with applications in several disciplines including physics, engineering, biology, and philosophy which primarily states that even small differences in initial condition (such as rounding errors in numerical computation) can result in huge Variation in the outcome. This makes it unpredictable. Due to Chaos, it is realized that even simple systems may produce complex results and behavior. Studying the different applications of chaos theory made us realize how it allows us to look at the universe from an entirely different perspective. The different applications and their extensions to different fields were interesting to dig in deep, yet there is still much left to be discovered.

Chaos has become a part of modern science. It is considered as one of the greatest triumphs of the 21 st century.It certainly is one of the greatest discoveries.

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Project Report

On

QUEUING THEORY

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

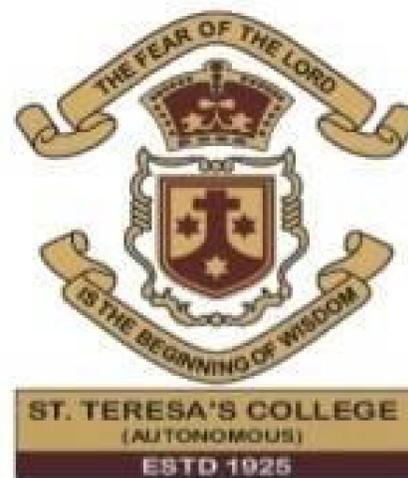
by

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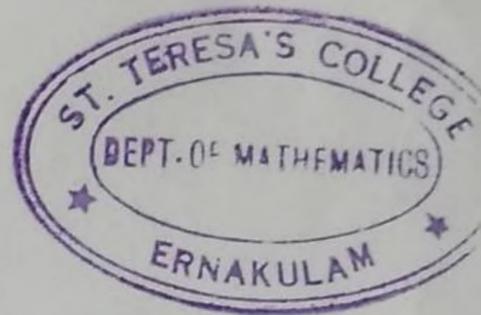


CERTIFICATE

This is to certify that the dissertation entitled, **QUEUEING THEORY** is a bonafide record of the work done by Ms. **JOSCELIA MARIA PHILIP** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

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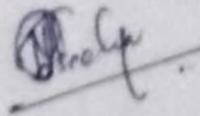
External Examiners

1: *[Signature]*

2:

DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Smt. Betty Joseph, Associate Professor, Department of Mathematics and Statistics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.



JOSCELIA MARIA PHILIP

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Date: 08.08.2022

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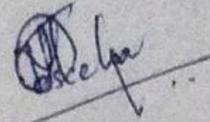
Firstly I thank God Almighty for giving me his grace to execute the project work successfully. I express my sincere gratitude to our guide Smt. Betty Joseph, Associate Professor of Department of Mathematics and Statistics, St. Teresa's College (Autonomous), Ernakulam, for her valuable guidance throughout the project. I would also like to mention Smt. Neenu Susan Paul, Department of Mathematics and Statistics, St. Teresa's College (Autonomous), Ernakulam, for her help and support.

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Ernakulam.

Date: 08.03.2022



JOSCELIA MARIA PHILIP

AB19BMAT036

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INTRODUCTION

Queuing theory is the mathematical study of formation, congestion and purpose of waiting lines .It is observed as a branch of Operations Research because the outcomes are often used for making business decisions about the measures needed to provide a service.

Queuing theory has its origin in 1909 when Professor A K Erlang Danish Mathematician and engineer published his fundamental paper in telephone traffic. He sought to determine how many circuits were needed to provide an acceptable level of telephone service for people not to be " on hold " for too long.



A queue is formed at a queuing system when either customers (human beings or physical entities) requiring service wait due to the number of customers exceeding the number of service facilities or service facilities do not work efficiently and take more time than prescribed to serve a customer.

Queuing theory can be applied to a variety of situations where it is not possible to predict accurately the arrival rate (or time) of customers and service rate (or time) of service facility or facilities. In particular, it can be used to determine the level of service (either the service

rate or the number of service facilities) that balances the following two conflicting costs.

(i) cost of offering the service

(ii) cost incurred due to delay in offering service

Chapter 1

BASICS OF QUEUING THEORY

1.1 Basic Definitions

1.1.1 Queue

A line or sequence of people or items awaiting their turn to be attended or for a service is called a queue.

1.1.2 Customer

A list of items or people that waits for a service is called customer.

1.1.3 Queue Length

Number of customers waiting in a system for service.

1.1.4 Server

Server provides service in the system.

1.2 The Basic Components of a Queue

1.2.1 Arrival process

Arrival defines the way customers enter the system , mostly the customers arrive randomly in between two adjacent arrivals.

1.2.2 Service and Departure process

It defines how long service will take, how many no. of servers are accessible, whether it is in series or parallel. Departure process is a Poisson process with rate that is statistically identical to the arrival process.

1.2.3 The number of servers

The number of servers available to serve the customers in the system. It may be single server or multi-server

1.2.4 The queuing discipline

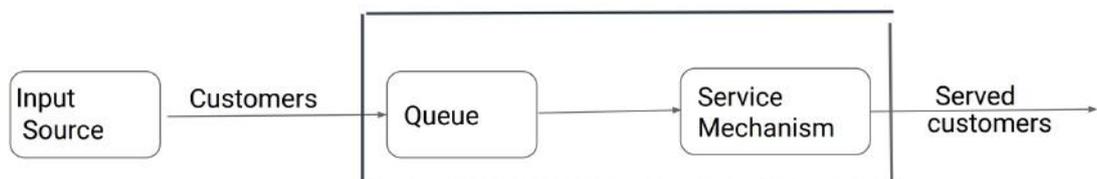
It represents the order in which the customers are selected from the queue for service.

1.2.5 The queue capacity

The number of customers/items the queue can hold

1.2.6 The size of the client population

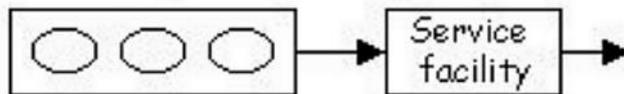
Also known as population size. The size of calling population can be finite or infinite. In case of large population, it is assumed as infinite



1.3 Types of Queues

1.3.1 Single server Single-phase

A waiting line in which single line of customers go through a single waiting line or phase and they are served by a single server. Queues in ATM is an example.



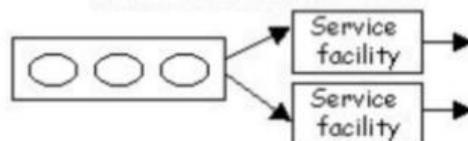
1.3.2 Single server Multi-phase

The system in Which there are multiple number of waiting lines or phase but only one server to serve. Queues in buffet restaurants is an example.



1.3.3 Multi server Single-phase

In this system there will be only one waiting line or phase and they are served by more than one servers. Queues in bank are commonly seen example.

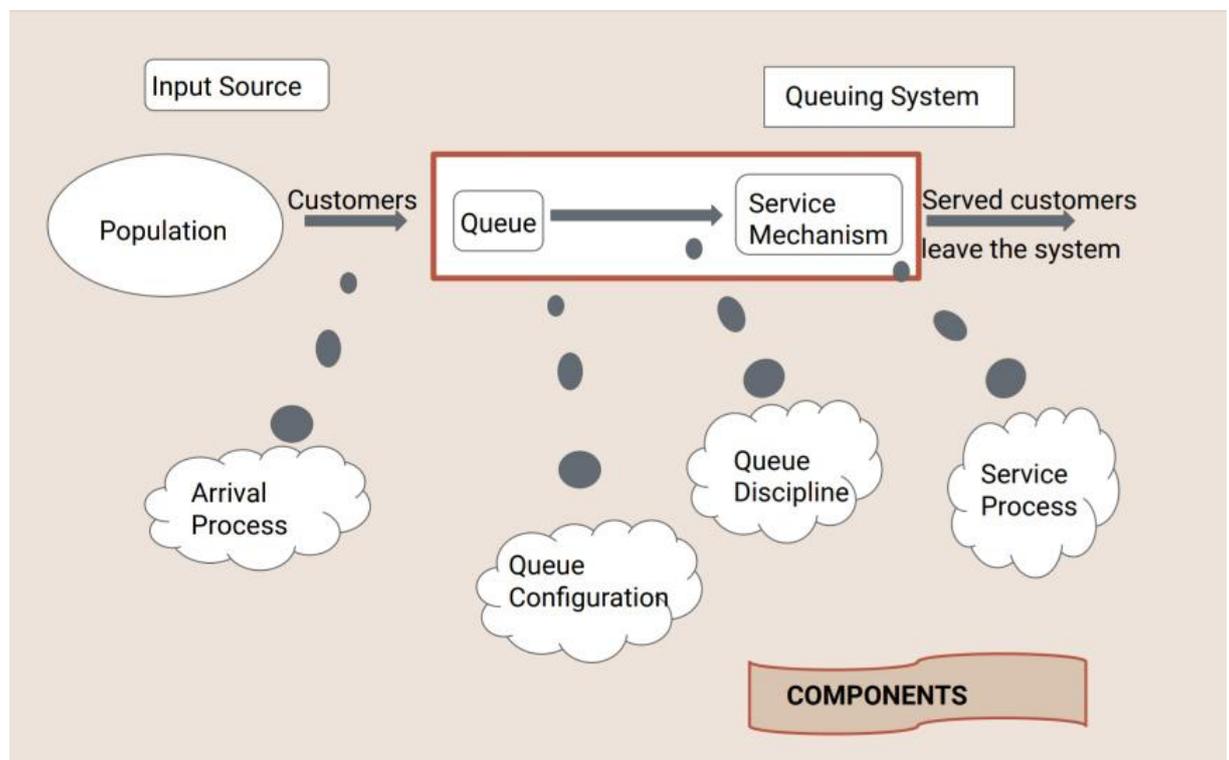


Multi server Multi-phase

Here we have two or more servers for serving multiple number of waiting lines. Supermarket queues are examples.



1.4 Queuing system



1.4.1 Queuing structure

It is the crucial element of queuing system, as it shows the queue discipline, which means the order in which the customers are picked from the queue for service.

1.4.2 Queuing system

Queuing systems are simplified mathematical models to explain congestion.

1.4.3 Components of Queuing systems

- **Input source** - The input source generates customers for the service mechanism. The most important characteristic of the input source is its size. It may be either finite or infinite.
- **Queue** - Queue represents a certain no. of customers waiting for a service.
- **Arrival process** - Arrival defines the way customers enter the system, mostly the arrivals are random intervals between two adjacent arrivals.
- **Queue configuration** - It refers to the queue in the system, their relationship to the servers. A queue may be a single queue or a multiple queue.
- **Queue discipline** - It indicates the order in which members of the queue are selected.

There are mainly three ways for queue discipline

(i) FIFO (First In First Out)

The first customer is served first or first item added will be the first one to be removed.

eg: Ticket supply in theaters

(ii) LIFO (Last In First Out)

The last customer is first or last item added will be the first one to be removed.

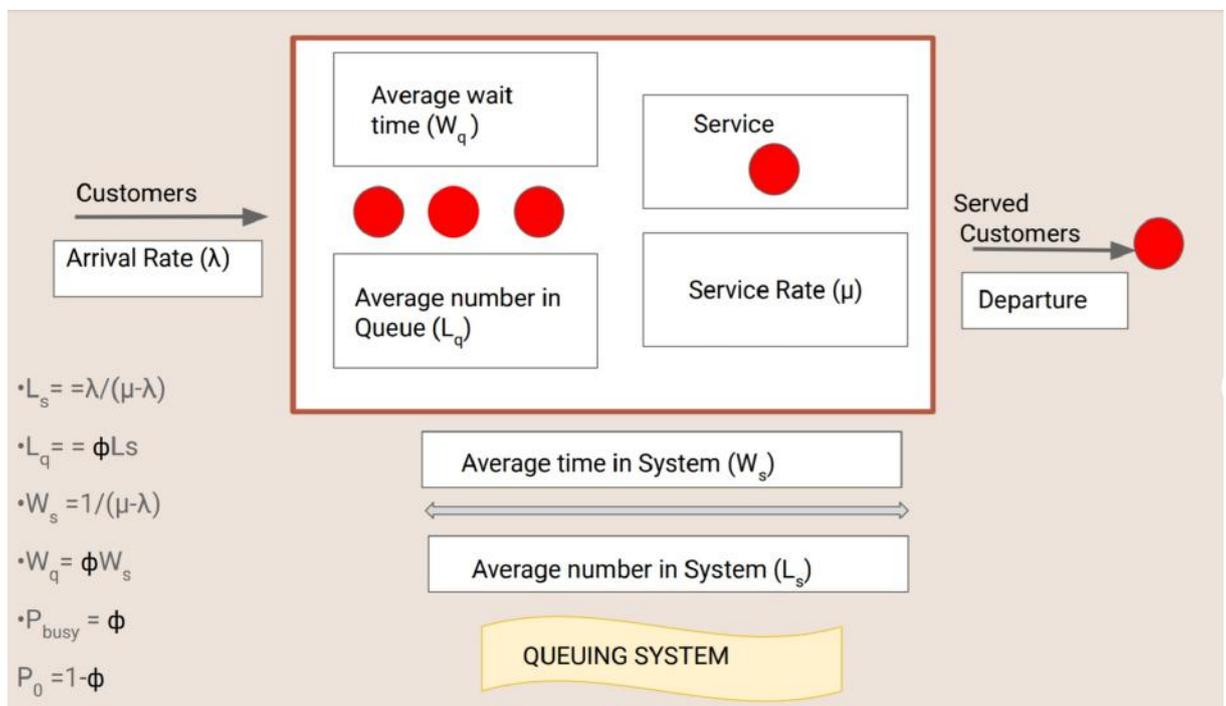
eg: A stack of plates that arrange in a pile

(iii) SIRO (Serve In Random Order)

In this system server selects one of the customers and service is provided randomly.

eg: Entering business

- Service process - It defines how long service will take, how many no. of servers available, whether the servers are in series or parallel.
- Service mechanism - Service represents some activity that takes time and that the customers are waiting for. it may be a real service carried on persons or machines. Typically a service takes random time.



1.5 Basic Terminologies

- Arrival rate (λ): Number of arrivals per unit time.
- Service rate (μ): Rate at which customers are served in the system.

- Utilisation factor (ϕ) : Average time the customers spends in the queue.

$$\phi = \frac{\lambda}{\mu}$$

- L_s = average no. of customers in the system
- P_0 = ideal probability = $1 - \phi$
- L_q = average no. of customers in the queue = ϕL_s
- W_s = average time in the system = $\frac{1}{\mu - \lambda}$
- W_q = average time in the queue = ϕW_s
- P_{busy} = standard busy probability = ϕ



Chapter 2

LAW AND NOTATION

2.1 Kendall's Notation

David George Kendall (15 January 1918 - 23 October 2007), well known English Statistician and Mathematician, known for his work on probability, statistical shape analysis, ley lines and queuing theory.



The standard system that describes and classifies a queuing node. D.G. Kendall proposed this in 1953. A general queuing system is denoted by

$$(P/Q/R) : (X/Y/Z)$$

The parameters of this notation are :

P - Arrival Rate distribution.

Arrival distribution can mainly be of Poisson distribution, Exponential distribution or Markov distribution.

Q - Service Rate distribution.

R - Number of servers.

X - Service discipline.

Y - Minimum number of customers permitted in the system.

Z - Size of the calling source of the customers.

2.1.1 Poisson Distribution

A discrete random variable **X** is said to be follow a Poisson distribution with the parameter λ if it's p.d.f is given by

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \text{ when } x = 0, 1, 2, \dots (\lambda \text{ greater than } 0)$$

$$f(x) = 0, \text{ elsewhere}$$

x-number of occurrences

x = 0, 1, 2...

e-Euler's number (e=2.71828...)

Mean = λ

Variance = λ

Skewness = $\frac{1}{\sqrt{\lambda}}$

Kurtosis = λ^{-1}

MGF = $\exp[\lambda(e^t - 1)]$

$\lambda = E(X) = Var(X)$

2.1.2 Exponential Distribution

A continuous random variable X is said to follow exponential distribution with parameter λ (λ greater than 0) if it's p.d.f is given by

$$f(x) = \lambda e^{-\lambda x}, \text{ when } x \text{ greater than or equal to } 0$$

$$f(x) = 0, \text{ elsewhere}$$

x - No. of occurrences, $x = 0, 1, 2, \dots$

e - Euler's number, $e = 2.71828\dots$

Mean = λ^{-1}

Variance = λ^{-2}

Skewness = 2

Kurtosis = 6

MGF = $(1 - \frac{t}{\lambda})^{-1}$

2.1.3 Markovian Distribution

A mathematical model for the time between job arrivals to a system. Markov process is a stochastic process which is used to analyse decision problems in which the occurrence of a specific event depends on the occurrence of the event immediately prior to the current event. Basically Markov process help us to identify

- (i) A specific state of the system being studied, and
- (ii) The state - transition relationship.

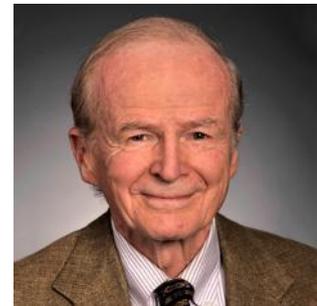
The occurrence of an event at a specified point in time (say, period n) put the system in a given state, say E_n . If, after the passage of one time unit, another event occurs (during time period $n+1$), the system has moved from state E_n to state E_{n+1} .

$$f(x) = \lambda e^{-\lambda x}, x \geq 0$$

2.2 Little's Law

Queuing Theory is a daily life applicable study that helps to work more easier in the queues. Waiting in line is a common process in daily life. Some theorems and laws were introduced to ensure the queues more effective. One of the theorem is Little Law.

John Little introduced Little Law in 1964. Initially, Little had not published the proof of the theorem. However in 1961, he published the proof.



Little law is a theorem that determines the average number of items in a stationary queuing system, based on the average waiting time of an item within a system and the average number of items arriving at the system per unit of time.

Little law states that the average number of customers in a queuing system is equal to their product of average arrival rate and average amount of time they spent in the system.

In other words, we can say that

Number of items in the queue = Arrival rate x Average time spent in the queue

$$L = \lambda W$$

L - stands for the number of items inside the queuing system. It is

also known as WIP which means work in progress.

λ - Arrival rate and departure rate of items in and out of the system.

W - Average amount of time an item spends in the system.

Chapter 3

QUEUING MODELS

3.1 Model 1 - $(M/M/1) : (GD/\infty/\infty)$

The parameters of the model :

- M - Arrival rate follows Poisson distribution.
- M - Service rate follows Poisson distribution.
- 1- Number of servers is one.
- GD - Service discipline is general.
- ∞ - Maximum number of customers permitted in the system is infinite.
- ∞ - Size of the calling source Infinite.

The steady-state formula to obtain the probability of having n customers in the system P_n are

$$P_n = (1 - \phi) \phi^n$$

$$L_s = \frac{\phi}{1-\phi}$$

$$L_q = L_s - \left(\frac{\lambda}{\mu}\right) = \frac{\phi^2}{(1-\phi)}$$

$$W_s = \frac{L_s}{\lambda} = \frac{\phi}{(1-\phi)\mu} = \frac{1}{\mu-\lambda}$$

$$W_q = \frac{L_q}{\lambda} = \frac{\phi}{\mu - \lambda}$$

Using the notation of the general model, we have

$$\lambda_0 = \lambda$$

$$\mu_0 = \mu$$

Also $\lambda_{eff} = \lambda$ and $\lambda_{lost} = 0$, because all arriving customers can join the system.

Letting $\rho = \frac{\lambda}{\mu}$, the expression for p_n in the generalised model reduces to

$$p_n = \rho^n p_0, n = 0, 1, 2, \dots$$

To determine the value of p_0 , we use the identity

$$p_0 (1 + \rho + \rho^2 + \dots) = 1$$

The sum of the geometric series is $\left(\frac{1}{1-\rho}\right)$, provided $\rho < 1$. Thus

$$\rho = 1 - \rho, \rho < 1$$

The general formula for p_n is thus given by the following geometric distribution

$$p_n = (1 - \rho) \rho^n, n = 1, 2, \dots (\rho < 1)$$

The mathematical derivation of p_n imposes the condition $\rho < 1$, or $\lambda < \mu$. If $\lambda \geq \mu$, the geometric series diverges, and the steady-state probabilities p_n do not exist. This result makes the intuitive sense, because unless the service rate is larger than the arrival rate, queue length will continually increase and no steady state can be reached.

The measure of performance L_q can be derived in the following manner :

$$\begin{aligned} L_s &= \sum_{n=0}^{\infty} np_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n \\ &= (1-\rho)\rho \frac{d}{d\rho} \sum_{n=0}^{\infty} \rho^n \\ &= (1-\rho)\rho \frac{d}{d\rho} \left(\frac{1}{1-\rho} \right) \\ &= \frac{\rho}{1-\rho} \end{aligned}$$

Because $\lambda_{eff} = \lambda$ for the present situation, the remaining measures of performances are computed using the relationships. Thus,

$$\begin{aligned} W_s &= \frac{L_s}{\lambda} = \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \\ W_q &= W_s - \frac{1}{\mu} = \frac{\rho}{\mu(1-\rho)} \\ L_q &= \lambda W_q = \frac{\rho^2}{1-\rho} \\ \bar{c} &= L_s - L_q = \rho \end{aligned}$$

Example : The arrival rate of customers at a banking counter follows Poisson distribution with a mean of 45 per hour. The service rate of the counter clerk also follows Poisson distribution with a mean of 60 per hour.

1. What is the probability of having 0 customers in the system?
2. What is the probability of having 5 customers in the system?
3. What is the probability of having 10 customers in the system?
4. Find L_s, L_q, W_s, W_q ?

Solution : We have

Arrival rate, $\lambda = 45$ per hour

Service rate, $\mu = 60$ per hour

Utilisation factor, $\phi = \frac{\lambda}{\mu} = \frac{45}{60} = 0.75$

1. $P_0 = 1 - \phi = 1 - 0.75 = 0.25$
2. $P_5 = (1 - \phi) \phi^5 = (1 - 0.75)0.75^5 = 0.593$
3. $P_{10} = (1 - \phi) \phi^{10} = (1 - 0.75)0.75^{10} = 0.0141$
4. $L_s = \frac{\phi}{1-\phi} = \frac{0.75}{1-0.75} = 3$ customers
- $L_q = \frac{\phi^2}{(1-\phi)} = \frac{0.75^2}{1-0.75} = 2.25$ customers
- $W_s = \frac{1}{\mu-\lambda} = \frac{1}{60-45} = 0.067$ hours
- $W_q = \frac{\phi}{\mu-\lambda} = \frac{0.75}{60-45} = 0.05$ hours

3.2 Model 2 - (M/M/C) : (GD/∞/∞)

The parameters of this model are given below :

- M - Arrival rate follows Poisson distribution.
- M - Service rate follows Poisson distribution.
- C- Number of servers is C.
- GD - Service discipline is general discipline.
- ∞ - Maximum number of customers permitted in the system is infinite.
- ∞ - Size of the calling source is infinite.

The steady-state formula to obtain the probability of having n customers in the system P_n and the formula for P_0, L_s, L_q, W_q, W_s are presented below:

$$P_n = \phi^n n! P_0, \text{ where } 0 \leq n \leq C$$

$$= \phi^n \frac{1}{C^{n-C} C!} P_0, n > C \text{ where } \frac{\phi}{C} < 1 \text{ or } \frac{\lambda}{\mu C} < 1$$

$$P_0 = \sum_{c+1}^{c-1} \frac{\phi^n}{n!} + \frac{\phi^C}{C!} \left[1 - \left(\frac{\phi}{C} \right) \right]^{-1}$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{\phi^n}{n!} + \frac{\phi^c}{C! \left[1 - \frac{\phi}{c}\right]} \right]^{-1}$$

The expression for L_q can be determined as follows:

$$\begin{aligned} L_q &= \sum_{n=c}^{\infty} (n - c) P_n \\ &= \sum_{k=0}^{\infty} K P_{k+c} \\ &= \sum_{k=0}^{\infty} K \frac{\phi^{k+1}}{c^k c!} P_0 \\ &= \frac{\phi^{c+1}}{c! c} P_0 \sum_{k=0}^{\infty} K \left(\frac{\phi}{c}\right)^{k-1} \\ &= \frac{\phi^{c+1}}{c! c} P_0 \frac{d}{d\left(\frac{\phi}{c}\right)} \sum_{k=0}^{\infty} \left(\frac{\phi}{c}\right)^k \\ &= \frac{\phi^{c+1}}{(c-1)!(c-\phi)^2} P_0 \end{aligned}$$

$$L_s = L_q + \phi$$

$$\begin{aligned} W_q &= \frac{L_q}{\lambda} \\ &= \frac{L_q + \phi}{\lambda} \\ &= \frac{L_q}{\lambda} + \frac{\phi}{\lambda} \\ &= W_q + \frac{1}{\mu} \end{aligned}$$

$$W_q = \frac{L_q}{\lambda}$$

Example: At a central ware-house, vehicles arrive at the rate of 18 per hour and the arrival rate follows Poisson distribution. the unloading time of the vehicles follows exponential distribution and the unloading rate is 6 vehicles per hour. There are 4 unloading crews. Find the following:

a) P_0 and P_3

b) L_q, L_s, W_q and W_s

Solution: we have,

Arrival rate, $\lambda = 18$ per hour

Unloading rate, $\mu = 6$ per hour

No. of unloading crews, $C = 4$

$$\phi = \frac{\lambda}{\mu}$$

$$= \frac{18}{6} = 3$$

a) Therefore P_0 is computed as :

$$\begin{aligned} P_0 &= \left[\sum_{n=0}^{c-1} \frac{\phi^n}{n!} + \frac{\phi^c}{C! [1 - \frac{\phi}{c}]} \right]^{-1} \\ &= \left[\sum_{n=0}^3 \frac{3^n}{n!} + \frac{3^4}{4! [1 - \frac{3}{4}]} \right]^{-1} \\ &= \left[\frac{3^0}{3!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4! (1 - (\frac{3}{4}))} \right]^{-1} \\ &= 0.0377 \end{aligned}$$

Now we have to compute P_3 , we have

$$P_n = \frac{\phi^n}{n!} P_0, 0 \leq n \leq C$$

Therefore,

$$\begin{aligned} P_3 &= \frac{3^3}{6} * 0.0377 \\ &= \mathbf{0.1697} \end{aligned}$$

b) L_q, L_S, W_q and W_s are computed as under:

$$L_q = \frac{\phi^{c+1}}{(c-1)!(c-\phi)^2} P_0$$

$$\begin{aligned}
&= \frac{3^5}{31 * 1} * 0.0377 \\
&= 1.53 = 2 \text{ vehicles} \\
L_s &= L_q + \phi \\
&= 1.53 + 3 \\
&= 4.53 = 5 \text{ vehicles} \\
W_q &= \frac{L_q}{\lambda} \\
&= \frac{1.53}{18} \\
&= 0.252 \text{ hour} = 5.1 \text{ minutes} \\
W_s &= W_q + \frac{1}{\mu} \\
&= 0.085 + \frac{1}{6} \\
&= 0.252 \text{ hours} = 15.12 \text{ minutes}
\end{aligned}$$

3.3 Model 3 - (M/M/1) : (GD/N/∞)

The parameters of this model are given below:

- M - Arrival rate follows Poisson's distribution.
- M - Service rate follows Poisson's distribution.
- 1 - Number of servers is one.
- GD - Service discipline is general discipline.
- N - Maximum number of customers permitted in the system is N.
- ∞ Size of calling source is infinite.

This model differ from (M/M/1):(GD/∞/∞) in that there is a limit N on the number in the system (maximum queue length=N-1) .Examples include manufacturing situations in which a machine may have a limited buffer space and a one-lane drive-in window in a fast-food restaurant .New arrivals are not allowed when the number of customers in the

system reaches N .Thus,

$$\lambda_n = \begin{cases} \lambda, n = 0, 1, \dots, N - 1 \\ 0, n = N, N + 1 \end{cases}$$

$$\mu_n = \mu, n = 0, 1, \dots$$

the value of P_0 is determined from the equation

$\sum_{n=0}^{\infty} P_n = 1$, which yields

$$P_0(1 + p + p^2 + \dots + p^N) = 1$$

or

$$p_0 = \begin{cases} \frac{1-\phi}{1-\phi^{N+1}}, \phi \neq 1 \\ \frac{1-\phi}{1-\phi^{N+1}}, \phi = 1 \end{cases}$$

The steady-state formula to obtain the probability of having n customers in the system P_n and the formula for P_0, L_s, L_q, W_s and W_q are represented below.

$$\begin{aligned} P_N &= \frac{1-\phi}{1-\phi^{N+1}} \phi^N, \phi \neq 1 \text{ and } N = 0, 1, 2, \dots, n \\ &= \frac{1}{N+1}, \phi = 1 \end{aligned}$$

The expected number of customers in the system is computed as

$$\begin{aligned}
 L_s &= \sum_{n=1}^N nP_n \\
 &= \frac{1-\phi}{1-\phi^{N+1}} \sum_{n=0}^N n\phi^n \\
 &= \left(\frac{1-\phi}{1-\phi^{N+1}}\right)\phi \frac{d}{d\phi} \sum_{n=0}^N \phi^n \\
 &= \left(\frac{1-\phi}{1-\phi^{N+1}}\phi\right) \frac{d}{d\phi} \left(\frac{1-\phi^{N+1}}{1-\phi}\right) \\
 &= \frac{\phi[1-(N+1)\phi^N + N\phi^{N+1}]}{(1-\phi)(1-\phi^{N+1})}, \phi \neq 1 \\
 &= \frac{N}{2}, \phi \neq 1 \\
 \lambda_{eff} &= \lambda(1 - P_N) \\
 L_q &= L_s - \frac{\lambda_{eff}}{\mu} \\
 &= L_s - \frac{\lambda(1 - P_N)}{\mu} \\
 W_q &= \frac{L_q}{\lambda_{eff}} \\
 &= \frac{L_q}{\lambda(1 - P_N)} \\
 W_s &= W_s + \frac{1}{\mu} \\
 &= \frac{L_q}{\lambda_{eff}} \\
 &= \frac{L_q}{\lambda(1 - P_N)}
 \end{aligned}$$

Example: Cars arrive at a drive-in restaurant with a mean arrival rate of 24 cars per hour and the service rate of the cars is 20 cars per hour. The arrival rate and the service rate follows Poisson distribution. The number of parking spaces for cars is only 4. Find the standard results of this system.

Solution:

Here ,Arrival rate, $\lambda = 24$ cars per hour

Service rate, $\mu = 20$ cars per hour

$N = 4$

$$\begin{aligned}\phi &= \frac{\lambda}{\mu} \\ &= \frac{24}{20} = \mathbf{1.2}\end{aligned}$$

Therefore we get,

$$L_s = \frac{\phi[1-(N+1)\phi^N + N\phi^{N+1}]}{(1-\phi)(1-\phi^{N+1})} = \frac{1.2[1-(4+1)1.2^4 + 4*1.2^5]}{(1-1.2)(1-1.2^5)} = 2.3 \text{ cars}$$

and

$$\begin{aligned}P_N &= \frac{1-\phi}{1-\phi^{N+1}}\phi^N \\ &= \left(\frac{1-1.2}{1-1.2^5}\right) * 1.2^4 = 0.2787 \text{ cars}\end{aligned}$$

the other results are:

$$\begin{aligned}\lambda_{eff} &= \lambda(1 - P_N) \\ &= 24(1-0.2787) \\ &= \mathbf{17.3112 \text{ per hour}}\end{aligned}$$

$$\begin{aligned}L_q &= L_s - \frac{eff}{\mu} \\ &= 2.36 - \left(\frac{17.3112}{20}\right) \\ &= \mathbf{1.494 \text{ cars}}\end{aligned}$$

$$\begin{aligned}W_q &= \frac{L_q}{\lambda_{eff}} \\ &= \frac{1.494}{17.3112} \\ &= \mathbf{0.0863 \text{ hours}} \\ &= \mathbf{5.2 \text{ min}}\end{aligned}$$

$$\begin{aligned}W_s &= \frac{L_s}{\lambda_{eff}} \\ &= \frac{2.36}{17.3112} \\ &= 0.1363 \text{ hours} \\ &= 8.2 \text{ min}\end{aligned}$$

Chapter 4

APPLICATIONS

4.1 Daily Life Applications

SITUATION	CUSTOMERS	SERVICE
Clinic	Patients	Doctors
Job interviews	Applicants	Experts
Railway station	Travelers	Ticket window
Bank counter	Account holders	Counter clerk
Airport runways	Planes	Runway
Telephone booth	Customers	Telephone
Ration shop	Ration card holders	Shop clerk
ATM counters	Customers	ATM machine
Toll plaza	Vehicles	Toll collectors
Emigration department	Travelers	Emigration officers
Traffic system	Vehicles	Signal point
Supermarkets	Customers	Workers
Computer center	Programs	Computer
Library	Students	Counter clerk
Maintenance shop	Breakdown machines	Machines
Photostat shop	Papers	Photostat machine

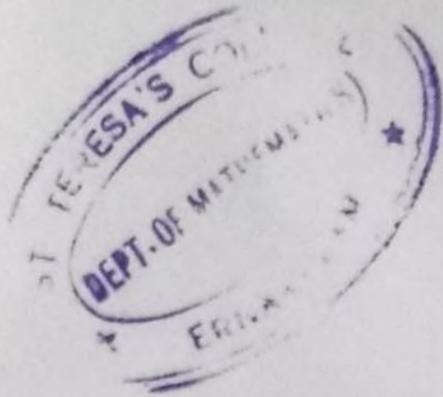
- Queuing theory has a important role in casualties of hospital
- It is useful in evaluating the impact of breakdown down of disasters
- It is really helpful in making business decisions
- If we consider the rain in monsoon season in India then the waiting days for getting the rain is a queue, here days are the customers and the sky is the server.
- As Covid-19 cases increasing day by day, vaccination centres need to provide more number of servers to avoid congestion of people.

CONCLUSION

Queues are very common in our society. Every person has to stand in a queue atleast once. Queuing theory helps in enhancing business strategies. From this project, we can conclude that, when the Average Service Rate μ is greater than Average Arrival rate λ , the customers are served at a faster rate than they arrive and the service will be fast. This theory gives a basic information for successfully designing queuing systems that acheives a healthy balance between arrival rate and service rate.

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Project Report

On

**A STUDY OF APPLICATION OF MATRICES
IN CRYPTOGRAPHY AND IMAGE
PROCESSING**

Submitted

in partial fulfilment of the requirements for the degree of

BACHELOR OF SCIENCE

in

MATHEMATICS

by

ANJALI A

(Register No.AB19BMAT023)

Under the Supervision of

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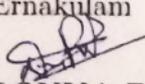
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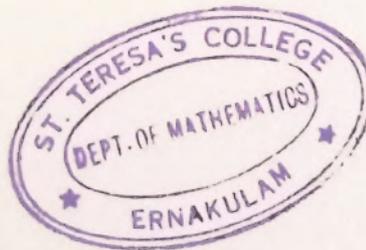


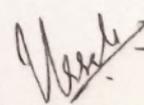
CERTIFICATE

This is to certify that the dissertation entitled, **A STUDY OF APPLICATION OF MATRICES IN CRYPTOGRAPHY AND IMAGE PROCESSING** is a bonafide record of the work done by Ms. **ANJALI A** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

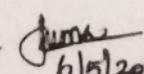
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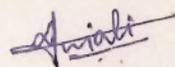
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DECLARATION

I hereby declare that the work presented in this project is based on the original work done by me under the guidance of MRS.DONNA PINHEIRO, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date: 11-03-2022



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Ernakulam.

Date: 11-03-2022

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AB19BMAT023

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INTRODUCTION

Linear algebra serves as a useful tool in cryptography, permitting the manipulation of multiple variables simultaneously to create a unique and reversible output.

With rapid demand of storing large amount of data, the space to store this data (in the form of files) in the hard drives of the computer systems or onto the servers of big companies is getting less compared to the amount of data that is to be stored. As a result, various compression techniques are in demand which can help to reduce the size of the data files.

In this project, we will be discussing how Linear algebra can be used in the compression of images and historic evidence of cryptology. Basically we will be discussing how SVD and Wavelet techniques are extensively used in image compression process resulting in saving computer's memory. The basic idea here is each image can be represented as a matrix and we apply linear algebra (SVD and Wavelet) on this matrix and get a reduced matrix out of this original matrix and the image corresponding to this reduced matrix requires much lesser storage space as compared to the original image.

Chapter 1

HISTORIC REVIEW OF CRYPTOGRAPHY

The first known evidence of the use of cryptography was found in an inscription carved around 1900 BC ,in the main chamber of the tomb in Egypt.Evolution of Cryptography is during and after the European renaissance,various Italian and Papal states led the rapid proliferation of cryptography techniques. Various analysis and attack techniques were researched in this era to break the secret codes. Improved coding techniques such as Vigenere Coding came into existence in the 15th century, which offered moving letters in the message with a number of variable places instead of moving them the same number of places. Historical accounts indicate that the Chinese,Egyptians, Indian,and greek encrypted messages in some way for various purposes.One famous scheme is called ceaser cipher, also known as substitution cipher,used by Julius Caesar,involved shifting letters in the alphabet, such as replacing A by C,B by D,C by E etc to encode a message.Substitution ciphers are too simple in design to be considered secure today.

Codes and ciphers were used extensively in world war I.Trench codes were used by field armies of most of the combatants (America, British, French, German) in world war I.The most important code breaking event of the war was the successful decryption by the Allies of the German "Enigma" cipher. Enigma, device used by the German military command to encode strategic messages before and during world war II.

There have been three well defined phases in the history of cryptography.

- **Period of manual cryptography**

Cryptography was limited by the complexity of what a code clerk could reasonably do aided by simple mnemonic devices. As a result, cipher were limited to at most a few pages in size. The security that could be archived was always limited.

- **The mechanization of cryptography**

The applicable technology involved either telephone and telegraph communications. These machines could realize far more complex operations. Could encrypt and decrypt faster and with less chance of error. The switch from electromechanical devices to electronic ones accelerated this trend. In 1999 the US government designed and fabricated a single silicon chip implementation of the Data Encryption Standard (DES). In a few seconds of operation, trillions of bits of cipher can be processed.

- **The dramatization of cryptography to the information age: digital signatures, authentication etc.**

The inevitable consequence of having to devise ways for electronic information to perform all of the functions that had historically been done with the aid of tangible documents.

Early cryptographic systems and applications

- **Egyptians, Hebrews, Babylonians, and Assyrians all devised protocryptographic systems.**
- **Spartans employed a cipher device called the scytale.**
- **Greeks were the inventors of the first transposition cipher.**
- **The Romans used monoalphabetic substitution with a simple cyclic displacement of the alphabet.**
- **Arabs devised and used both substitution and transposition ciphers.**

- **European cryptology was developed by the papal States and the Italian city-states.**

Chapter 2

CRYPTOGRAPHY TECHNIQUE AND ILLUSTRATION

2.1 DIFFIE-HELLMAN MECHANISM

Diffie-Hellman key exchange also known as exponential key exchange, is a key exchange protocol that enables two parties communicating over public channel to establish a mutual secret transmitted over the internet. It enables the users to use a public key to encrypt and decrypt their data using symmetric cryptography.

2.1.1 ADVANTAGES

- The sender and the receiver don't need any prior knowledge.
- Once the key are exchanged, the communication of data can be done through an insecure channel.
- The sharing of the secret key is safe.

2.1.2 PROCEDURE

First we choose two individuals A and B each of them has a private value, which they does not reveal to anyone. Let the private values be a

and b respectively. Both of them creates their own cipher texts,

$$A = q^a \text{ mod } p$$

$$B = q^b \text{ mod } p$$

where p - prime number

q - Any non zero integer

a and b - private values

They exchange their cipher texts.

Both A and B find the same private key K , using the equation,

$$K = B^a \text{ mod } p$$

$$K = A^b \text{ mod } p$$

2.1.3 ILLUSTRATION

Alice and Bob wanted to exchange ATM number through mail but they are worried that there could be a third party hacker. So they choose to use matrices in exchanging ATM number.

ATM number 3690 5314 4384 9402

The problem was that they were far from each other. So the only means they can use to share the secret key is through public transport. Due to sensitivity of the information, it was required that this secret key should be shared as securely as possible.

$$A = g^a \text{ mod } p(\text{Alice}) \text{ and } B = g^b \text{ mod } p(\text{Bob})$$

$$K = B^a \text{ mod } p(\text{Alice}) \text{ and } K = A^b \text{ mod } p(\text{Bob})$$

where a and b are the secret values exclusively known to Alice and Bob respectively.

They both know $p = 23$ and generator, $g = 11$

Table 2.1: The method of generating the elements of the key matrix

Alice	Bob	K
$a=2 \ A=11^2 \bmod 23 = 6$ $K = 5^2 \bmod 23 = 2$	$b=5 \ B=11^5 \bmod 23 = 5$ $K = 6^5 \bmod 23 = 2$	2
$a=3 \ A=11^3 \bmod 23 = 20$ $K = 6^3 \bmod 23 = 9$	$b=2 \ B=11^2 \bmod 23 = 6$ $K = 20^6 \bmod 23 = 9$	9
$a=7 \ A=11^7 \bmod 23 = 7$ $K = 2^7 \bmod 23 = 13$	$b=10 \ B=11^{10} \bmod 23 = 2$ $K = 7^{10} \bmod 23 = 13$	13
$a=5 \ A=11^5 \bmod 23 = 5$ $K = 6^5 \bmod 23 = 2$	$b=2 \ B=11^2 \bmod 23 = 6$ $K = 5^2 \bmod 23 = 2$	2
$a=2 \ A=11^2 \bmod 23 = 6$ $K = 11^2 \bmod 23 = 6$	$b=1 \ B=11^1 \bmod 23 = 11$ $K = 6^1 \bmod 23 = 6$	6
$a=3 \ A=11^3 \bmod 23 = 20$ $K = 13^3 \bmod 23 = 12$	$b=4 \ B=11^4 \bmod 23 = 13$ $K = 20^4 \bmod 23 = 12$	12
$a=3 \ A=11^3 \bmod 23 = 20$ $K = 9^3 \bmod 23 = 16$	$b=6 \ B=11^6 \bmod 23 = 9$ $K = 20^6 \bmod 23 = 16$	16
$a=7 \ A=11^7 \bmod 23 = 7$ $K = 8^7 \bmod 23 = 12$	$b=8 \ B=11^8 \bmod 23 = 8$ $K = 7^8 \bmod 23 = 12$	12
$a=2 \ A=11^2 \bmod 23 = 6$ $K = 5^2 \bmod 23 = 2$	$b=5 \ B=11^5 \bmod 23 = 5$ $K = 6^5 \bmod 23 = 2$	2
$a=0 \ A=11^0 \bmod 23 = 1$ $K = 6^0 \bmod 23 = 1$	$b=2 \ B=11^2 \bmod 23 = 6$ $K = 1^2 \bmod 23 = 1$	1
$a=8 \ A=11^8 \bmod 23 = 8$ $K = 19^8 \bmod 23 = 9$	$b=9 \ B=11^9 \bmod 23 = 19$ $K = 8^9 \bmod 23 = 9$	9
$a=6 \ A=11^6 \bmod 23 = 9$ $K = 5^6 \bmod 23 = 8$	$b=5 \ B=11^5 \bmod 23 = 5$ $K = 9^5 \bmod 23 = 8$	8
$a=5 \ A=11^5 \bmod 23 = 5$ $K = 9^5 \bmod 23 = 8$	$b=6 \ B=11^6 \bmod 23 = 9$ $K = 5^6 \bmod 23 = 8$	8
$a=3 \ A=11^3 \bmod 23 = 20$ $K = 8^3 \bmod 23 = 6$	$b=8 \ B=11^8 \bmod 23 = 8$ $K = 20^8 \bmod 23 = 6$	6
$a=2 \ A=11^2 \bmod 23 = 6$ $K = 1^2 \bmod 23 = 1$	$b=0 \ B=11^0 \bmod 23 = 1$ $K = 6^0 \bmod 23 = 1$	1
$a=5 \ A=11^5 \bmod 23 = 5$ $K = 6^5 \bmod 23 = 2$	$b=2 \ B=11^2 \bmod 23 = 6$ $K = 5^2 \bmod 23 = 2$	2

Both A and B values are exchanged. Hence the generated secret key is,

$$K = \begin{bmatrix} 2 & 6 & 2 & 8 \\ 9 & 12 & 1 & 6 \\ 13 & 16 & 9 & 1 \\ 2 & 12 & 8 & 2 \end{bmatrix}$$

$$K^{-1} = \begin{bmatrix} 0.0544 & -0.0445 & 0.1217 & -0.1449 \\ -0.1439 & 0.1632 & -0.1128 & 0.1424 \\ 0.1573 & -0.2196 & 0.1335 & -0.0371 \\ 0.1800 & -0.0564 & 0.0208 & -0.0613 \end{bmatrix}$$

Plain text,

$$T = \begin{bmatrix} 3 & 5 & 4 & 9 \\ 6 & 3 & 3 & 4 \\ 9 & 1 & 8 & 0 \\ 0 & 4 & 4 & 2 \end{bmatrix}$$

To encode matrix T, with secret key K,

$$X = K * T = \begin{bmatrix} 2 & 6 & 2 & 8 \\ 9 & 12 & 1 & 6 \\ 13 & 16 & 9 & 1 \\ 2 & 12 & 8 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 & 4 & 9 \\ 6 & 3 & 3 & 4 \\ 9 & 1 & 8 & 0 \\ 0 & 4 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 60 & 62 & 74 & 58 \\ 108 & 106 & 104 & 42 \\ 16 & 126 & 176 & 183 \\ 150 & 62 & 116 & 70 \end{bmatrix}$$

The encoded message is sent to bob through public transport. For bob to get the intended message he is required to decode the message sent by alice by multiplying the encoded message with an inverse of K.

To decode the encoded matrix,

$$T = K^{-1} * X = \begin{bmatrix} 0.0544 & -0.0445 & 0.1217 & -0.1449 \\ -0.1439 & 0.1632 & -0.1128 & 0.1424 \\ 0.1573 & -0.2196 & 0.1335 & -0.0371 \\ 0.1800 & -0.0564 & 0.0208 & -0.0613 \end{bmatrix} \begin{bmatrix} 60 & 62 & 74 & 58 \\ 108 & 106 & 104 & 42 \\ 16 & 126 & 176 & 183 \\ 150 & 62 & 116 & 70 \end{bmatrix}$$

$$T = \begin{bmatrix} 3 & 5 & 4 & 9 \\ 6 & 3 & 3 & 4 \\ 9 & 1 & 8 & 0 \\ 0 & 4 & 4 & 2 \end{bmatrix}$$

After multiplying the encrypted message with an inverse, bob was able to get the exact ATM number in a secured manner. A virtual external force cannot hack the information sent to bob since they doesn't know the secret key.

Chapter 3

IMAGE COMPRESSION

USING SVD

3.1 THEORY

In Linear algebra, the Singular Value Decomposition(SVD) is a factorization of a real or complex matrix.It generalize the eigendecomposition of a square normal matrix with an orthonormal eigenbasis to any $m \times n$ matrix.It is related to the polar decomposition.

What SVD does is, it split a matrix into three important sub matrices to represent the data.

Given matrix A , where the size of A is $m \times n$ where m represents the number of rows in the matrix, and n represents the number of columns, A can be broken down into three sub matrices.

$$A = U\Sigma V^T$$

Where U is of size $m \times m$, Σ is of size $m \times n$ and is diagonal, and V^T is of size $n \times n$. It is required for matrix multiplication that the size of the columns of the first matrix must match up with the size of the rows of the second matrix.When you multiply a matrix of size $a \times b$ and a matrix of size $b \times c$, the resulting matrix will yield a matrix of size $a \times c$. So, abstracting the matrices into their size components, we can see that this multiplication will yield a matrix of the same size:

$$m * n = [(m * m)(m * n)](n * n)$$

$$m * n = (m * n)(n * n)$$

$$m * n = (m * n)$$

Now, the interesting part of these matrices " $U\Sigma V^T$ " are the data is arranged in such a way that the most important data is stored on the top. U is a matrix that holds important information about the rows of the matrix, and the most important information about the matrix is stored on the first column. V^T is a matrix that holds important information about the columns of each matrix, and the most important information about the matrix is stored on the first row. Σ is a diagonal matrix which will only have at most "m" important values, the rest of the matrix being zero. Since the important number of this matrix are only stored on the diagonal, we will ignore this for size comparisons.

3.2 ILLUSTRATION

Now we will get into the math and theory behind what we just described above. We will go through an example to solve the equation $A=U\Sigma V^T$.

Given $A = \begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix}$ find the SVD:

The first thing we need to find in this computation is the matrix $A^T * A$. The superscript T stands for "transpose" i.e, for A^T , row one becomes column one.

$$A^T * A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Once you find $A^T A$, you will need to find its eigenvalues and eigenvectors. To find the eigenvalues of the matrix, you need to compute the

determinant of $(A^T - \lambda I)$ and solving for λ , where I is the identity matrix. First let's find

$$\begin{aligned} A^T A - \lambda I &= \begin{bmatrix} 5 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 5-\lambda & 3 & 0 \\ 3 & 5-\lambda & 0 \\ 0 & 0 & -\lambda \end{bmatrix} \end{aligned}$$

Now we need to compute the determinant of this matrix.

$$\begin{aligned} \begin{vmatrix} 5-\lambda & 3 & 0 \\ 3 & 5-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} &= 0 \begin{vmatrix} 3 & 5-\lambda \\ 0 & 0 \end{vmatrix} - 0 \begin{vmatrix} 5-\lambda & 3 \\ 0 & 0 \end{vmatrix} + (-\lambda) \begin{vmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{vmatrix} \\ &= -\lambda((5-\lambda)(5-\lambda) - (3)(3)) \\ &= -\lambda(\lambda^2 - 10\lambda + 16) \end{aligned}$$

Now we can solve this to find our eigenvalues:

$$-\lambda(\lambda^2 - 10\lambda + 16) = 0$$

$$-\lambda(\lambda - 2)(\lambda - 8) = 0$$

Therefore our eigenvalues are 8, 2 and 0. We will want to keep these numbers in descending order.

With this information, we can find an important value " σ " which is the square root of the eigenvalues. We ignore zero for the " σ " term. Therefore:

$$\sigma_1 = \sqrt{8} = 2\sqrt{2}$$

$$\sigma_2 = 2$$

These values are the important values along the diagonal of matrix " Σ ".

Next we need to find the normalized version of the corresponding eigenvectors to each of the eigenvalues. To find an eigenvalue, replace λ with the corresponding eigenvalue in the equation $(A^T A - \lambda I)$. Then find the nullspace of that resulting matrix:

When $\lambda = 8$, the resulting matrix yields:
$$\begin{bmatrix} -3 & 3 & 0 \\ 3 & -3 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

To find the null space of this matrix, we need to find some vector " \vec{v} " that when multiplied by the matrix, will yield the zero vector. The vector though cannot be the zero vector itself. There are complicated methods to solve for the null space, however in this example, you might be able to just see it.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

As you can see, when we multiply this vector by the matrix, it will yield the zero vector:

$$\begin{bmatrix} -3 & 3 & 0 \\ 3 & -3 & 0 \\ 0 & 0 & -8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now that we found " \vec{v} ", we need to compute the normalized version, and then solve for the remaining eigenvectors in a similar way. To find the normalized version, you multiply the vector by the reciprocal of the square root of the sum of the squared rows. So for this example, $1^2 = 1$ and the sum of the squared rows is 2. Therefore the normalized version is:

$$\vec{v}_1 = 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Now we need to find the normalized eigenvalues for $\lambda = 2$ and $\lambda = 0$.

When $\lambda = 2$, the resulting matrix yields:

$$\begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

The normalized version is:

$$v_2 = 1/\sqrt{2} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

When $\lambda = 0$, the resulting matrix yields:

$$\begin{bmatrix} 5 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The normalized version is:

$$V_3 = 1/\sqrt{1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ or just } \rightarrow \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now that we have found our eigenvalues denoted by " \vec{v} " we need to find this term " \vec{u} " which can be found using the equation.

$$A \vec{v} = \sigma \vec{u} \text{ or } 1/\sigma A \vec{v} = \vec{u}$$

$$1/\sigma A \vec{v}_1 = \vec{u}$$

$$1/2\sqrt{2} \begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \vec{u}_1$$

$$1/4 \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \vec{u}_1$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \vec{u}_1$$

Calculating the next term :

$$1/\sigma A \vec{v}_2 = \vec{u}_2$$

$$1/\sqrt{2} \begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} 1/\sqrt{2} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \vec{u}_2$$

$$1/2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \vec{u}_1$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \vec{u}_2$$

Since there are no more σ terms, we can stop there. Now with all of the values we have found, we can complete the three matrices in the equation $A=U\Sigma V^T$.

First is the matrix U which can be found by making the first column in the matrix the first term and the second column the second term. Therefore:

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Next is the matrix Σ which is the same size as the matrix A and can be found by placing the σ terms along the diagonal of a matrix of that size, and then filling in the rest of the matrix with zeros.

$$\Sigma = \begin{bmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

Lastly is the matrix V^T which can be found by first computing V and then transposing it, by turning the columns into the corresponding rows. V is found similar to U in the fact that the columns of V are the corresponding \vec{v} terms. Remember to multiply the constant used to scale

the vector through the entire term before adding it to the matrix.

$$V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So now we have the finished equation $A = U\Sigma V^T$.

$$\begin{bmatrix} 2 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

You can also multiply the terms together to show that the equation holds true. Now to move onto the point, what are the real world applications. We will now see how an image, whether grayscale or coloured is being compressed in Matlab by SVD method. The following is an illustration of image compression for a grayscale and a coloured image, along with the codes used.

3.3 MATLAB CODE FOR GRAYSCALE IMAGES

This section contains instructions for Matlab which will open an image file, turn the image into a grayscale format, grab the image data and build a matrix representing each pixel value as 0-255 as data on the matrix. It will then compute the SVD on the matrix and display varying different modes and levels of precision based on the image compression, as well as an error graph at the end on how accurate the image got based on the difference from the original image. It will also save these resulting images on your computer. To upload an image, replace the "image.jpg" with the filepath, name, and data type of the image you wish to use. The following will give you modes 1,10,... To edit these, change the value of N in the loops.

```
close all
```

```
clear all
```

```
clc
```

Reading and converting the image

```
inImage=imread('image.jpg');
```

```
inImage=im2gray(inImage);
```

```
inImageD=double(inImage);
```

```
imwrite(uint8(inImageD), 'original.jpg');
```

Decomposing the image using singular value decomposition

```
[U,S,V]=svd(inImageD);
```

Using different number of singular values (diagonal of S) to compress and reconstruct the image

```
dispEr = [];
```

```
numSVals = [];
```

For N=1

```
N = 1
```

Storing the singular values in a temporary variable

```
C = S;
```

discard the diagonal values not required for compression

```

C(N+1:end,:)=0;
C(:,N+1:end)=0;
Construct an Image using the selected singular values
D=U*C*V';
display and compute error
figure;
buffer = sprintf('Image output using %d singular values', N)
imshow(uint8(D));
imwrite(uint8(D), sprintf('%dbw.jpg', N));
title(buffer);
error=sum(sum((inImageD-D).^2));
Storing values for display
dispEr = [dispEr; error];
numSVals = [numSVals; N];
    
```

For N=2

```

N=2
C = S;
C(N+1:end,:)=0;
C(:,N+1:end)=0;
D=U*C*V';
figure;
buffer = sprintf('Image output using %d singular values', N)
imshow(uint8(D));
imwrite(uint8(D), sprintf('%dbw.jpg', N));
title(buffer);
error=sum(sum((inImageD-D).^2));
dispEr = [dispEr; error];
numSVals = [numSVals; N];
end
    
```

Now we take an image and discuss the effect that different iterations

will have on it.

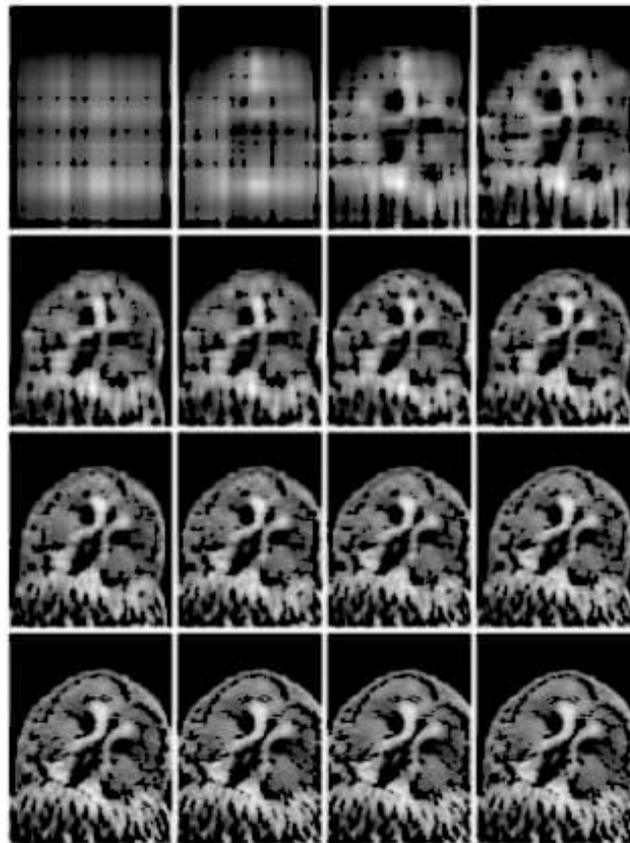


Figure 3.1: Image using modes (1,2,4,6)(8,10,12,14)(16,18,20,25)(50,75,100,Original image)

From the above figure, the resemblance to the original image can be seen from mode 50 onwards. By negating the size of Σ , since it is so miniscule, we can calculate,

Original image = 68000 bytes

Mode 100 = 62000 bytes

Mode 75 = 58000 bytes

Mode 50 = 52000 bytes

By using the code in matlab, we can even graph the error involved in compression.

Display the error graph

```
figure;  
title('Error in compression');
```

```

plot(numSVals, dispEr);
grid on
xlabel('Number of Singular Values used');
ylabel('Error between compress and original image');
    
```

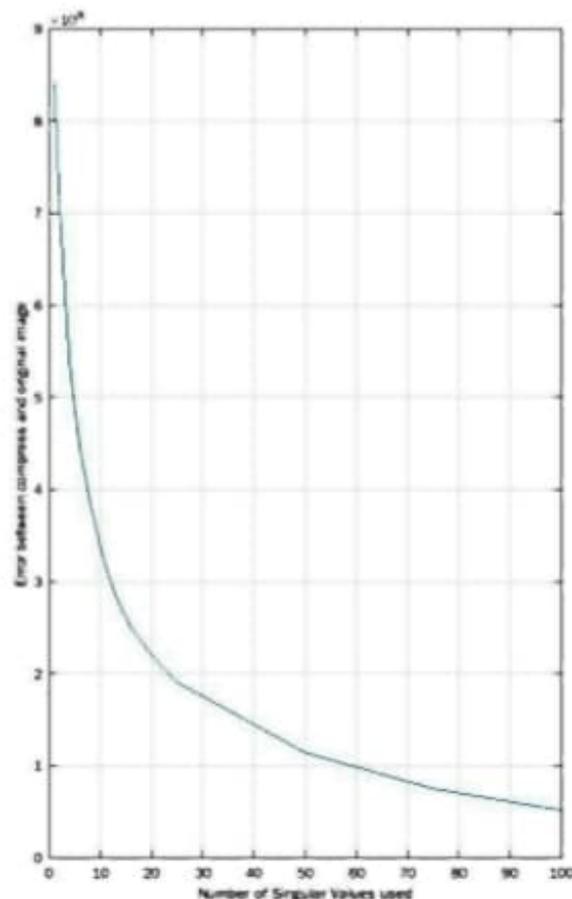


Figure 3.2: Image using modes (1,2,4,6)(8,10,12,14)(16,18,20,25)(50,75,100,Original image)

Now we can see that this works for a grayscale image, but what about a colored image? Would this still have the same application for image compression? The answer is a surprising yes, but it does require a few more calculations. The difference between a grayscale image and a colored image is that you are now storing 3 bytes of information per pixel rather than 1 byte per pixel. This is because the red, green, and

blue pixel values are now different rather than the same, so we have to represent each individually.

3.4 MATLAB CODE FOR COLORED IMAGES

The following are the instructions for Matlab which will Open an image file, and then split the image into three separate images; a red-scale, a green-scale, and a blue-scale image. It will then plot the pixel data from these images into a matrix, representing values 0-255 based on the pixel saturation. It will then compute the SVD on each of these scaled images, save them on the computer, display the corresponding scaled images, and then it will also merge these images back together to form a colored image, also displaying and saving the image as well. The following will give you modes 1,10,... To edit these, change the value of N in the loops.

```
close all
clear all
clc
filename = 'image.jpg';
[X,map] = imread(filename);
figure('Name', 'ORIGINAL component of the imported image');
imshow(X);
imwrite(X, 'original.jpg');
R = X( :,:,1 );
G = X( :,:,2 );
B = X( :,:,3 );
Rimg = cat(3, R, zeros(size(R)), zeros(size(R)));
Gimg = cat(3, zeros(size(G)), G, zeros(size(G)));
Bimg = cat(3, zeros(size(B)), zeros(size(B)), B);
figure('Name','RED component of the imported image');
imshow(Rimg);
imwrite(Rimg, 'red.jpg');
figure('Name','GREEN component of the imported image');
imshow(Gimg);
```

```
inwrite(Gimg, '!green.jpg');  
figure('Name','BLUE component of the imported image');  
imshow(Bimg);  
inwrite(Bimg, '!blue.jpg');
```

```
Red =double(R);  
Green = double(G);  
Blue = double(B);
```

For $N=1$

```
N = 1;  
Compute values for the red image  
[U, S, V]=svd(Red);  
C = S;  
C(N+1:end,:)=0;  
C(:,N+1:end)=0;  
Dr=U*C*V';
```

Rebuild the data back into a displayable image and show it

```
figure;  
buffer = sprintf('Red image output using %d singular values', N);  
Rimg = cat(3, Dr, zeros(size(Dr)), zeros(size(Dr)));  
imshow(uint8(Rimg));  
inwrite(uint8(Rimg), sprintf('%dred.jpg', N));  
title(buffer);
```

Compute values for the green image

```
[U2, S2, V2]=svd(Green);  
C = S2;  
C(N+1:end,:)=0;  
C(:,N+1:end)=0;  
Dg=U2*C*V2';
```

Rebuild the data back into a displayable image and show it

figure;

```
buffer = sprintf('Green image output using %d singular values', N);
```

```
Gimg = cat(3, zeros(size(Dg)), Dg, zeros(size(Dg)));
```

```
imshow(uint8(Gimg));
```

```
imwrite(uint8(Gimg), sprintf('%dgreen.jpg', N));
```

```
title(buffer);
```

Compute values for the blue image

```
[U3, S3, V3]=svd(Blue);
```

```
C = S3;
```

```
C(N+1:end,:)=0;
```

```
C(:,N+1:end)=0;
```

```
Db=U3*C*V3';
```

Rebuild the data back into a displayable image and show it

figure;

```
buffer = sprintf('Blue image output using %d singular values', N);
```

```
Bimg = cat(3, zeros(size(Db)), zeros(size(Db)), Db);
```

```
imshow(uint8(Bimg));
```

```
imwrite(uint8(Bimg), sprintf('%dblue.jpg',N));
```

```
title(buffer);
```

Take the data from the Red, Green, and Blue image

Rebuild a colored image with the corresponding data and show it

figure;

```
buffer = sprintf('Colored image output the%d singular values', N);
```

```
Cimg = cat(3, Dr, Dg, Db);
```

```
imshow(uint8(Cimg));
```

```
imwrite(uint8(Cimg), sprintf('%dcolor.jpg', N));
```

```
title(buffer);
```

For $N=2$

$N=2$

Recompute modes for the red image

```
C = S;  
C(N+1:end,:)=0;  
C(:,N+1:end)=0;  
Dr=U*C*V';  
figure;  
buffer = sprintf('Red image output using %d singular values', N);  
Rimg = cat(3, Dr, zeros(size(Dr)), zeros(size(Dr)));  
imshow(uint8(Rimg));  
imwrite(uint8(Rimg), sprintf('%dred.jpg', N));  
title(buffer);
```

Recompute modes for the green image

```
C = S2;  
C(N+1:end,:)=0;  
C(:,N+1:end)=0;  
Dg=U2*C*V2';  
figure;  
buffer = sprintf('Green image output using %d singular values', N);  
Gimg = cat(3, zeros(size(Dg)), Dg, zeros(size(Dg)));  
imshow(uint8(Gimg));  
imwrite(uint8(Gimg), sprintf('%dgreen.jpg', N));  
title(buffer);
```

Recompute modes for the blue image

```
C = S3;  
C(N+1:end,:)=0;  
C(:,N+1:end)=0;  
Db=U3*C*V3';  
figure;
```

```
buffer = sprintf('Blue image output using %d  
singular values', N);  
Bimg = cat(3, zeros(size(Db)), zeros(size(Db)), Db);  
imshow(uint8(Bimg));  
imwrite(uint8(Bimg), sprintf('%dblue.jpg', N));  
title(buffer);
```

Take the data from the Red, Green, and Blue image
Rebuild a colored image with the corresponding data and show it
figure;

```
buffer = sprintf('Colored image output using %d singular values', N);  
Cimg = cat(3, Dr, Dg, Db);  
imshow(uint8(Cimg));  
imwrite(uint8(Cimg), sprintf('%dcolor.jpg', N));  
title(buffer);  
end
```

Now we take an image and discuss the effect that different iterations will have on it.

First, we need to take a colored image, and split it into three new images, a red-scale, a green-scale, and a blue-scale image.

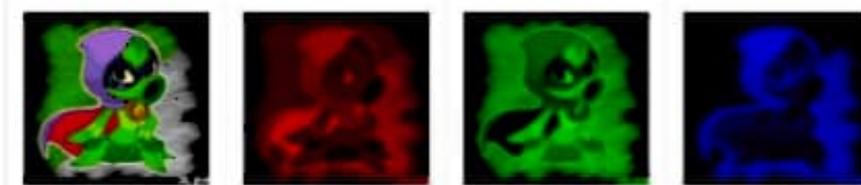


Figure 3.3: Original image and its Red, Green and Blue components

Changing the no. of modes, to 1, 5, 10, 15 & 20 we get,

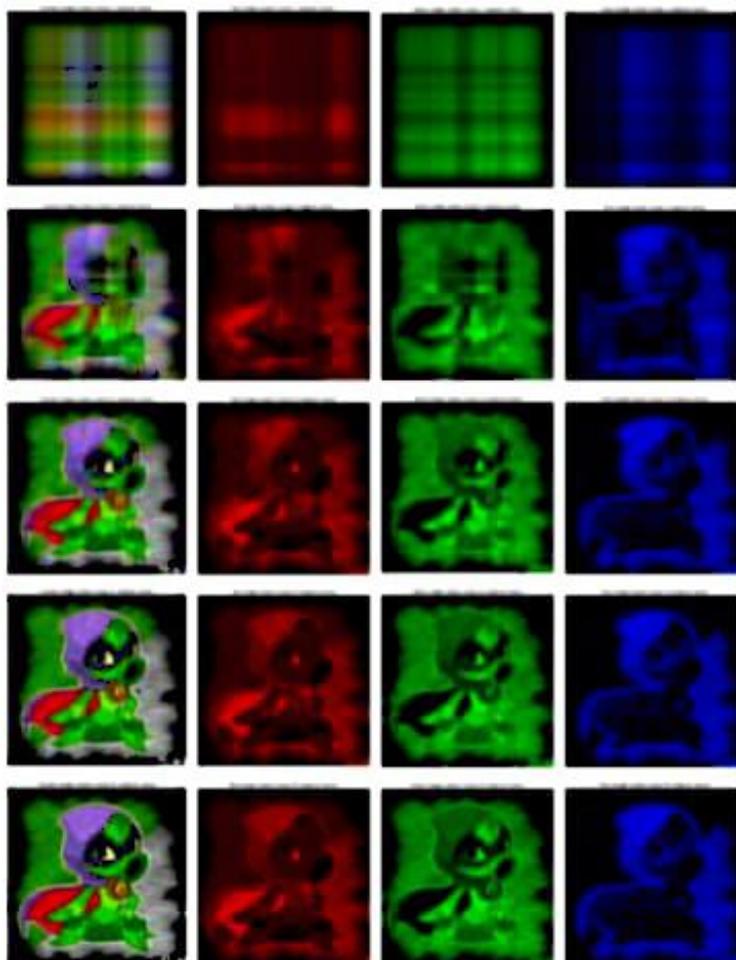


Figure 3.4: Image using modes(1,5,10,15,20)

Just by taking 20 modes, we can get an idea of what we are looking at. Going on like this we will get the image as follows.

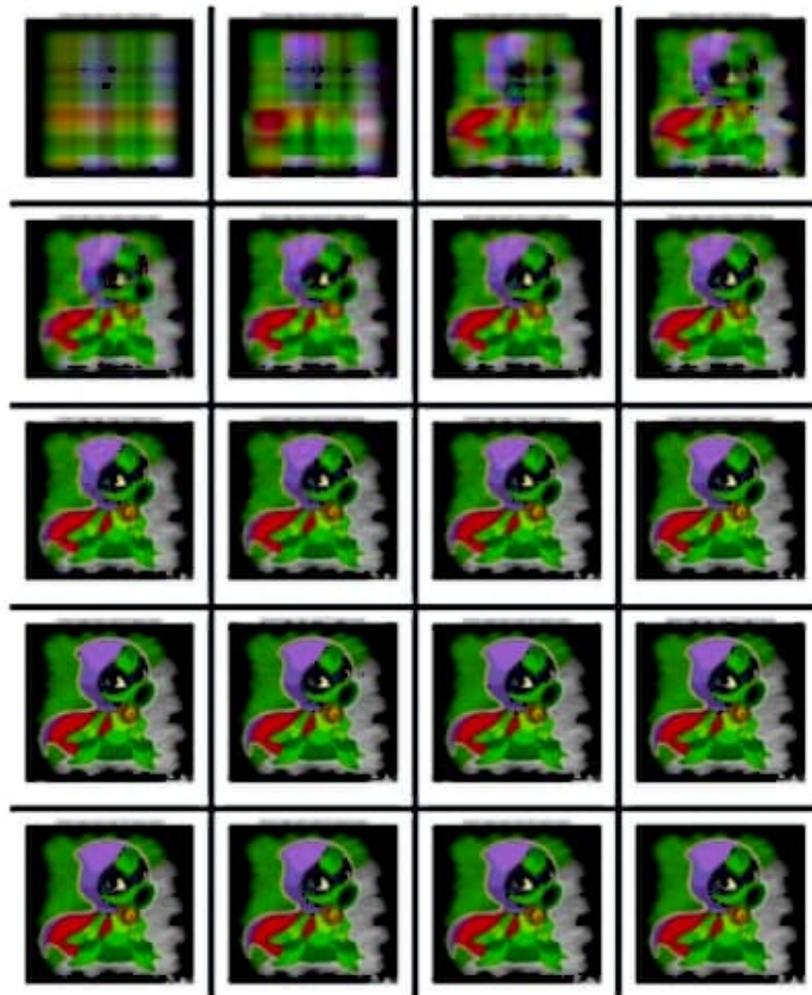


Figure 3.5: Image using modes(1,2,4,6)(8,10,12,14)(16,18,20,25)(50,75,100,Original image)

From the above figure, the resemblance to the original image can be seen from mode 50 onwards. By negating the size of Σ , since it is so miniscule, we can calculate,

Original image = 878,000 bytes

mode 200 = 152,000 bytes

Mode 100 = 149,000 bytes

Mode 75 = 143,000 bytes

Mode 50 = 133,000 bytes

Since the image has 3 layers, the graph would be three dimensional. So the error is a little more difficult to plot. But just as in case of a grayscale image, from the figure, here too we can observe that the error will be so miniscule as it closes to mode 100.

Chapter 4

PRELIMINARY STUDY ON HAAR WAVELET

Wavelets provide a powerful and remarkably flexible set of tools for handling fundamental problems in science and engineering, such as signal compression, object detection and fingerprint compression, image enhancement, image recognition, diagnostic heart trouble and speech recognition. Here, we are going to discuss on wavelet application in the field of Image Compression . Wavelet image compression is performed with various known wavelets with different mathematical properties.

Haar-wavelet compression is an efficient way to perform both lossless and lossy image compression. It relies on averaging and differencing values in an image matrix to produce a matrix which is sparse or nearly sparse.(A sparse matrix is a matrix in which a large portion of its entries are 0. A sparse matrix can be stored in an efficient manner, leading to smaller file sizes.)Using a wavelet transform, the wavelet compression methods are enough for representing transients, such as percussion sounds in audio, or high-frequency components in two-dimensional images, Wavelets are functions which allow data analysis of signals or images, according to scales or resolutions. The processing of signals by wavelet algorithms in fact works much the same way the human eye does; or the way a digital camera processes visual scales of resolutions, and intermediate details. But the same principle also captures cell phone signals, and even digitized color images used in

medicine. Wavelets are of real use in these areas, for example in approximating data with sharp discontinuities such as choppy signals, or pictures with lots of edges. While wavelets is perhaps a chapter in function theory, we show that the algorithms that result are key to the processing of numbers, or more precisely of digitized information, signals, time series, still-images, movies, color images, etc. Thus, applications of the wavelet idea include big parts of signal and image processing, data compression, fingerprint encoding, and many other fields of science and engineering. This is focuses on the processing of color images with the use of custom designed wavelet algorithms, and mathematical threshold filters. The computer generated images which serve to illustrate our ideas and our algorithms, and also with the resulting, compressed images.

The advantages of Haar-wavelet transform are it is conceptually simple and fast. It is memory efficient, since it can be calculated in place without a temporary array. It is exactly reversible without the edge effects that are a problem with other wavelet transforms. It provides high compression ratio and high PSNR (Peak signal to noise ratio). And it increases detail in a recursive manner.

CONCLUSION

In Diffie Hellman mechanism, the matrix to generate the key is transferred publicly and the key is generated by both the parties using the secret elements that are exclusively known to them. Even though the secret elements are different, the same key matrix can be generated at the same time in different locations. Since elements used to generate the key matrix are secret, no external sources can crack the message, even if they get the cipher text. This ensures safe transfer of the sensitive data.

An image of $m \times n$ pixels, is data represented on a matrix of order $m \times n$, visually displayed through pixels of red, green and blue. This data can be manipulated through the use of the SVD theorem to calculate a level of precision close to the original without storing as much data. The SVD allows us to store $(\#modes)(m+n)$ information instead of $(m \times n)$ information, where the size of the image is $m \times n$, or $3(\#modes)(m+n)$ when the image is in color instead of $3(m \times n)$.

By the Haar wavelet techniques, the image compression was developed fastly and the promising results obtained concerning the reconstructed image quality as well as protection of important image details, on the other hand achieves a high compression rate.

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FINAL YEAR M.Sc. PHYSICS
PROJECT REPORT
2021-2022

**TEMPERATURE CONTROLLED SYNTHESIS OF ZINC
FERRITES AND ITS NANOCOMPOSITES FOR
PHOTOCATALYTIC AND ANTIBACTERIAL
APPLICATIONS**

PROJECT REPORT

Submitted by
FATHIMA NOURIN V. A.
Register No: AM20PHY005

Under the guidance of
Ms. MINU PIUS

In partial fulfillment of the requirement for the award
Of

**MASTER DEGREE OF SCIENCE IN
PHYSICS**



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ERNAKULAM, KOCHI-682011**

**DEPARTMENT OF PHYSICS AND CENTRE FOR RESEARCH
ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM**



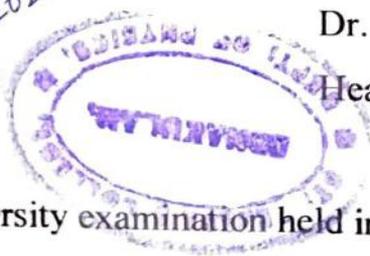
M.Sc. PHYSICS PROJECT REPORT

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Year of Work : 2020-2022

This is to certify that the project “**TEMPERATURE CONTROLLED SYNTHESIS OF ZINC FERRITES AND ITS NANOCOMPOSITES FOR PHOTOCATALYTIC AND ANTIBACTERIAL APPLICATIONS**” is the work done by **Fathima Nourin V A.**

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CERTIFICATE

This is to certify that the project report titled “**TEMPERATURE CONTROLLED SYNTHESIS OF ZINC FERRITES AND ITS NANOCOMPOSITES FOR PHOTOCATALYTIC AND ANTIBACTERIAL APPLICATIONS**” submitted by **FATHIMA NOURIN V A**, towards partial fulfillment of the requirements for the award of the degree of Masters of Physics is a record of bonafide work carried out by them during the academic year 2021-2022

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DATE: 13-06-22

**TEMPERATURE CONTROLLED
SYNTHESIS OF ZINC FERRITES AND ITS
NANOCOMPOSITES FOR
PHOTOCATALYTIC AND ANTIBACTERIAL
APPLICATIONS**

DECLARATION

I, **FATHIMA NOURIN V A**, Register No.AM20PHY005, hereby declare that this project entitled “**TEMPERATURE CONTROLLED SYNTHESIS OF ZINC FERRITES AND ITS NANOCOMPOSITES FOR PHOTOCATALYTIC AND ANTIBACTERIAL APPLICATIONS**”, is an original work done by me under guidance of **Ms. Minu Pius**, Assistant Professor, Department of Physics and Centre for Research St. Teresa's College (Autonomous), Ernakulam in partial fulfillment for the award of the Degree of Master of Physics, I further declare that this project is not partly or wholly submitted for any other purpose and the data included in the project is collected from various sources and are true to best of my knowledge.

PLACE: Ernakulam


FATHIMA NOURIN V A

DATE: 13 - 06 - 22

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ABSTRACT

The project deals with the synthesis of Zinc Ferrite nanoparticles at different calcinations temperatures and characterization to obtain the optimized sample. This sample is made to form nanocomposite with Silver and Silver Chloride. The synthesis of Zinc Ferrite is done by co-precipitation method using chloride precursors and NaOH as precipitating agent. The nanocomposite is made in the same procedure by adding Silver precursors at different stages of synthesis. Thus different samples are obtained the photo catalytic ability and antibacterial ability of the samples is evaluated. The nanocomposite of Zinc ferrites shows a maximum degradation of 98% efficiency which makes them very useful as a photo catalyst. The sample can be retrieved by applying external magnetic field due to the super paramagnetic nature.

The antibacterial properties show that the Zinc Ferrite and silver chloride nanocomposite have higher antibacterial properties. Thus the composite sample has a great potential to be used for targeted drug delivery and antibacterial applications.

The study focuses on the effect of calcination temperature on different samples and the variations in properties due to their different composition.

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CHAPTER 1

INTRODUCTION

1.1 Ferrite Nanoparticles

Ferrite is a type of ceramic compound composed of iron oxide (Fe_2O_3) combined chemically with one or more additional metallic elements. Ferrites are hard, brittle, iron-containing and polycrystalline—i.e., made up of a large number of small crystals. They are composed of iron oxide and one or more other metals in chemical combination. Ferrites can have several different types of crystalline structures, including spinel, garnet, perovskite, and hexagonal. They are ferrimagnetic, meaning they can be magnetized or attracted to a magnet, and are electrically nonconductive. In ferrites the magnetic moments of constituent atoms align themselves in two or three different directions. Ferrites are exceptional magnetic materials that exhibit a strong magnetic property, relatively low conductivity, low eddy current and dielectric losses, and high permeability. Ferrite nanoparticles (FNPs) belong to a broad group of magnetic nanoparticles (MNPs). Ferrite's electrical and magnetic features are influenced by various factors, including the method of preparation, elemental composition, sintering temperature, cation distribution, particle size, etc.

Because of their large surface to volume ratio, ferrite nanoparticles exhibit fascinating and significantly different magnetic properties than those of bulk materials. Ferrite nanoparticles have wide applications in several areas such as biomedical, wastewater treatment, catalyst and electronic device.

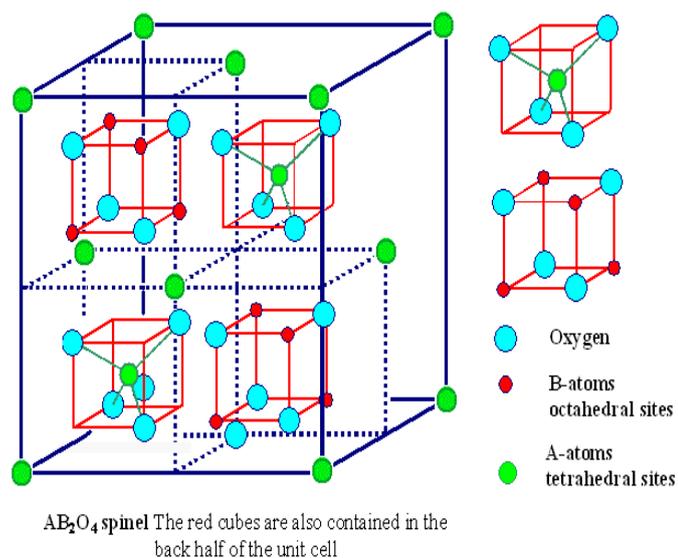
In terms of biomedical applications, they are used for diagnosis (magnetic resonance imaging, biosensors), cancer treatment, drug delivery systems, and magnetic hyperthermia.

1.2 Relevance of Zinc Ferrites

Zinc ferrites are a series of synthetic inorganic compounds of zinc and iron (ferrite) with the general formula of $Zn_xFe_{3-x}O_4$. Reducing the particle size of zinc ferrites, from micron to nanometer size, results in a significant change of their properties. Although bulk zinc ferrite is an antiferromagnetic material; when reduced to nanosize, they exhibit super paramagnetic behavior. Zinc ferrite nanoparticles have unique properties such as chemical and thermal stability due to smaller particle size and higher surface area. It exhibits unique structural, opto-electrical, magnetic and photo catalytic activities, and has high electromagnetic performance, mechanical hardness and moderate saturation magnetization.

1.3 Structure of Zinc Ferrites

Zinc ferrites are normal spinel structures which are cubic close-packed oxides with eight tetrahedral and four octahedral sites per formula unit. Zinc ferrite ($ZnFe_2O_4$) possesses an AB_2O_4 structure with tetrahedral A site occupied by Zn^{2+} ions and octahedral B site with Fe^{3+} ions in a face-centered cubic unit cell.



1.4 Synthesis Techniques

Many methods can be employed for the synthesis of zinc ferrite nanoparticles. The most common among those are sol-gel synthesis and co-precipitation, which are both bottom-up synthesis processes.

Sol-gel synthesis is also known as wet chemical method. The sol-gel method is a conventional and industrial method for the synthesis of nanoparticles with different chemical composition. The basis of the sol-gel method is the production of a homogeneous sol from the precursors and its conversion into a gel. In this method, the molecular precursor is dissolved in water or alcohol and converted to gel by heating and stirring by hydrolysis. The solvent in the gel is then removed from the gel structure and the remaining gel is dried. The properties of the dried gel depend significantly on the drying method. After the drying stage, the produced gels are powdered and then calcined.

Co-precipitation is a phenomenon where a solute that would normally remain dissolved in a solution precipitates out on a carrier that forces it to bind together, rather than remaining dispersed. In the process of co-precipitation, chemical similarities between a carrier and a solute allow the two to bind in some way. The binding pulls the solute out of the solution as the carrier forms crystals or other structures. These can potentially be skimmed out or removed in other ways, leaving a purified solution behind.

There is also biological synthesis method, which involves the use of biological reducing and stabilizing agents such as plant extracts, bacteria, fungi, fruit extracts and natural biopolymer which is safer and safer and environmental friendly. Sono-chemical method and solution combustion method are also some of the synthesis techniques. The solution combustion technique is useful in preparation of high-quality ferrite nanoparticles.

1.5 Relevance of calcination temperature in co-precipitation method

Calcination refers to thermal treatment of a solid chemical compound whereby the compound is raised to high temperature without melting under restricted supply of ambient oxygen for the purpose of removing impurities or volatile substances.

Calcination temperature is the temperature at which calcination is performed. Samples to be studied are calcined at higher temperatures and it is seen that as the calcination temperature is increased, the crystalline nature of the nanoparticles was improved and had an effect on the size and shape of the crystal formed. Calcined materials at higher temperatures, exhibited greater activity as a result of larger particle size and high crystallinity. Calcination temperatures allowed obtaining materials with good optical, morphological and magnetic properties that enable their efficient use in various fields.

1.6 Applications of Zinc Ferrite Nanoparticles

Sl.No.	Title	Application
1	Fabrication and characterization of self-assembled zinc ferrite nanospheres for biomedical applications.(2022)	Biomedical application
2	Zinc ferrite nanoparticles from industrial waste for Se (IV) elimination from wastewater.(2022)	Waste water treatment
3	Biogenic synthesis enhanced structural, morphological, magnetic and optical properties of zinc ferrite nanoparticles for moderate hyperthermia applications.(2021)	Hyperthermia application
4	"Anti-bacterial and wound healing-promoting effects of zinc ferrite nanoparticles(2021)	Antibacterial and wound healing effect

1.7 Relevance of Zinc Ferrite Nanocomposites

Zinc ferrite nanoparticles alone have appreciable properties and applications in different fields but are limited. Zinc ferrite nanocomposites are used in order to enhance the properties, enabling it for more applications.

Sl.No.	Title	Application
1	Eco-friendly synthesis of cobalt-zinc ferrites using quince extract for adsorption and catalytic applications: An approach towards environmental remediation. (2022)	Adsorptive removal of pollutants, catalytic decomposition of the H ₂ O ₂ and low-frequency hyperthermia.
2	Controllable synthesis of zinc ferrite nanostructure with tunable morphology on polyaniline nanocomposite for super capacitor application. (2022)	Various applications from electronic devices to transportation.
3	Modulation of magneto electric coupling through systematically engineered spin canting in nickel–zinc ferrite (2022).	Used in modulation of the Magneto Electro coupling in spinel-structured oxides.
4	Recent advances on synthesis, characterization and high frequency applications of Ni-Zn (ferrite nano particles. (2021)	Microwave devices, power transformers in electronics, rod antennas

CHAPTER 2

EXPERIMENTAL METHODS

In the present study, Zinc Ferrite nanoparticles and composite of silver and silver chloride with Zinc Ferrite nanoparticles are synthesized by co-precipitation method and the temperature dependence on the properties are evaluated. The co-precipitation technique is a promising method to synthesize easily reproducible and pure nanomaterials at low temperature and low cost.

2.1 Synthesis of Zinc Ferrite Nano Particles

The Nano particles of Zinc Ferrites are synthesized using chloride precursors and sodium hydroxide as precipitating agent at room temperature and subsequent precipitate is subjected to calcinations over a temperature range of 100 to 800°C.



(Chloride precursors)

Ferric Chloride hexahydrate ($\text{FeCl}_3 \cdot 6\text{H}_2\text{O}$), Zinc Chloride (ZnCl_2) and sodium hydroxide (NaOH) combined in the stoichiometric ratio 2:1:8 to form Zinc Ferrite (ZnFe_2O_4), Sodium Chloride (NaCl) and water (H_2O).

Optimized concentration:



The required weight of salts are found using the formula,

$$W = \frac{(\text{Molecular mass (u)} \times \text{Volume (ml)} \times \text{Molar concentration (M)})}{1000(\text{ml})}$$

Sl. No.	Compound	Molecular mass(u)	Volume (ml)	Molar Concentration (M)	Required Weight (g)
1	FeCl ₃ .6H ₂ O	270.3	300	0.1	8.109
2	ZnCl ₂	136.28	300	0.05	2.044
3	NaOH	40	600	0.4	9.6

The required weights are measured using electronic balance. The Ferric chloride, Zinc Chloride, NaOH powders are stirred in distilled water for half hour. The precipitating agent NaOH are added dropwise into this mixed solution of ferric chloride and zinc chloride subjected to constant magnetic stirring at a temperature of 80°C. After 2 hours the precipitation begins. The pH of the mixture should be 10-12 range. The solution is kept in a hydrothermal unit at a temperature of 100°C for 90 minutes for digestion. The obtained precipitate is washed with distilled water and acetone and centrifuged at a rate of 6000 rpm, 3-4 times. This sample is kept for drying for 2 hours. Then the sample is grinded for 30 minutes in a mortar and pestle. The powdered precipitate is subjected to calcinations at a temperature range of 100 to 800°C in a muffle furnace for 6 hours for obtaining different samples ZF100, ZF200, ZF300.....ZF800. The calcined and uncalcined samples are characterized using different techniques and optimized sample is chosen for composite synthesis.

2.2 Synthesis of Zinc Ferrite nanocomposites

Chapter 2 Experimental Methods

The composite with silver and silver chloride are prepared by combining the optimized sample of Zinc ferrite nanoparticles with silver nitrate (AgNO_3) and trisodium citrate (TSC).

Optimized concentration: AgNO_3 : TSC = 0.01M: 0.1M

Sl. No.	Compound	Molecular mass(u)	Volume (ml)	Molar Concentration (M)	Required Weight (g)
1	$\text{FeCl}_3 \cdot 6\text{H}_2\text{O}$	270.3	200	0.1	5.406
2	ZnCl_2	136.28	200	0.05	1.363
3	NaOH	40	500	0.4	8
4	AgNO_3	169.87	350	0.01	0.595
5	TSC	294.1	350	0.1	10.294

The synthesis procedure of Zinc ferrite is done as discussed above and the silver nitrate and trisodium citrate mixture is added at different stages of precipitation for obtaining different samples. Calcination is done by keeping the samples taken in crucibles in muffle furnace for 6 hours.

- a) Slow mixing of silver nitrate and trisodium citrate with the precipitate of Zinc Ferrite sample. Calcination at temperature 500°C - 800°C . Samples obtained are labeled as CS500, CS600, CS700, and CS800.
- b) Sudden mixing of silver nitrate and trisodium citrate after the precipitate is formed. Calcination at 700°C and sample is CS1.

- c) Mixing of silver precursor is done after centrifugation of zinc ferrite precipitate. Samples are calcined at temperature of 700°C. Sample is named as CS2.

2.3 Characterizations

The structural, optical, thermal and magnetic characterizations of the zinc ferrite nanoparticles and nanocomposite samples are done and the properties are compared.

2.3.1 X-ray Diffraction (XRD) Analysis

X-ray diffraction techniques are used to study the structure of solids and crystalline materials. The crystalline nature, the chemical composition, lattice parameters, crystallite size etc., can be obtained from the XRD data. This method involves the elastic scattering of an X-ray beam from a sample and the intensity of the beam as a function of scattered angle is studied.

The most common methods in X-ray diffraction are the powder diffraction method. In this method the X-ray diffraction on powder or microcrystalline samples are used for structural characterization of materials using a powder diffract meter.

The sample contains a large number of crystallites or grains with random orientations. When a monochromatic beam falls upon the atoms in the Bragg's plane scattered radiations will emerge from each atom in all directions. Strong amplification of emitted signal occurs at very specific angles, where the scattered waves interfere constructively. This effect is called diffraction. The angle between the incident and scattered beam is 2θ .

Constructive interference takes place when the Bragg's law is satisfied, i.e.

$$2d\sin\theta = n\lambda \quad (2.1)$$

Where n is the integer describing the order of reflection, λ is the wavelength of radiation and d is the interplanar spacing.

X-ray diffraction pattern

XRD pattern is a plot of intensity of the radiations over the y axis and angle between incident and scattered radiation on the x axis. The peaks shown in the plot depicts the structure of the sample. The appearance of broad peaks implies the amorphous nature of material and short range order, whereas the sharp peaks is due to crystalline nature of the material. The lattice parameters, interplanar spacing, crystallite size can be calculated by knowing the miller indices corresponding to each peak. The structure of the sample is determined from the lattice parameters.

The crystallite size D can be calculated using Debye-Scherrer equation;

$$D = \frac{0.94 \lambda}{B\cos\theta} \quad (2.2)$$

Where B and θ represents the full width at half maximum and diffraction angle respectively.

XRD is one of the widely used tools for the structural characterization of nanomaterials of any sizes. The observed change in position of diffraction peaks of each sample gives the variations of crystalline structure and parameters.

2.3.2 Fourier Transform Infrared (FTIR) Studies

FTIR spectroscopy is a dispersive method, where measurements are performed over a broad spectrum instead of a narrow band of frequencies. It is a spectroscopic method used to evaluate the chemical bonds in a material by producing an infra-red absorption spectrum. The infrared radiations interact with the molecules, changing the dipole moment, causing vibrations. The vibrations can induce stretching, contraction, or bending in the sample. These results in the absorption of infra-red radiation over a specific wave number range and the absorbed radiations are recorded. The correlation between bond wave number and position with chemical structure is used to identify the functional groups of a material.

The FTIR system consists of the source that produces the infra-red radiations, followed by an interferometer, and a detector. A spectrometer obtains an infrared spectrum by collecting the interferogram of a sample signal with an interferometer, which measures all the infrared frequencies simultaneously transmitted or reflected from the sample surface. The beam arrives at the detector and is measured by the detector. These detected interferograms are decoded by a computer with a mathematical technique in terms of Fourier Transform and a spectrum is produced.

FTIR spectrum consists of transmittance or absorbance and wave number dependence. Generally a wave number range of 400 – 4000 cm^{-1} is analyzed. The fingerprint regions in the spectrum are unique for a material and can be used to distinguish between compounds.

FTIR spectroscopy can be used for quantitative and qualitative analysis of solid, liquid or gaseous samples, without destroying the sample. This method is applicable to organic or inorganic materials.

2.3.3 TGA and DSC

Thermal analysis describes the techniques which are used to analyze the changes occurring in a substance when it is heated or cooled. This method consists of different methods, which are distinguished from one another by the property which are being measured.

Thermo Gravimetric Analysis (TGA)

TGA is a technique by which the mass of a given sample is monitored as a function of time or temperature as the sample specimen is subjected to a controlled temperature. The analysis is performed using TG analyzer. This consists of a sample pan where the sample is taken supported by a precision balance. This setup measures the weight of the sample throughout the experiment. A sample purge gas controls the sample environment.

The TGA curve is a plot describing time or temperature along the abscissa and Weight loss or weight along y axis. The change in sample composition, thermal stability, Oxidative properties, and Volatile content of the material can be determined from TGA curve.

Differential Scanning Calorimetry (DSC)

The difference in heat flow rate between a sample and inert reference as a function of time and temperature is measured. DSC gives the information of heat flow into and out of the sample.

The heat flows into the sample as a result of heat capacity (heating), melting, evaporation, endothermic process etc. and heat flows out of the sample as a result of heat capacity (cooling), crystallization, oxidation and exothermic processes.

Heat from heater is supplied to the sample and the reference through heat sink and heat resistor. Heat sink has enough heat capacity compared to the sample. In case the sample occurs endothermic or exothermic phenomena such as transitions and reactions these are compensated by the heat sink, keeping the temperature difference between sample and reference constant.

The DSC curve displays the time or temperature along x axis and heat flow rate along y axis. The peaks in the curve represent endothermic and exothermic processes.

DSC curve measures enthalpy change, compensates heat release or absorbed during thermal event, oxidation of the sample etc.

Simultaneous DSC – TGA Curve (SDT)

SDT measures both heat flow and weight loss of a material as a function of temperature or time in a controlled atmosphere. Thus productivity is increased and interpretation of the results is simplified.

This provides information about endothermic and exothermic processes which are having no associated weight loss and which involve weight loss respectively.

2.3.4 Vibrating Sample Magnetometer (VSM) Studies

The magnetic properties of the samples are analyzed by using Vibrating Sample Magnetometer which operates on the Faraday's Law of Induction. Faraday's law states that a change in magnetic field will create an electric field, and by measuring the electric field the changing magnetic field can be analyzed. VSM studies provide information about the magnetic behavior of materials.

The sample is subjected to a constant magnetic field, which causes the magnetization inside the material to be polarized. So that, the material can generate the magnetic field by itself even after the removal of external magnetic field. Then the sample is made to vibrate in an alternating magnetic field, this variation in the magnetic field will produce an electric field. This induces current in the coil which can be measured to determine the magnetization of the material.

VSM studies indicate whether the materials can be magnetized, the nature of magnetism exhibited and the extent of magnetization the sample.

The VSM data can be used to study the hysteresis by drawing the M-H curve. The M-H curve represents magnetic field along the x-axis and the magnetization along the y axis. The hysteresis of the curve determines the type of magnetism exhibited by the sample.

2.3.5 Ultra Violet and Visible (UV –VIS) Spectroscopic Studies.

UV – VIS spectroscopy is a spectroscopic method which studies the optical characteristics of a material. UV-VIS spectroscopy performs optical spectroscopy. This method involves the interaction of light with matter in the ultraviolet and visible region.

When a material is irradiated with a beam of radiations, the particles absorb energy to excite into higher energy levels which results in the loss of energy by a certain amount. The variation of intensity is recorded by the detector. The material only absorbs certain wave lengths, the measured absorbance or transmittance gives information about the optical characteristics.

The instrument used is known as spectrophotometer. The source produces radiations of wavelength 200-800 nm, which is passed to a monochromator which separates all the wavelengths of the beam into single beam. The beam separator separates the beam and guide into two directions. There will be two cuvettes made of quartz or glass which consists of sample and reference solution. The change in intensity of radiation of the sample is compared with the reference. Both of these are connected to a detector. This measures the light passing through the sample and compares it to the intensity of light before it passes through the sample. The ratio is called the transmittance ratio. The spectrophotometer can be configured to measure the reflectance also. The transmittance or reflectance with wavelength is recorded and spectrum is obtained.

The spectrum consists of wavelength along x axis and absorbance along y axis. The wavelength at maximum absorption bands will give information about the structure of molecule or ion, and the extent of absorption is proportional with the amount of species absorbing the light.

The amount of light absorbed and concentration of the sample has a linear relation explained by Beer – Lambert's law.

$$A = \log_{10}(I_0/I) = \epsilon.c.L \quad (2.3)$$

Where A is the measured absorbance, I_0 and I are the intensities of incident and transmitted beams, L is the path length through the sample and c is the concentration. ϵ is the molar absorptivity or extinction coefficient. As the concentration increased the absorption also increases.

A tauc plot is drawn with x axis representing $h\nu$ and $(\alpha h\nu)^2$ along y axis, where α is the absorption coefficient. The value of exponent is determined by the type of transition. The value 2 implies direct transitions. The plot follows a linear regime over a specific range, which marks the onset of

absorption. By extrapolating this linear region on the x axis, the band gap can be determined.

2.4 Applications

2.4.1. Photo catalytic Degradation of Methylene Blue

The chemical name of methylene blue is Methylthioninium Chloride, which is used as a dye and as a medication. It is a cationic dye. This is used to stain cells in biological applications, paints, textiles etc. The heavy dose of methylene blue is highly toxic. Based on recent studies it has a high potential to become carcinogenic. The dye is expected to be significantly bio accumulate.

The degradation of dye is done using photo catalysis. Photo catalysis is a chemical process that takes place in the presence of light with the activity of a photo catalyst. This method is employed for the complete degradation of pollutants or chemicals which are harmful and cause pollution. The ability of the sample to degrade the dye in presence of sunlight is studied with time. The degradation efficiency is calculated and compared with the different samples.

The dye in the presence of synthesized samples is subjected to sunlight exposure for a period of 4 hours. 3 ml of the solution is taken after every 30 minutes and the variation of absorbance spectrum of the methylene blue dye is studied using UV-VIS-NIR Spectrophotometer. The degradation efficiency is calculated by knowing the maximum value of absorbance of dye kept at dark and that of the sample.

$$\text{Degradation efficiency (\%)} = \frac{C_0 - C_s}{C_0} \times 100 \quad (2.4)$$

Where C₀ and C_s represents the maximum value of absorbance of dye in dark and sample in sunlight.

2.4.2 Antibacterial Studies

Antibacterial activity is the potential for inhibiting the growth of bacteria. Antibacterial activity is the most important phenomenon which gives adequate protection against microorganisms. The antibacterial property of the sample is studied by well diffusion method for both gram positive and gram negative bacteria.

The agar medium is prepared on a sterile glass petri plates and solidified then swabbed using sterile cotton with reference bacterial strain. These agar plates are punched using sterile tips of sample dispersed in water via sonification and poured with a micropipette. These plates are allowed to stand for some time and were incubated. The zone of inhibition is measured. The measured zone of inhibition determines the antibacterial properties of the sample .

CHAPTER 3

RESULTS AND DISCUSSIONS

3.1 Characterization and Analysis of Zinc Ferrite

Nanoparticles

Sample	Calcination Temperature (°C)
ZF Unc	Uncalcined
ZF100	100
ZF200	200
ZF300	300
ZF400	400
ZF500	500
ZF600	600
ZF700	700
ZF800	800

3.1.1 X-Ray Diffraction (XRD) Analysis

The samples of Zinc Ferrite Nanoparticles are characterized using X-ray diffraction (XRD) studies. XRD pattern are obtained using AXS D8 Advance X-ray diffractometer with Cu-K α radiation ($\lambda=1.5406 \text{ \AA}$) scanning in the range $2\theta = 10-80^\circ$. The X-ray generator is operated at 40kV and 35mA.

The structure and size of Zinc Ferrite nanoparticles are determined from the XRD pattern. The crystal structure of Nanoparticles is compared with the JCPDS file. For cubic structure, the lattice constants are equal, $a=b=c$.

The XRD pattern for the different samples are obtained as shown in figure 3.1 The emergence of the peaks can be seen for the samples calcined at 400^oand above. Thus the crystallization of the Zinc Ferrite nanoparticles takes place around 400^oC and the most intense peak (311) is obtained for the sample calcined at 800^oC. Therefore the crystalline growth is favored with increase in temperature.

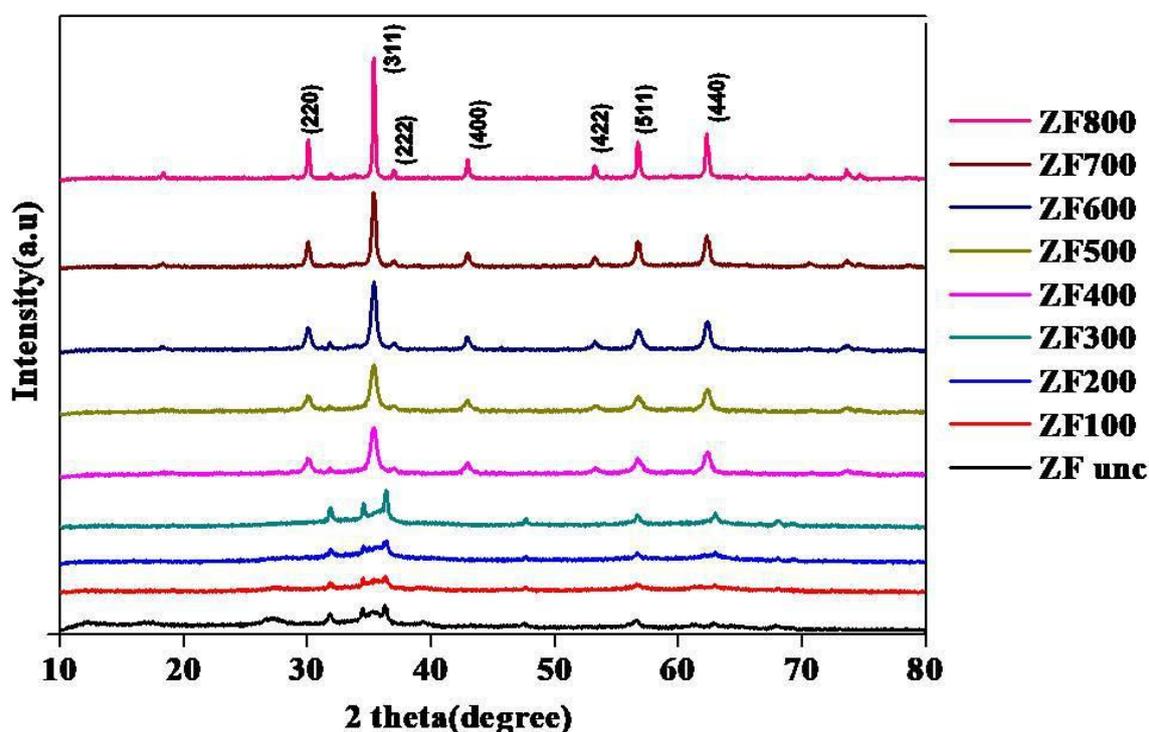


Figure 3.1 XRD patterns of Zinc Ferrite nanoparticles at different calcination temperature

The characteristic peaks are obtained at 2θ values 30.047°, 35.384°, 42.968°, 53.25°, 56.6° and 62.285° corresponding to the reflection planes (220),(311),(400), (422),(511) and(440) respectively. The most intense peak is obtained for the 311 plane (35.384^o). These peaks matches with the JCPDS(Card No.82-1049), confirming the formation of spinel cubic structure with the Fd3m space group, with lattice constant $a = 8.441 \text{ \AA}$. The sharp and well defined peaks indicate the formation of Zinc Ferrite

nanoparticles and crystallinity for the samples ZF400-ZF800. The increase in intensity can be attributed to the increased crystallite size.

The crystallite size was calculated using Debye -Scherer equation and are tabulated. The crystallite size varies from 15.16 nm to 38.78 nm. The crystallite size is found to increase with temperature and maximum value is obtained for ZF800.

The lattice parameter values are consistent with the single-crystalline cubic spinel form of Zinc Ferrite Nanoparticles. The absence of any other peaks implies the phase purity of the sample.

Table 3.1 the crystallite size and lattice parameters corresponding to the reflecting plane (311)

Sample	Diffracting Angle (2θ)	β (rad)	Crystallite size D(nm)	Interplanar spacing D (A$^\circ$)	Lattice constant a (A$^\circ$)
ZF400	35.333	0.00878	16.58	2.53823	8.418
ZF500	35.327	0.0096	15.16	2.53865	8.420
ZF600	35.362	0.00808	18.01	2.53623	8.412
ZF700	35.338	0.0062	23.49	2.5379	8.417
ZF800	35.352	0.00375	38.78	2.53694	8.414

3.1.2 Thermo gravimetric Analysis And Differential Scanning Calorimetry (TGA –DSC) Analysis

The thermal properties of the uncalcined samples are analyzed using TGA-

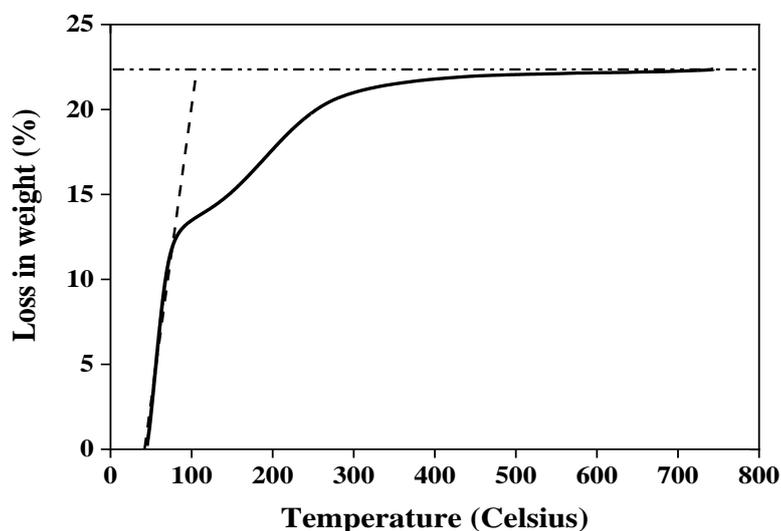
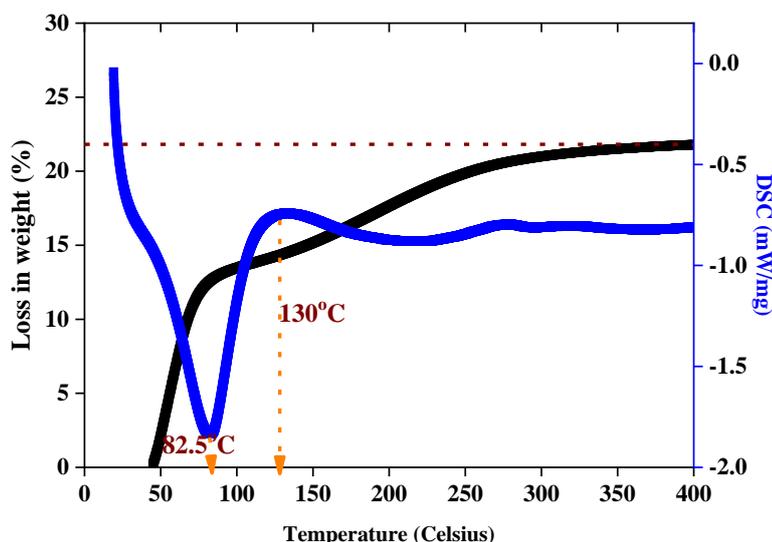


Figure 3.2 Weight loss curve of uncalcined Sample.

DSC method. TGA curve indicates the beginning of weight loss at a temperature around 42°C. The weight loss is calculated as a function of temperature over a range of temperature 0°C to 750°C. The maximum weight loss is about 22%.



The weight loss can be seen from temperature of 77°C to 130°C due to the desorption of water, in the sample, the corresponding endothermic curve is obtained at 82.5°C. The weight loss from 130°C to 300°C is arising due to the decomposition of Iron and Zinc hydroxides. From 400°C to 700°C no significant weight can be seen, stating the thermal stability of the material.

The weight loss from 130°C to 300°C is arising due to the decomposition of Iron and Zinc hydroxides. From 400°C to 700°C no significant weight can be seen, stating the thermal stability of the material.

3.1.3 Fourier Transform Infrared Spectroscopy (FTIR)

Analysis

The FTIR studies are conducted for the samples using Fourier transform infrared Spectrometer (thermo Nicolet, Avatar 370) in the range 400 to 4000 cm^{-1} . The FTIR spectrum reveals the characteristic peaks at 400.5 cm^{-1} and 533 cm^{-1} for the samples from ZF400 to ZF800. These two peaks corresponds to the metal oxide vibrations along octahedral (Fe-O) and (ZnO) tetrahedral sites respectively. There is a weak band around 1400 cm^{-1} for samples ZF100 – ZF300 that corresponds to the bending of the –OH bonds. This indicates the presence of feeble water content in these samples.

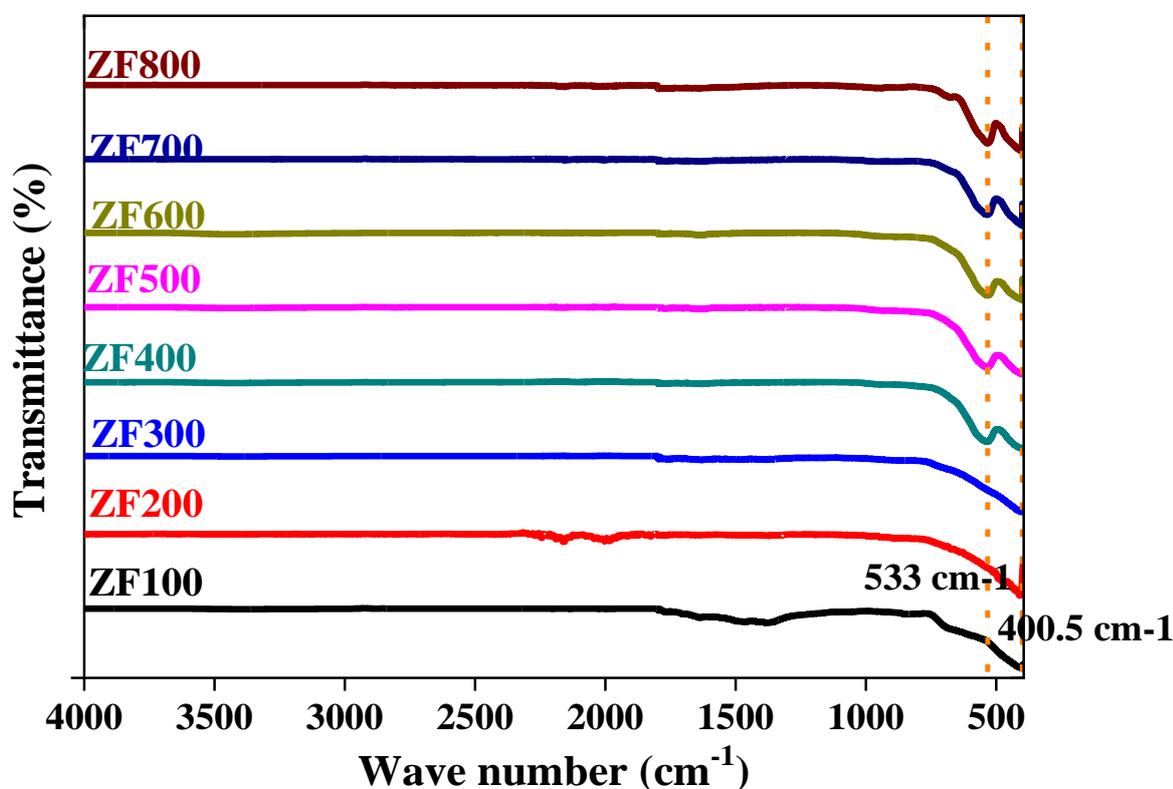


Figure 3.4 FTIR spectrum of different Samples

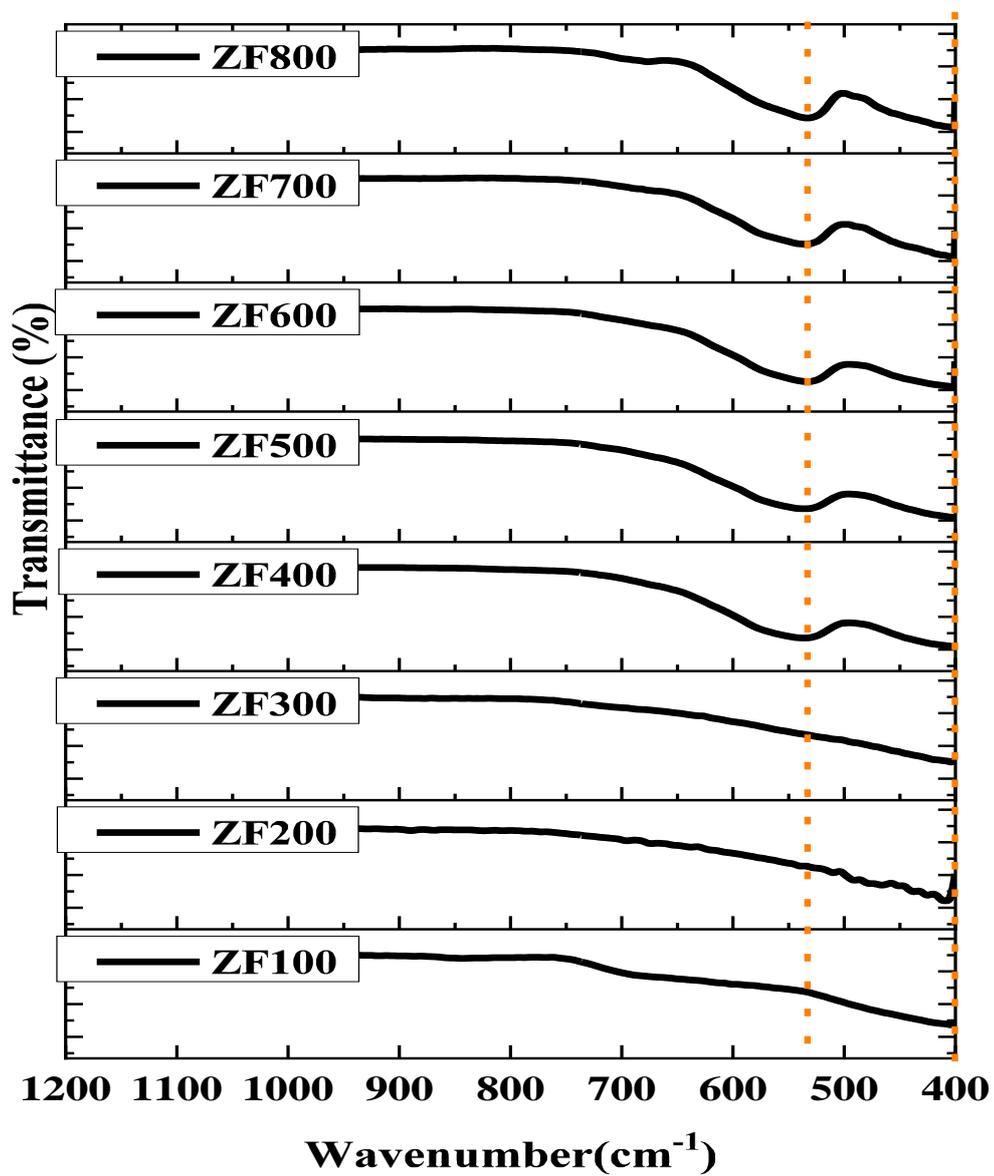


Figure 3.5 FTIR Spectrum showing shift in peaks

3.1.4 Vibrating sample Magnetometer (VSM) Analysis

The magnetic properties of the samples are determined by Vibrating Sample Magnetometer (Lakeshore VSM 7410) at room temperature with an applied. The MH curve, reveals that the samples, ZF400 – ZF800 are having super paramagnetic nature. The characteristic S shaped curve for super paramagnetism is more pronounced for ZF800. The least squareness ratio indicates the formation of single domain particles. Thus the magnetic materials can be retrieved by applying magnetic field. field of -15k – 15kOe.

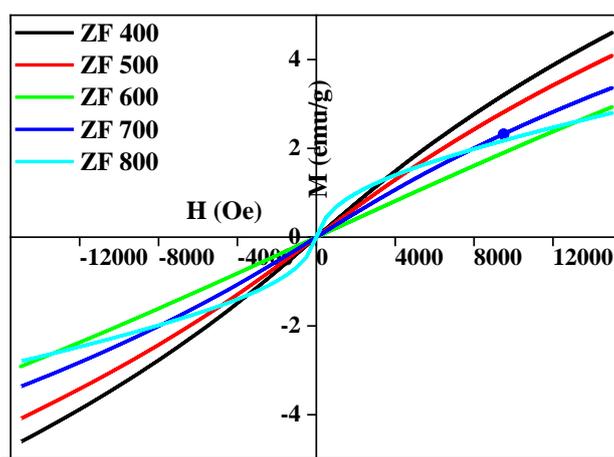


Figure 3.6 M-H curve for ZF Nanoparticles

Sample	Ms (emu/g)	Mr (emu/g)	Squareness ratio(Mr/Ms)
ZF400	4.612	5.18E-05	1.12E-05
ZF500	4.091	1.76E-04	4.31E-05
ZF600	2.923	7.23E-05	2.47E-05
ZF700	3.3666	1.93E-04	5.72E-05
ZF800	2.798	5.33E-05	1.90E-05

3.1.5 UV-VIS-NIR Analysis

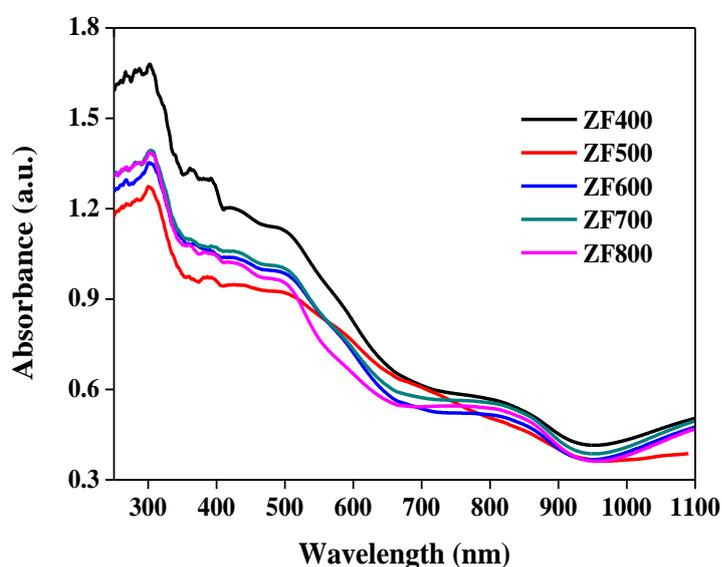


Figure 3.7 Absorption spectrum of Zinc Ferrite nanoparticles

The UV-VIS-NIR absorption studies are conducted using Cary 5000 UV-VIS-IR absorption spectrophotometer in the spectral range 220-1980 nm. The maximum absorption is shown by sample ZF400. The range of maximum absorption is about 300nm for all samples. There is a red shift in absorption edge with an increase in calcination temperature which is expected for an increase in crystallite size. Zinc ferrite nanoparticles are found to exhibit direct band gap. Band gap ranges from 1.67 to 1.78 eV. The least band gap is obtained for sample calcined at 500°C

Table 3.8 Band gap of ZF Nanoparticles.

Sample	Band Gap
ZF400	1.78
ZF500	1.67
ZF600	1.72
ZF700	1.75
ZF800	1.74

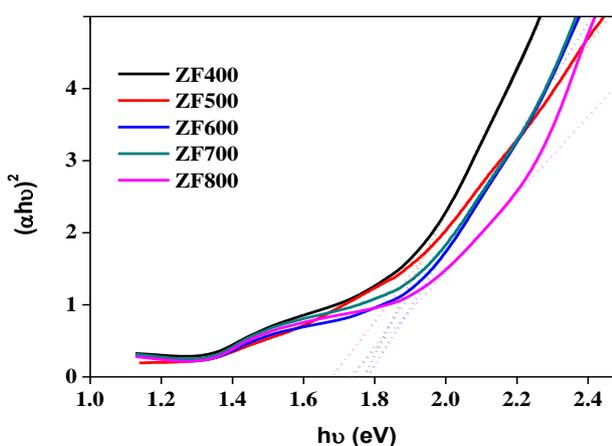


Figure 3.8 Tauc Plot of ZF Nanoparticles

3.2 Applications Of Zinc Ferrite Nanoparticles

3.2.1 Photo catalytic Degradation Of Methylene Blue Dye

The different samples were tested for degradation of methylene blue dye (10ppm) in darkness for 15 minutes and 4 hours in sunlight. Dosage of samples used as photo catalyst was 1g/L.

The absorption spectrum of each of the samples is taken for every 30 minute interval for about 3 hours.

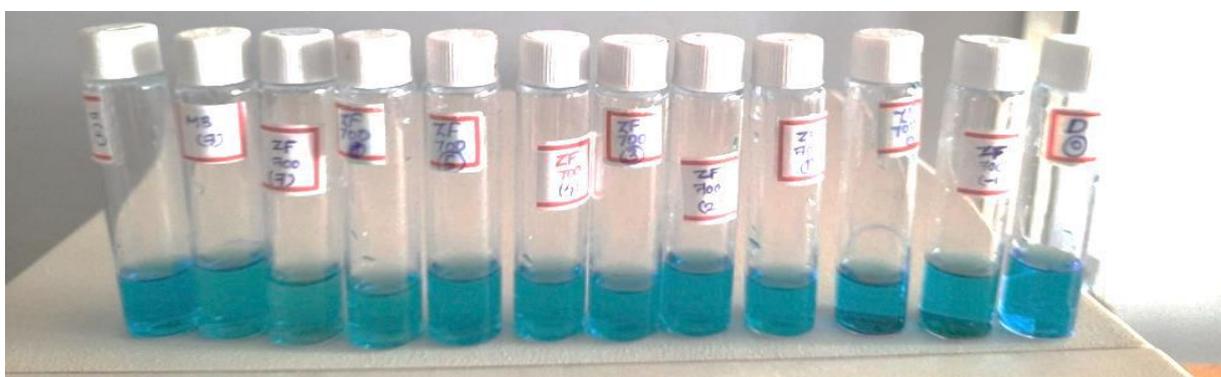


Figure 3.9 The sample ZF700 at different time of sunlight exposure in the order D(7),MB(7),ZF700(7),ZF700(6), ZF700(5), ZF700(4), ZF700(3).....ZF700(0), D(0)

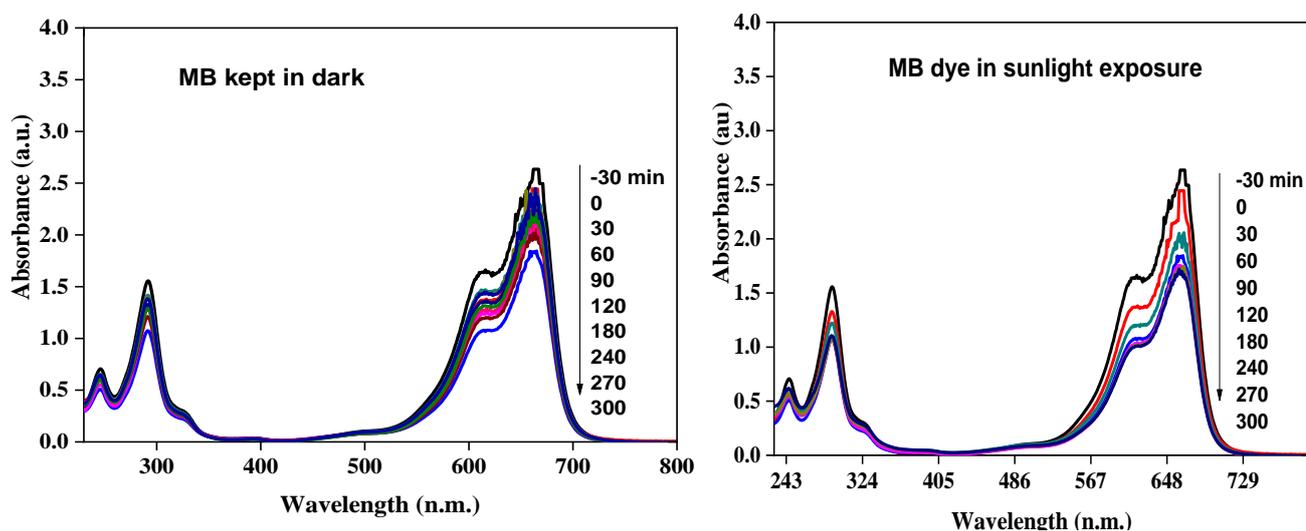
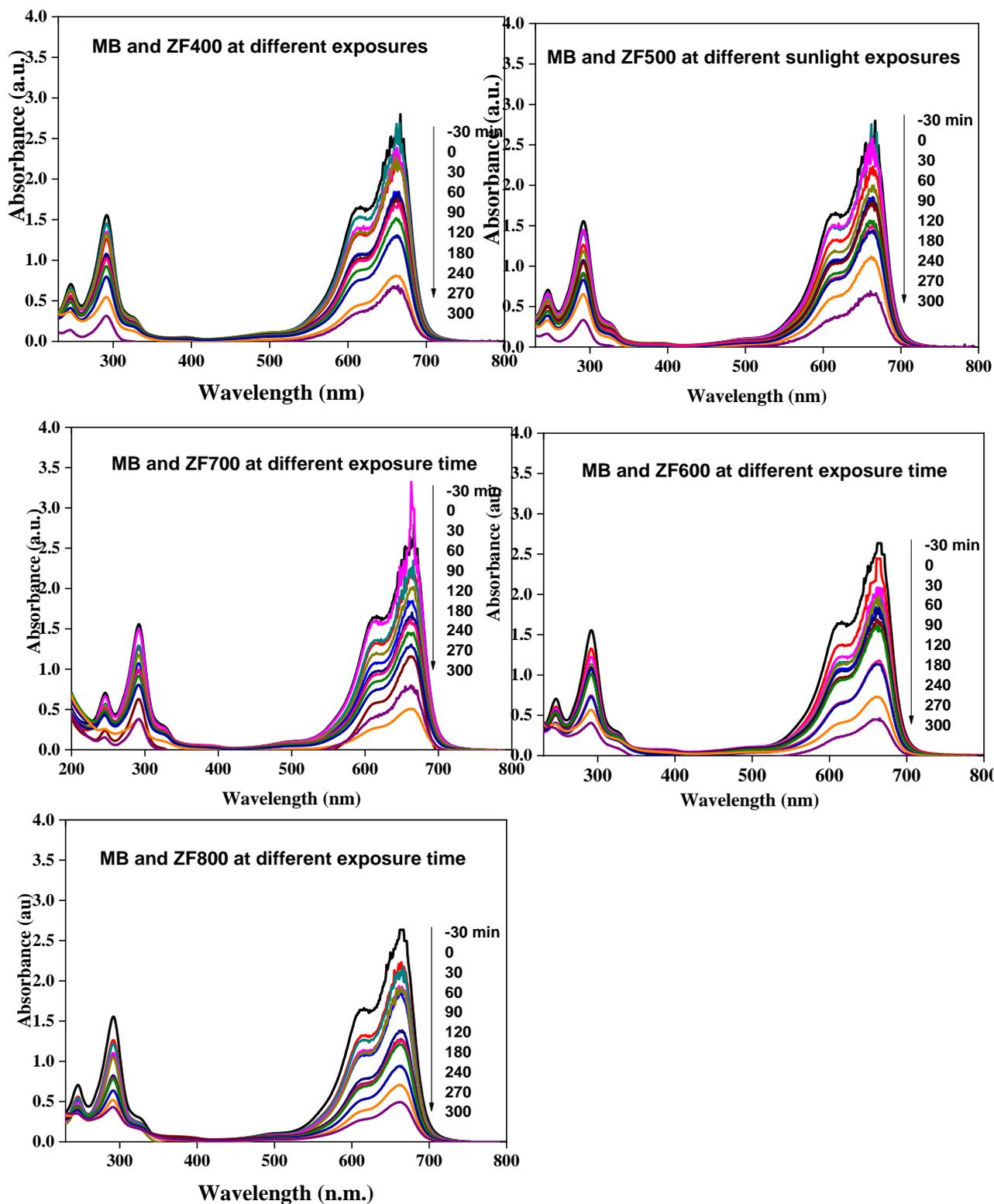


Figure 3.10 Absorption of Dye in dark and in sunlight at different time intervals



The maximum absorption peak wavelength of Methylene blue is about 664 nm. The degradation of dye by different samples for a time period of 4 hours was studied. The maximum degradation was observed with ZF800 about 2 hours exposure to sunlight marked as sample set 4 – ZF800(4).

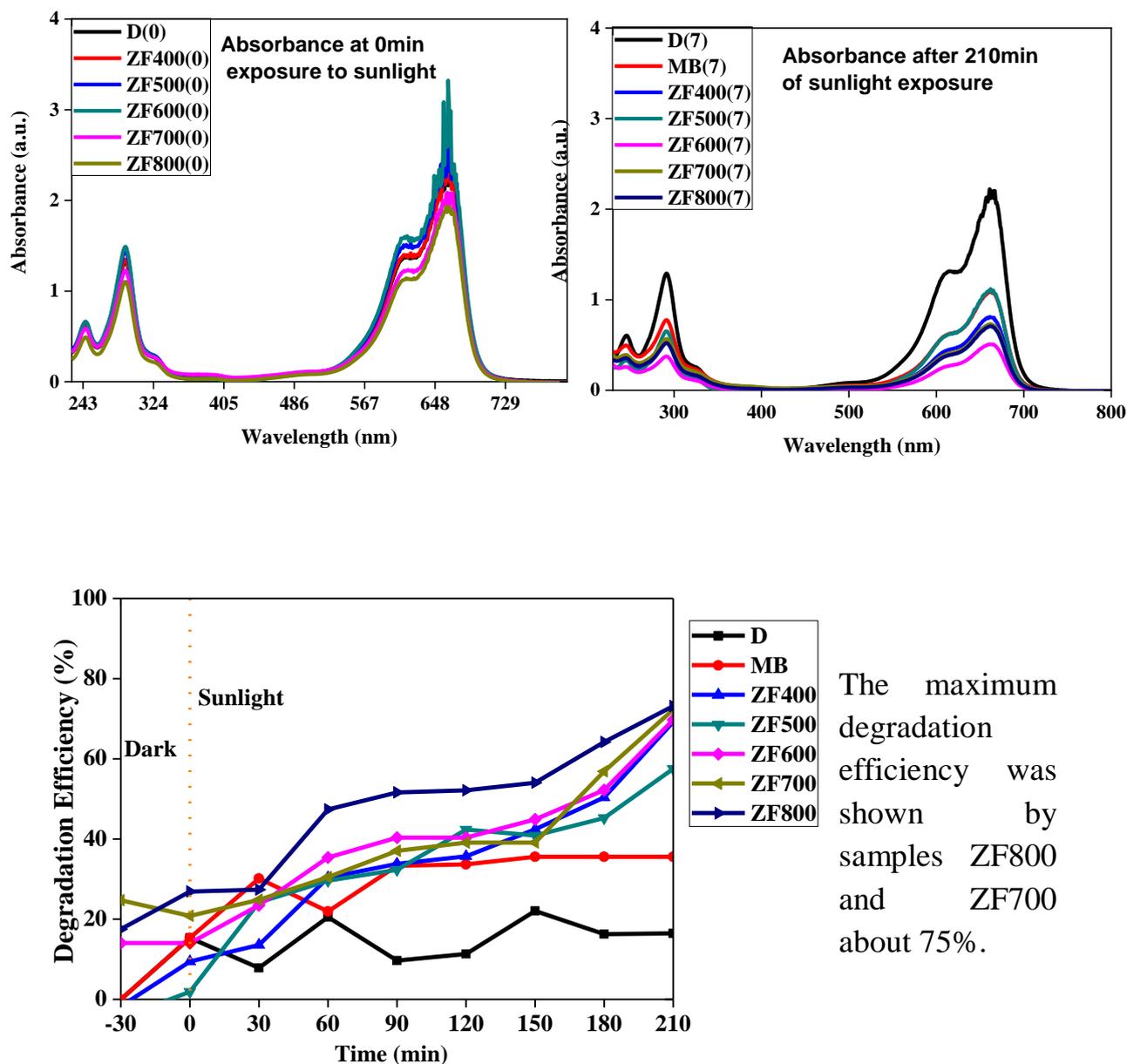
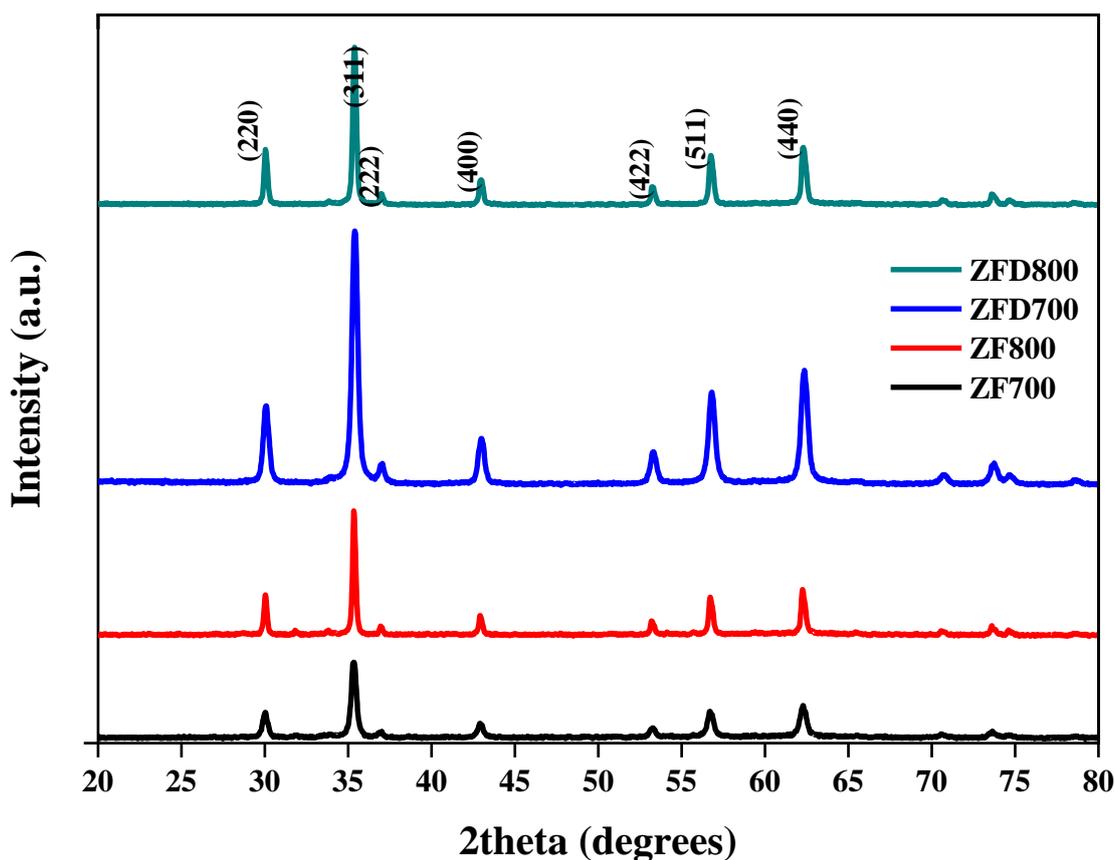


Figure 3.11 Degradation Efficiency of Zinc Ferrite Nanoparticles

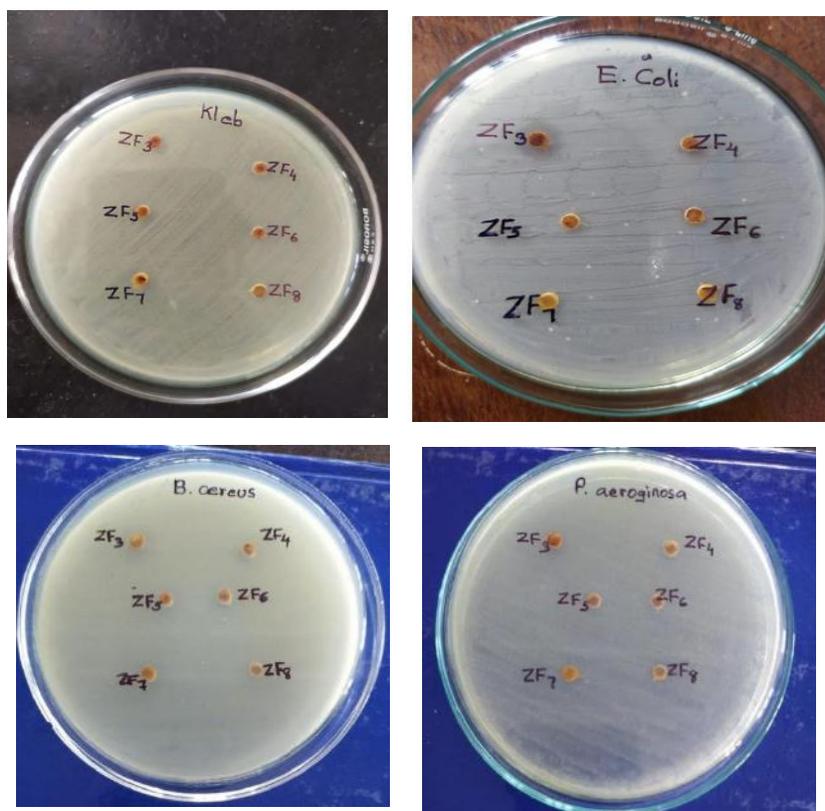
During a period of 210 minutes, the absorbance decreases considerably for all the samples. The absorbance is minimum for the sample ZF800. This ability to degrade the dye makes it useful as a photo catalyst.

3.2.2 Retrieval of Photo catalyst

The samples of Zinc Ferrite nanoparticles can be retrieved after photo degradation by applying external magnetic field. This is due to the super paramagnetic nature of Zinc Ferrite nanoparticles. The samples retrieved are again characterized by XRD, no significant change in crystallite size and properties are observed.



3.2.3 Antibacterial Studies



Antibacterial studies are conducted for zinc ferrite nanoparticles using gram positive (Staph. aureus, Enterococcus, Bacillus) and gram negative bacteria (E. coli, Klebsiella p, Pseudomonas) using Well diffusion methods. Only the sample ZF800 shows antibacterial activity against Enterococcus with a zone of inhibition 5 mm. No significant antibacterial activity was shown by the other samples. So, the Zinc Ferrite nanoparticles have no considerable antibacterial properties.

3.3 Characterization and Analysis Of Zinc Ferrite/AgCl-Ag Nanocomposites

Samples	Calcination temperature (°C)	Addition of silver precursors	Addition time
CS 700	700	Drop by drop	After precipitation
CS 800	800	Drop by drop	After precipitation
CS 1	700	Rapid	After precipitation
CS 2	700	Rapid	After centrifugation

3.3.1 X-Ray Diffraction (XRD) Analysis

The structure and size of Zinc Ferrite Composites are determined from the XRD spectrum. The XRD pattern for the different samples is obtained as shown in figure 3.11. The characteristic peak (311) corresponding to Zinc Ferrite nanoparticles is the most intense peak in all samples. This peak has a maximum intensity for sample CS800, suggesting more crystalline nature.

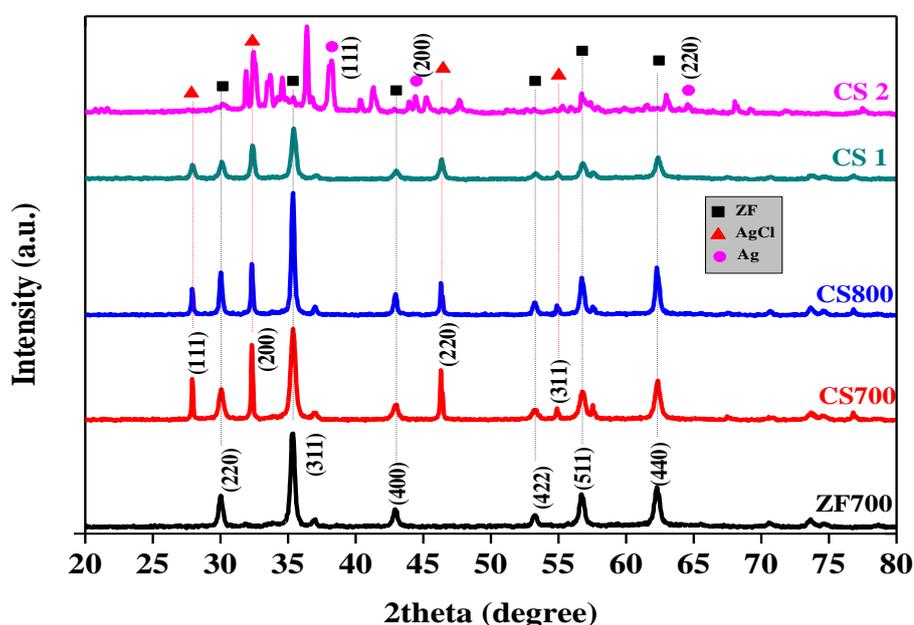


Figure 3.11 XRD Pattern of ZF/AgCl-Ag

The samples CS1, CS700, and CS800 have peaks corresponding to the reflecting planes (111), (200), and (220). These data indicates that the sample CS1, CS700 and CS800 are Zinc Ferrite and Silver Chloride nanocomposites. This is verified with the JCPDS File No:01-071-5209. This represents Fm-3m (225) lattice space group with lattice constant 5.54630 Å.

The Sample CS2 has the peaks corresponding to Silver. This is in agreement with JCPDS File No: 00-004-0783. Thus the sample CS2 is Zinc ferrite and Silver nanocomposite. The space group is Fm-3m (225) and 4.08620 Å is the lattice parameter.

Table 3.4 Crystallite size and Lattice Parameter for nanocomposites

Sample	ZF (311)		AgCl (200)		Ag(111)	
	D(nm)	a (Å)	D(nm)	a (Å)	D(nm)	a (Å)
CS700	20.39	8.408	-	-	-	-
CS800	31.95	8.4	51.06	5.534	-	-
CS1	21.95	8.39	50.57	5.53	-	-
CS2	42.23	8.1819	30.87	5.528	-	-
ZF700	23.49	8.417	24.55	5.5107	41.96	4.0813

The sample CS2 has got maximum crystallite size and this corresponds to Zinc Ferrite and Silver nanocomposite.

3.3.2 Fourier Transform Infrared Spectroscopy (FTIR)

Analysis

The FTIR spectrum reveals the characteristic peaks at 400.5 cm^{-1} and 533 cm^{-1} for the all the composite samples. These two peaks corresponds to the metal oxide vibrations along octahedral (Fe-O) and (ZnO) tetrahedral sites respectively. In addition to these peaks there are peaks at vibrational frequencies 636 cm^{-1} , 865 cm^{-1} , 1451 cm^{-1} , 2974 cm^{-1} and 3420 cm^{-1} .

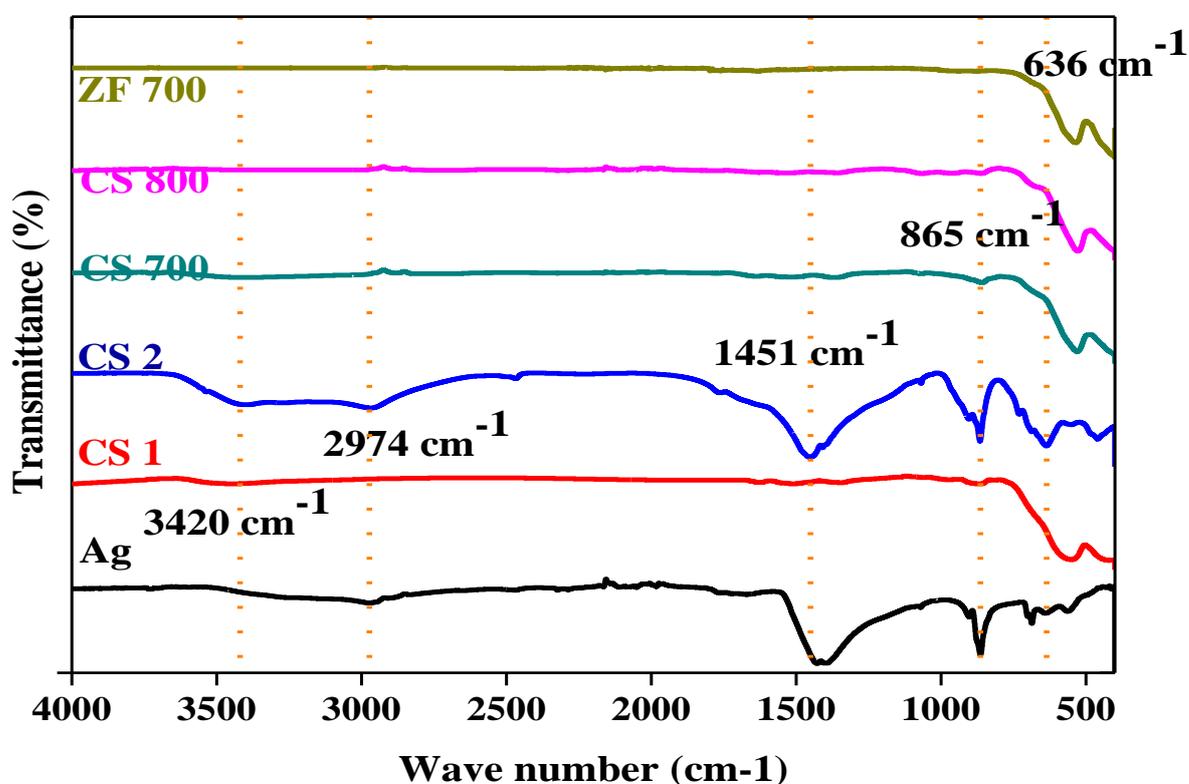


Figure 3.12 FTIR spectrum of ZF/AgCl-Ag Nanocomposite

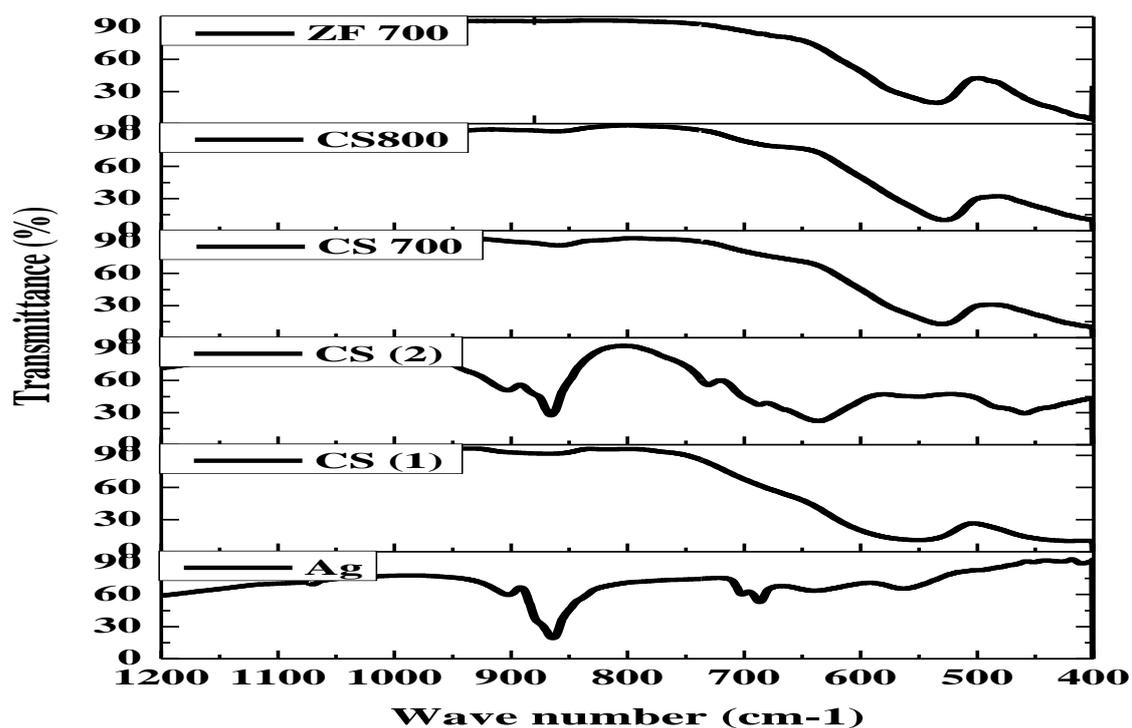


Figure 3.13 FTIR spectrum showing wavelength Shift

Table 3.5 Vibrating frequency of ZF/AgCl-Ag nanocomposite.

Sl.No	Wave number (cm ⁻¹)	Chemical Bond
1	400.5	Fe-O vibration at octahedral site.
2	533	Zn-O Vibration at tetrahedral site
3	1451	-OH- stretching
4	636	-C=C- Stretching
5	865	-CH ₃ - deformations
6	2974	-CH- Stretching
7	3420	-OH- Stretching

3.3.3 Vibrating Sample Magnetometer (VSM) Studies

The VSM studies reveal that all the samples are having super paramagnetic nature. Compared to the Zinc Ferrite nanoparticles the magnetization is less. But compared to the paramagnetic silver these composite samples have 10⁻⁴ times squareness ratio. The absence of hysteresis suggests that the material can be easily magnetized. Thus the magnetic materials can be retrieved by applying magnetic field.

Table 3.6 Magnetic parameters of ZF/AgCl Composites

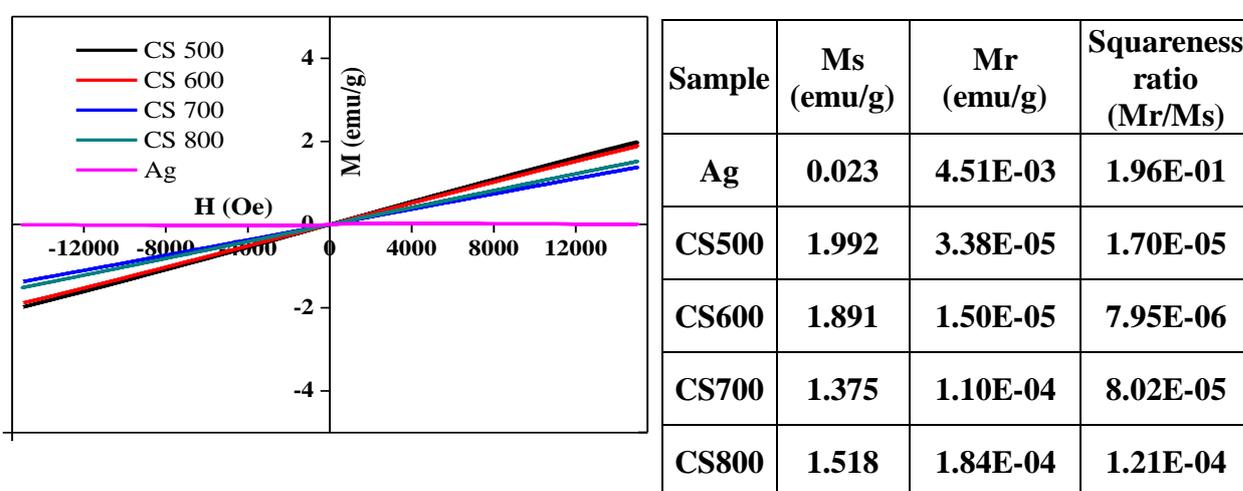


Figure 3.14 M-H curve of ZF/AgCl Composites

3.4 Applications Of Zinc Ferrite Nanocomposites (ZnFe₂O₄/AgCl-Ag)

3.4.1 Photocatalytic Degradation Of Methylene Blue Dye

The different samples were tested for degradation of methylene blue dye (10ppm) in darkness for 15 minutes and 3 hours in sunlight. Dosage of dye was 1g/L.

The absorption spectrum of each of the samples are taken for every 30 minute interval

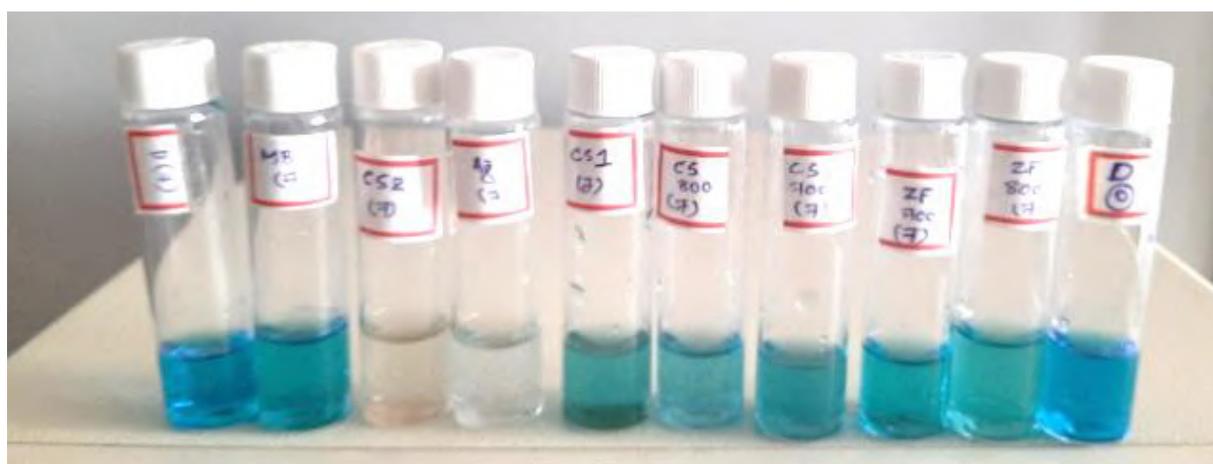


Image 3.16 Photo degradation of methylene blue dye by ZF/AgCl-Ag nano composite at the seventh exposure(210 min) is compared with pure silver nanoparticles and zinc ferrite nanoparticles given in the ordered(7), MB(7), CS2(7), Ag(7), CS1(7), CS800(7), CS700(7), ZF700(7), ZF(800), D(0)

Absorption Spectrum of Samples at Different Intervals of Time

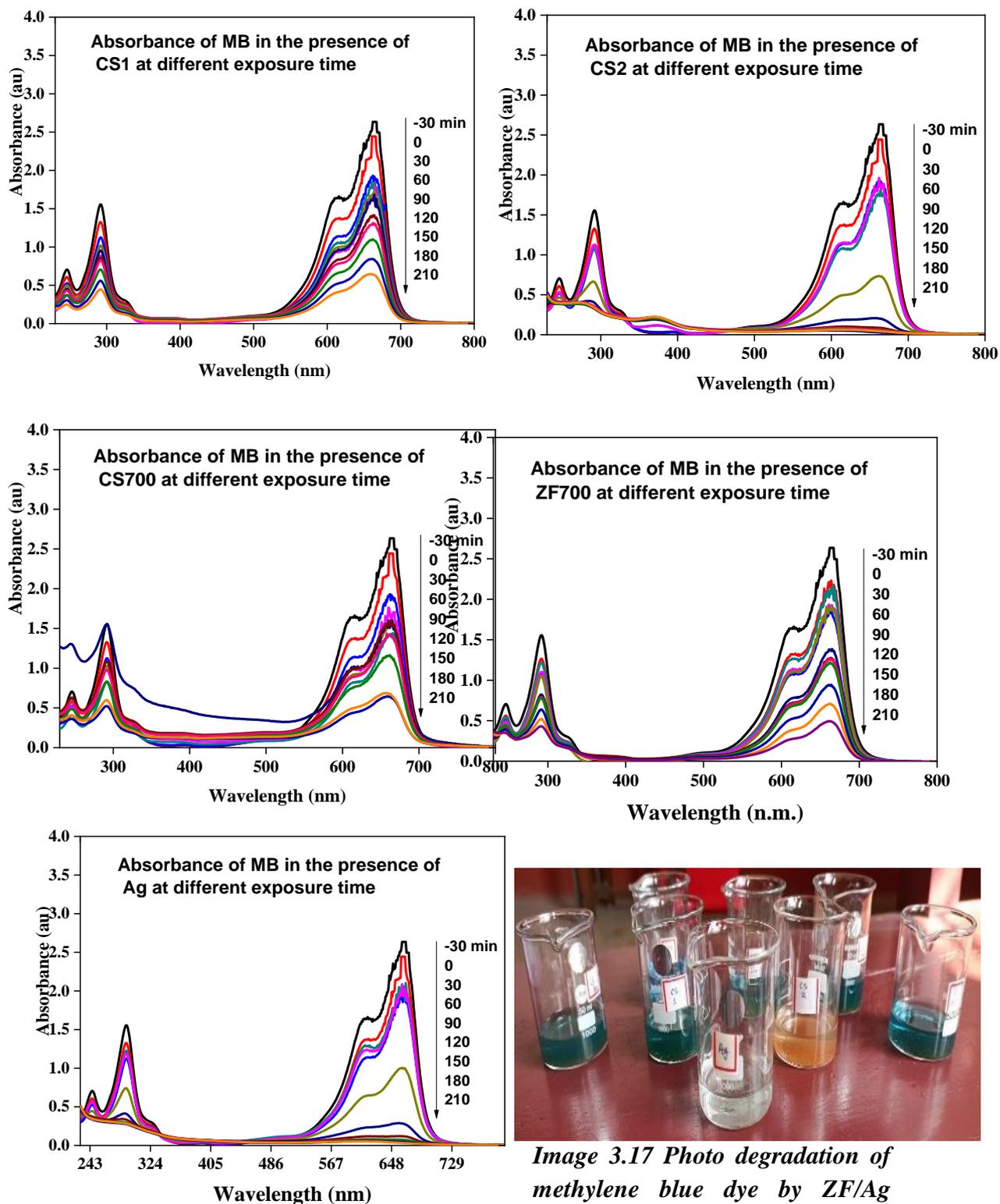


Image 3.17 Photo degradation of methylene blue dye by ZF/Ag nano composite (Sample CS2)

Absorption Spectrum of Different Samples in Same Intervals of Time

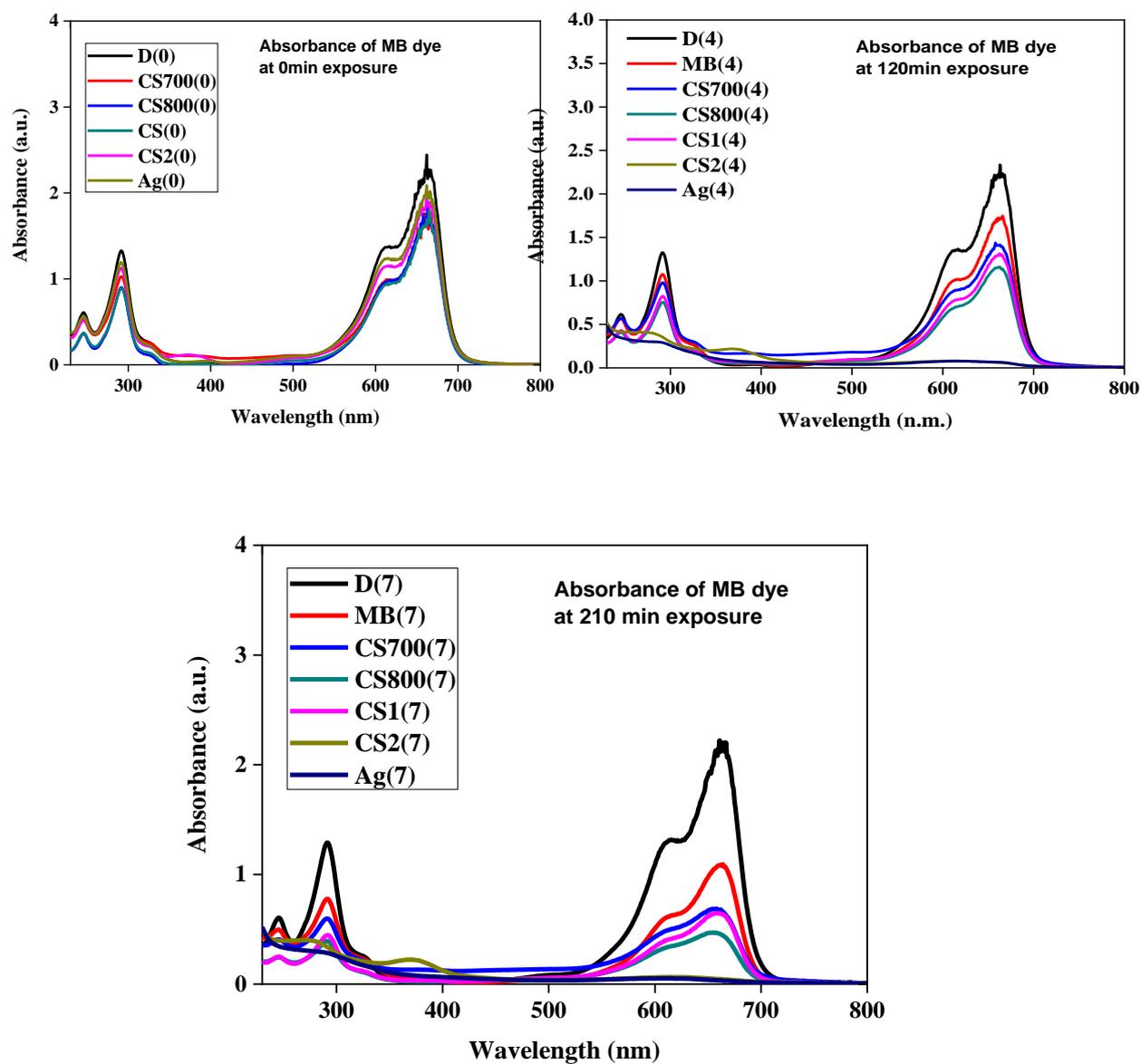


Figure 3.17 Absorbance of MB dye in the presence of different photo catalyst at 0 min, 120 min and 210 min sunlight exposure

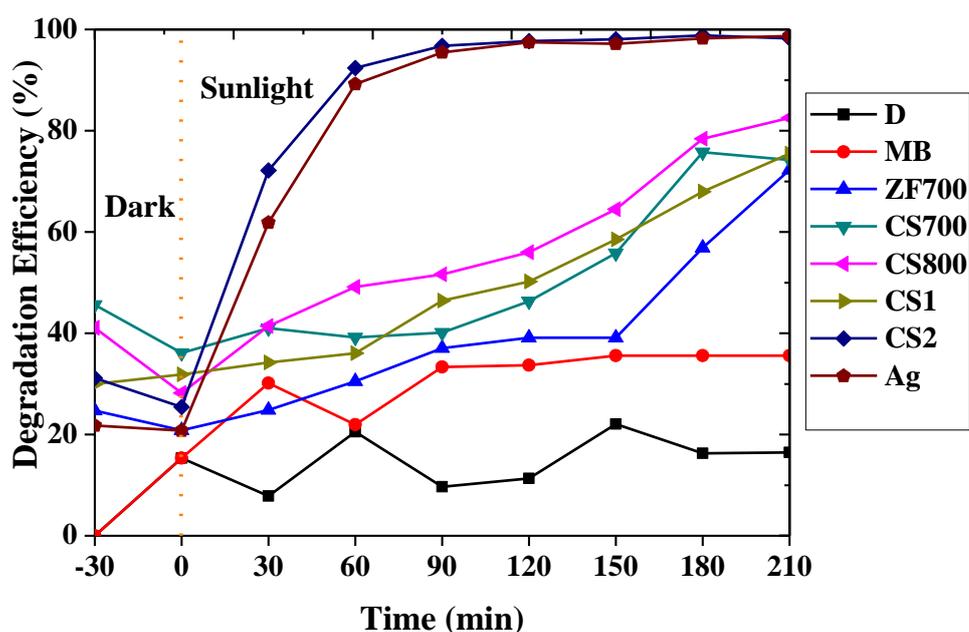


Figure 3.18 Degradation Efficiency of Zinc Ferrite Nanoparticles

The photo catalytic effects of different samples were compared. Among these CS2 and Ag have maximum degradation efficiency (98%). Therefore they are the best samples showing photo degradation of methylene blue dye in the presence of sunlight.

Table 3.7 Degradation Efficiency of Composite Nanoparticles

Sample	Degradation Efficiency (%)
CS700	75
CS800	82
CS1	76
CS2	98
Ag	97

3.4.2 Antibacterial Studies

Activity against bacteria was tested for different samples using well diffusion method. Both gram positive and gram negative bacteria are selected.

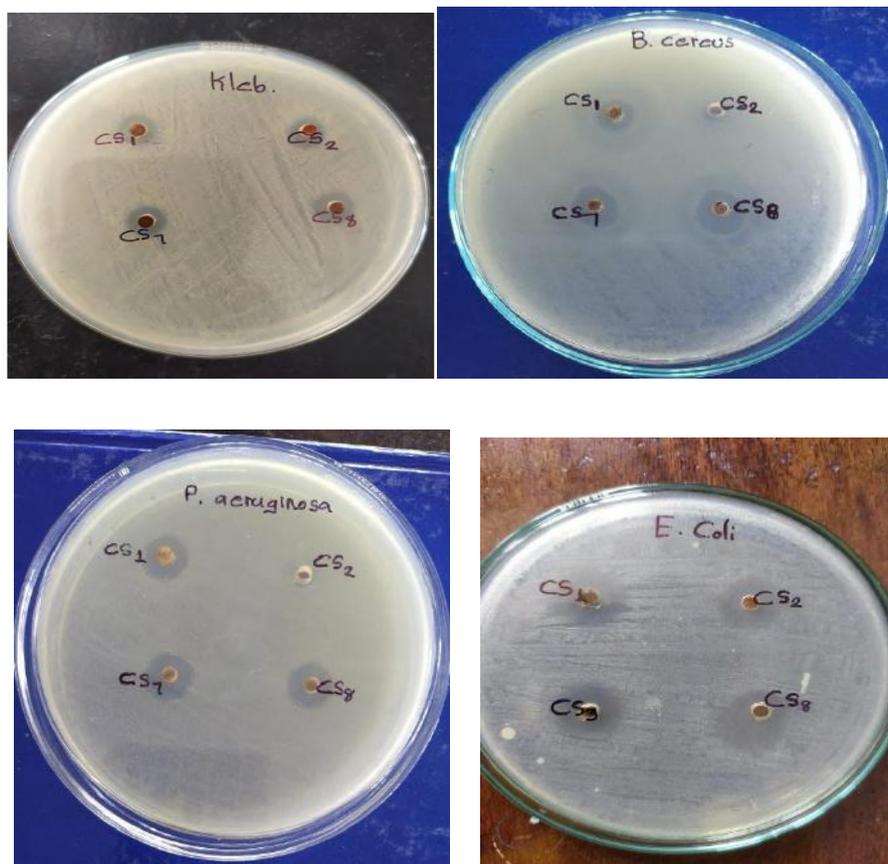


Image 3.18 Well Diffusion Method to study Antibacterial Properties

Table 3.9 The zone of inhibition against gram positive and gram negative bacteria

Samples	E.Coli	Klebscella p.	Pseudomonas	Enterococcus	Bacillus	Staph.Auereus
CS700	13	7	12	10	12	9
CS800	14	7	11	10	13	8
CS1	13	10	12	10	11	9
CS2	11	7	-	6	6	4

The composite sample shows antibacterial property. The maximum activity is shown by CS800.

CHAPTER 4

CONCLUSION

Zinc ferrite nanoparticles and its nanocomposite forms with silver and silver chloride were synthesized at different calcination temperatures using co-precipitation method. Structural characterizations confirmed the formation of cubic lattice of spinel ferrite with crystallites of zinc ferrites in the range of 15-38nm. Optical and magnetic characterization revealed the visible light active and super paramagnetic nature of the synthesized samples. Samples calcined at higher temperatures exhibited greater activity as a result of improved crystallinity. Photo degradation of methylene blue dye using the synthesized samples was analyzed. Slow degradation kinetics was observed for zinc ferrite nanoparticles with degradation efficiency of about 80% but only within 240 minutes of solar irradiation. Antibacterial analysis by well diffusion method showed no zone of inhibition for zinc ferrite nanoparticles making it inefficient for this application. Nanocomposite enhanced the properties making it suitable for applications. Rapid degradation kinetics was observed for nanocomposites with a degradation efficiency of 98% within 120 minutes of solar irradiation. Because of magnetic properties, nanocomposites can be simply recovered from reaction and reused almost without loss of catalytic activity. For nanocomposites, Zone of inhibition formation was seen both for gram negative and gram positive bacteria making it suitable for antibacterial applications. The zinc ferrite nanocomposites raise potential scope as a broad spectrum antibacterial agent against the evolution of drug-resistant bacteria, which facilitates targeted delivery and retrieval after disinfection by making use of magnetic field.

FUTURE SCOPE

The photo catalytic activity can be studied for different sample dosages, dye concentrations and different exposure intensity. We can also use arc lamps instead of direct solar irradiance and study the degradation.

The antibacterial investigations can be done using different methods like micro titer assay method, or Kirby Bauer disc diffusion method. Optical concentration which gives the density of bacterial strains can hence be found.

Applications of zinc ferrite nanocomposites in biomedical fields like magnetic hyperthermia, nanomedicine and as MRI contrast agents can be studied.

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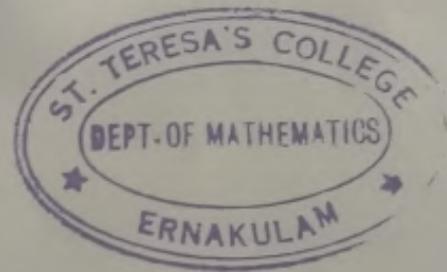
CERTIFICATE

This is to certify that the dissertation entitled, **MATHEMATICS BEHIND RUBIK'S CUBE** is a bonafide record of the work done by Ms. **VISHNUPRIYA M S** under my guidance as partial fulfillment of the award of the degree of **Bachelor of Science in Mathematics** at St. Teresa's College (Autonomous), Ernakulam affiliated to Mahatma Gandhi University, Kottayam. No part of this work has been submitted for any other degree elsewhere.

Date: 07-03-2022

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DECLARATION

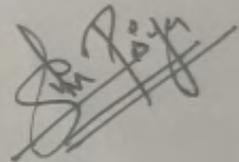
I hereby declare that the work presented in this project is based on the original work done by me under the guidance of Dr. Susan Mathew Panakkal, Assistant Professor, Department of Mathematics, St. Teresa's College(Autonomous), Ernakulam and has not been included in any other project submitted previously for the award of any degree.

Ernakulam.

Date: 07-03-2022

VISHNUPRIYA M S

AB19BMAT012



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Firstly, I thank God Almighty for his grace to execute this project works successfully. I express my deep sense of gratitude to our guide Dr. Susan Mathew Panakkal, Department Of Mathematics And Statistics, St. Teresa's College, Ernakulam, for her valuable guidance and suggestions

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Ernakulam.

Date: 07-03-2022

VISHNUPRIYA M S

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Chapter 1

INTRODUCTION

Rubik's cube is a 3D combination puzzle. It is widely considered to be the world best selling toy. First it was invented by Hungarian sculptor. The concept of Rubik's Cube is derived from China Luo book. That can be converted into jiucong map. In 1974 Erno Rubik was developed into modern Rubik's Cube. And it is also called magic Cube. It won a special award for German game and awards for best toy in US, France and UK. There are several features in Rubik's Cube structure such as permutations and combinations, rotation, cycle and its mechanism can be used in various science field such as mechanics field in physics, math's. The Rubik's cube structure has a few elements for example turn, change and blends and cycle and balance, which were treated as actual models or instruments to study specific scientific issues or were considered by utilizing scientific hypothesis or strategies in certain spaces. All things consider the standards of Rubik's cube are contained in various scientific frameworks that include stages and combo- countries, balances and cyclicity.

1.1 LITERATURE REVIEW

Lach and Sakshaug (2004) performed a take a look at the usage of those methods, which supplied quite a few thrilling results. They noted "Games are clearly motivating and amusing, video games facilitate individualization of evaluation and instruction, and video games make the summary greater concrete." They additionally concluded that gambling arithmetic video games ought to enhance college students' spatial abilities and algebraic reasoning. This take a look at had each a manage institution and an experimental institution. Both businesses consisted of

higher middle-magnificence college students who scored above the thirtieth percentile in math at the Stanford Achievement Test. At the start and of of entirety of the take a look at each the manage and experimental businesses have been given a pre-take a look at and a post-take a look at. Both businesses scored further withinside the pre-take a look at. The researchers located a fine statistically big distinction of their experimental institution's pre-take a look at and post-take a look at ratings. There became no statistically big distinction among the pre-take a look at and post-take a look at ratings of the manage institution. Instructional sport gambling in lecture rooms is strongly supported via way of means of Blum and Yocom (1996). They declare that video games can boom motivation and permit college students a amusing manner to Practice talents they have got already learned. Klein and Freitag (1991) and Olsen and Platt (1992) located that video games helped college students' trouble fixing talents due to the fact the video games gave them a risk to clear up issues in a non-threatening environment.

1.2 PRELIMINARY

1.2.1 GROUP

Group G is a set of points with an operation, $*$, that relates every pair of elements x and y such that the following properties are satisfied:

1. **Closure:** $\forall x, y \in G, x * y = z, z \in G$.
2. **Associativity:** $\forall x, y, z \in G, \text{ we have } x *(y * z) = (x * y) * z$.
3. **Identity:** \exists an element 'a' $\in G$, such that $\forall x \in G, x*1 = 1*x = x$.
4. **Inverse:** $\forall x \in G, \exists x^{-1}$, such that $x * x^{-1} = 1$.

1.2.2 ABELIAN GROUP

A group is said to be Abelian, if it is commutative.

i.e. $x * y = y * x$

1.2.3 PERMUTATION

A Permutation is an invertible mapping of a finite set N onto itself.

1.2.4 CYCLE

A Cycle is a subset of a permutation in which the affected elements, E , can ordered, and every element of E is sent to another element of E .

1.2.5 TRANSPOSITION

A cycle of length two is known as Transposition.

1.2.6 PARITY

Parity is the property of an integer of whether it is even or odd.

1.2.7 ORDER

Order of the group is the number of elements.

1.3 ORIGIN AND DEVELOPMENT

The concept of Rubik's Cube is derived from China Luo-book that can be converted into jiucong map. Ten-order magic square is a certain configuration Combination of numbers $1, 2, 3, \dots, (n^2)$ in a n -th order square, which makes the sum of the numbers in Each row, each column and two diagonals $n((n^2) + 1)/2$. It is called the magic square constant. Rubik's Cube The constant of the third-order cube is 15. The rearrangement of the nine places is one-dimensional and three-order The cube and the game have eight movable chess pieces placed in nine places Create another pattern by moving the pieces in order to complete Rearrange's mode change nine Places. It is a game developed from Jiucong around the Yuan Dynasty in China. From that point forward, the improvement of the block moved from the request to the aspect. In the Qing Dynasty, Chinese researchers set forward the possibility of utilizing computerized pieces of three-layered wizardry

squares. Indeed, this was a three-layered second-request block model. Different further developed solid shapes are planned into Grow the Rubik's Cube family after the Rubik's Cube was created. As a rule, Rubik's Cube can be isolated into two kinds of Classification: 3D squares and molded blocks. One 3D shape alludes to the block in the container structure, not has changed, yet the request for the solid shapes has been expanded. There are unique Various variations of the Rubik's Cube with up to 33 layers, With $2 \times 2 \times 2$ (pocket size/smaller than normal shape), standard $3 \times 3 \times 3$ Solid shape, $4 \times 4 \times 4$ (Rubik's Revenge/Rubik's Cube), and $5 \times 5 \times 5$ (Teacher's Cube) is the most renown exceptional.

$17 \times 17 \times 17$ "Preposterous" block (recorded toward the finish of 2011) Until December 2017, the biggest scale (and the most costly) save, cost in excess of 2,000 US dollars) business deals 3D square. There is a functioning plan of $22 \times 22 \times 22$ 3D square.

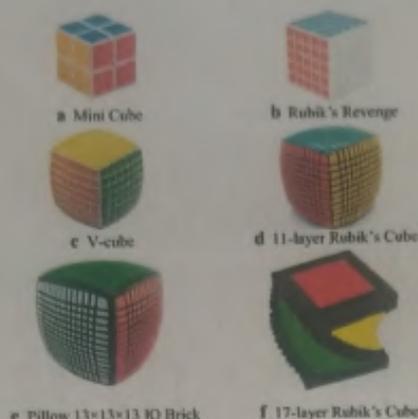
Also exhibited in January 2016, and The $33 \times 33 \times 33$ 3D shape was sent off in December 2017. Chinese manufacturer ShengShou has been producing cubes in all sizes from $2 \times 2 \times 2$ to $10 \times 10 \times 10$ (as of late 2013), and also produced an $11 \times 11 \times 11$ model. The uncommonly molded 3D shape alludes to the block family other than the cubic 3D square. The primary state of uniquely molded shape is different, including polyhedron cubes, sphere block, tetrahedron 3D square, reflect solid shape, gear cube, cake 3D square, etc.

1.4 TYPES OF RUBIK'S CUBE

The Magic Cubes are divided into two broad categories: Cubic Cube and Specially Shaped Cube.

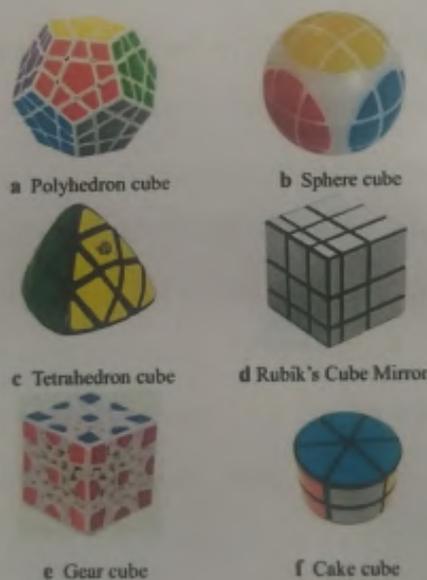
1.4.1 CUBIC CUBE

A Cubic cube refers to a cube in a box structure that is not changed but the order of the cube increases. There are different variation of Rubik's cube to $33 \times 33 \times 33$. Different types of cubic cube involves $2 \times 2 \times 2$ (Pocket/mini cube), $3 \times 3 \times 3$ (Standard cube), $4 \times 4 \times 4$ (Revenge/Master cube), $5 \times 5 \times 5$ (Professor's cube), and so on.



1.4.2 SPECIALLY SHAPED CUBE

The specially shaped cube refers to the cube family besides the cubic cube. The structural shape of the cube is diverse including polyhedron cube, sphere cube, tetrahedron cube, mirror cube, cake cube, gear cube and so on.



Chapter 2

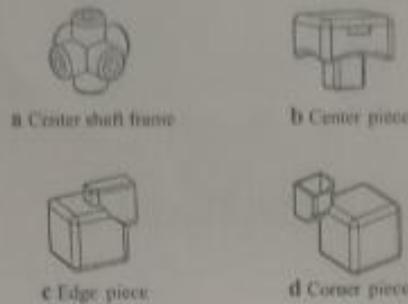
BASIC CHARACTERISTICS

The three main characteristics of a Rubik's cube are:

2.1 STRUCTURAL AND ROTATIONAL

The structure of the Rubik's Cube mechanism is the most exciting thing of the Rubik's Cube. At first, the cube seemed completely impossible to work. Few people have proposed possible mechanisms, and few people have proposed realities. Later, many of the Rubik's Cube mechanisms were based on mortise connections between components. The cubes are fixed together and there are no key components in any respect. This connection is difficult to make with sufficient accuracy, so the face can be turned over without difficulty. A Cube ($3 \times 3 \times 3$) consists of 26 scaled down cubes, also called "pieces" or "cube lets" as well as a center shaft frame, 6 center pieces, 8 corner pieces, and 12 edge pieces. Each consists of a hidden internal extension that interlocks with the others while permitting them to circulate to unique locations. The central component can be divided into two components. The middle piece and the middle shaft frame are related through spring-loaded screws the system of assembling different pieces separately. The constraints on the edges are formed by way of structural restrictions and the pressure locking of two adjoining centers. The restrictions on the corner pieces are shaped by the use of structural constraints and force locking of three adjoining edges. They all contact each other via planes and curved surfaces. The internal extensions of the corner piece and the brink piece are firmly hooked together. When force is applied, these collets-forming layers can flexibly rotate around the axis. The feature of Rubik's Cube, where small pieces are connected by mutual

mosaics.



2.2 PERMUTATION AND COMBINATION

The concept of permutation and combination was put forward to meet the needs of the number of people, that is, the idea of permutation and combination. It can describe the spatial coordinate system of the Rubik's Cube, which has a one-to-one correspondence with the Cartesian coordinate system. These suggest that the rotation transformation of the Rubik's cube Incorporates the idea of permutation and combination. The Rubik's Cube achieves the desired state and various color combinations by rotating the blocks. The Rubik's Cube has six sides. First, every face has the same color, and each face has nine small outer surfaces. They may usually be fifty-four outer surfaces. Each side of the Rubik's Cube is composed of shadows with different shapes after each side is randomly rotated several times. The transformation among different configurations of the Rubik's cube may be used as a model the use of the concept of permutation and combination to pursue the variety of solutions. The characteristics of combination transformation have inspired many product design thinking, especially the actual realization and structural design of industrial product modularization.

2.3 CYCLIC CHARACTERISTICS

Loop is one of the easy traits of Rubik's cube. The range of instances the Rubik's cube rotates is referred to as the cycle period. The Rubik's Cube is returned to its original state from its original state through a certain number of operations and specified operations. Periodic characteristics can be divided into two types: periodic and non-periodic. If the cycle period is consistent, the cycle is periodic. If the movement cycle is variable, the cycle is non-periodic.

Chapter 3

GROUP THEORY

3.1 GROUPS

The different transformations and configurations of the cubes form subgroups of permutation groups produced by different horizontal and vertical rotations of the puzzle. Front (F) means to rotate the front 90 degrees clockwise. Counterclockwise rotation is indicated by lowercase letters (f) or addition (F'). Indicates a 180-degree turn by adding a superscript 2 (F²) or just moving followed by a 2 (F2).

THEOREM: A group of operations on the cube can be moved from the unresolved state to the resolved state by moving in various combinations in the group under the series operation. Then we call this group a cube group.

Proof

Group G is composed of a group of objects and binary operator's * on objects satisfying the following four conditions.

Operation * is closed, so for any group elements h and g in G, $h * g$ is also in G. The operation * is associative, so for any element f, g, and h, $(f * g) * h = f * (g * h)$. There is a unit element $e \in G$ such that $e * g = g * e = g$. Each element in G has an inverse g^{-1} Relative to the operation such that $g * g^{-1} = g^{-1} * g = e$. Rubik's cube is not Abelian, the sequence of moves acting on cube is not commutative. For example (RU) is not equal to (UR).

3.2 PERMUTATION

The different movement sequences of the cube elements can be regarded as the arrangement or rearrangement of the cube. Note that different movement sequences of moving the cube elements can be seen as an arrangement or rearrangement of the cube. The number of possible permutations of squares on the Rubik's Cube seems daunting. There are 8 corner pieces, which can be arranged in $8!$ ways. Each corner can be arranged in 3 directions, providing 3^8 possibilities for each arrangement of corner blocks. There are 12 edge pieces, which can be arranged into $12!$ ways. Each edge block has 2 possible directions, so each arrangement of edge blocks has 2^{12} arrangements. But in the Rubik's Cube, only $1/3$ of the arrangements makes the corner cube rotate correctly. The arrangement of edge blocks that has the same edge flip direction as the original cube is only $1/2$. Only $1/2$ among them has the correct cube rearrangement parity.

The number of Rubik's cube configuration species is about,

$$(8! \cdot 3^8 \cdot 12! \cdot 2^{12}) \div (3 \cdot 2 \cdot 2 \cdot 2) = 43,252,003,274,489,856,000 \text{ ,43 quintillion.}$$

3.3 PARITY

THEOREM: The cube always has even parity, or an even number of cubbies exchanged from the starting position.

Proof

Let $P(n)$: after n rotations, there is an even number of cubbies exchanged. We assume $P(n)$ to show $P(n) \rightarrow P(n+1)$.

After n moves the cube has an even number of cubbies exchanged. Since the $n+1$ move will be a face turn, there will be an even number of cubbies flipped. There was already an even number exchanged, and so an even parity of cubbies exchanges is preserved overall. Since any permutation of the Rubik's cube has even parity, there is no move that will exchange a single pair of cubbies. This means that when two cubbies are exchanged, we know there must be other cubbies exchanged as well.

THEOREM: If the cube starts at the solved state, and one move sequence P is performed successively, then eventually the cube will return to its solved state.

Proof

Let P be any cube move sequence. Then at some number of times m that P is applied, it recycles to the same arrangement k , where $k < m$ and m is the soonest an arrangement appears for the second time. So $P^k = P^m$. Thus if we show that k must be 0, we have proved that the cube cycles back to P^0 , the solved state. If $k = 0$, then we are done, since $P^0 = 1 = P^m$. Now we prove by contradiction that k must be 0. If $k > 0$: if we apply P^{-1} to both P^k and P^m . We get the same thing, since both arrangements P^k and P^m are the same. Then $P^k \cdot P^{-1} = P^m \cdot P^{-1} \rightarrow P^{k-1} = P^{m-1}$. But this is contradictory, since we said that m is the first time that arrangements repeat, so therefore k must equal 0 and every move sequence eventually cycles through the initial state again first before repeating other arrangements.

3.4 LAGRANGE'S THEOREM

Lagrange's theorem states that for finite groups, the size of any subgroup divides the order of the group.

Lagrange says that if you repeat a sequence of moves, i.e., take successive powers, you will evidently get back where you started.

A simple example is to look at the power of F . You will notice that the power of F gives you a set of 4 elements $\{F, F^2, F^3, F^4\}$, where $F^4=I$. Here I use to mean "move" without doing anything. In addition, Lagrange stated that the Rubik's Cube must be divisible by 4. Because the subgroup generated by F , namely $\{F, F^2, F^3, I\}$, has 4 different elements.

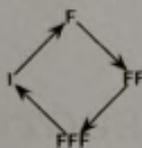
3.5 CAYLEY'S GRAPH

We can in-depth understand the structure of groups and subgroups is a Cayley's diagram. It would be ridiculous to draw a Cayley's diagram for R . If there are 43

trillion vertices, we will look at the Cayley's graphs of some small subgroups of R .

If the two groups have the same Cayley's diagram, they have essentially the same structure, which is called isomorphism. The two isomorphic groups will have the same order and the same effect on the cube. For example, performing FFRR has the same effect as rotating a cube, so that L is now facing forward, and then performing RRBB.

The following is the Cayley graph for the subgroup generated by F :



The moves $\phi = FF$ and $\rho = RR$ generate the following graph (note that $\phi^2 = \rho^2 = 1$):



Chapter 4

CRYPTOGRAPHY

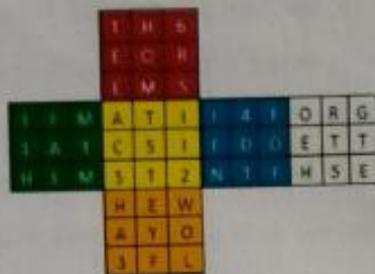
4.1 MITCHELL'S CRYPTOGRAPHY

West Virginia University Economics Professor Douglas W. Mitchell (Douglas W. Mitchell) submitted a suggested cryptographic system using Rubik's Cube toys to the Math's Magazine, Cryptologia, 1992. Replacement and transposition encryption are the two most basic forms of encoding and decoding messages. Generally, the use of transposed cipher text creation mechanisms is not efficient in generating letters in the way required for encoding and decoding. This type of system is designed to apply multiple alphabet substitutions, that is, the arrangement of alphabets, rather than multi-picture replacement, or position arrangement.

4.2 MECHANISM OF MITCHELL'S CRYPTO-SYSTEM

The original Rubik's Cube, a 3×3 arrangement of 27 mini cube faces, had six faces, nine cubes each. The rotation of the cube allows movement around three axes rotating the face by a multiple of 2 will restore the cube to its original shape, but as the cube changes. Mitchell's password system involves some sneaky steps to repair the plain text. The first step of the process requires writing number "1" in the upper left corner of the cube face. The number "2" can be on a square chosen arbitrarily on the other side, and so on, until all six sides have a characteristic number as its representative identifier. Take the top line, and then you can write the plain text on the rest of the cube surface, starting from the top line and writing from left to right. Below is the first cube written in quotation marks

"IFMATHEMATICSISTHEWAYOFLIFEDONTFORGETTHEOREMS".



An important feature of Cube is that the selected message can be encrypted in a lot of ways. After the "1" is marked on the first side, there are $5!$ Use the above numbering system to select the order of the remaining faces. The position of the remaining plain text letters depends on Choose which of the 120 options to complete the Cube sorting. Next One aspect of sorting is the sorting of the nine individual cubes, and the placement Place the remaining 8 letters after the orientation number. After that, the direction of the numbers on the square will change the direction of the rest of the numbers. Letters distributed on the surface of the cube. The spin key is used to create the resulting "hybrid". Essentially, this disrupts the cube in a way that letters and numbers are more mixed, and creates a "new" state of the cube. Fix the number "1" (toward the user) to three categories rotations by raw, by column, and by level. The rotation categories, row, column, and level, can be twisted to count as the movement of the mixed cube. The outer one or two rows, two columns or several layers can be moved by $/2$, or $3/2$. Because one or two layers can be twisted, there are three sizes Rotation and three shafts that can be twisted; the total number of different twists is $2 \times 3 \times 3 = 18$ possible moves.

4.3 ENCRYPTION AND DECRYPTION

Using cipher text, people can read letters from the cube in a pre-arranged order, from the initial "1" on the cube. In order to provide some additional "randomness", encryption the key can contain information about how to read the opening square Cipher text. According to the fixing system, the cube can be positioned like this, all While represents each face as a specific letter (A = top face, B = bottom face, Etc.), put a key extension or set of extra letters at the beginning

of the exchange. Rotating the key will imply the order of reading the cipher text. Mitchell described a six-letter extension, such as "AFDEBC", which tells the recipient to read First is the text on the top surface of the cube, then the surface represents "F", etc., until it is copied To the paper.

In addition, Mitchell proposed that the order of reading letters be determined by Additional letters in the key extension: "Suppose X stands for reading from left to right, top to bottom Rank first, middle row second, bottom row last, and Y stands for reading from top to bottom, First, the left column, then the center column, then the right column, and so on." This is after it's easy to create and decipher. The decoder will copy the cipher text onto the cube Follow the order of key expansion. The rotated key will read backwards, this will involve replacing right with left and down with up. Plain text can Read from the face marked "1", then "2" and so on. Mitchell points out the direction the face after the first one will be one of four possibilities, the correct one to read Understandable plain text. For rotary keys, we can use the following criteria: R = row, C = column, L = level. 1, 2, 3 will represent a clockwise rotation of a face in multiples of 2.

Chapter 5

APPLICATION AND CURRENT RESEARCH STATUS

5.1 APPLICATION

Rubik's Cube, "Huarongdao" as constructed by Chinese, and "Independent diamond" constructed by the French are known as three major intellectual toys of the world. Although the Rubik's Cube was constructed just over 40 times ago, it has been popular with people around the world, and an adding number of cube suckers and scholars have come hooked on Rubik's Cube's charm. The main reason is that Rubik's Cube contains the esoteric fine principles and an each-encompassing metamorphosis, which attracts curious people to explore the mystifications of Rubik's Cube. These experimenters studied the description of gyration of the Rubik's Cube, explored recovery algorithms and the fine principles of the Rubik's Cube, and used the Rubik's Cube as a model to study the scientific problems in multidisciplines.

As seen from the below studies, with the exploration and operation of Rubik's Cube, it isn't only an intellectual toy. It has changed from a exploration object to an object of significance in multidisciplinary exploration. These studies and operations are substantially grounded on the abstract characteristics of the Rubik's Cube structure. The exploration to explore the internal medium law of Rubik's

Cube has just begun. The medium principle of Rubik's Cube structure has yet to have an in- depth study.

With the nonstop enhancement in the demand of mechanical products, classical mechanisms are moving forward to ultra modern complex mechanisms, which changed from fixed topology to variable topology and from weak coupling to strong coupling. Variable topology and strong coupling of the cell body gradationally attracted attention. Compared with a resembling medium and simple multi loop coupled medium, the structure of Rubik's Cube has a advanced complex degree of freedom, the rack of Rubik's Cube structure is connected to a number of end-effectors, and the connection between rod and rod is non-continuous. Still, Rubik's Cube has great operation eventuality in the artificial field for its features, including high space application, a large number of combinations, sophisticated sports form, and multi module structure. The operation of Rubik's Cube in the ministry assiduity has the following three ideas.

The multi-end effectors of Rubik's Cube and the ability of permutation and combination can be applied to mechanical design. The Rubik's Cube medium can be used as the main structure of multifunctional mechanical products. The use of the gyration of the cube medium can achieve the asked position or form of movement, so that different ends of the selector can complete a task in order in a certain position, or make different ends of the selector work together to complete a job with orderly cooperation.

According to the kinetic brace characteristics of the Rubik's Cube structure that are different from the common structure, the cube kinetic dyads can be integrated into the current artificial medium. If such a clever design operation can replace the robot globular joint, the problem of a small gyration angle of the globular joint being limited by the mechanical structure will be answered, therefore expanding the robot's work space, especially that of the resembling mechanism. If this design

can be applied to a machine tool holder or the manipulator of a mechanical arm, the flexibility of setting and operation will be greatly bettered. The Rubik's Cube structure has high space application.

If this specified can be applied to other mechanical products, I'll promote the miniaturization process of mechanical products.

In addition, with the development of the world's aero-space assiduity, there will be more and more deep- space disquisition systems. Some features of the Rubik's Cube structure including modular, rotating, multifunctional, multipurpose, and recyclable functions can be applied to the design of deep- space disquisition spacecraft or vehicles.

According to the below operation ideas, the application of the cell structure also requires introductory theoretical exploration. The movement of the product to achieve a certain direction involves the exploration of degrees of freedom order to achieve the needed form of movement and specific state related to the structural mathematical expression and the metamorphic parcels. There are some problems with the Rubik's Cube medium, including a certain degree of flexibility, a large number of face connections, a number of movement directions of special locales, and numerous further. Posterior exploration on specific static, disunion, control, and other motifs will be involved.

With farther disquisition, implicit operations of the Rubik's Cube bear the study of medium problems of the cube structure. This can lay a theoretical foundation for operations of the cell medium and promote special cube mechanisms from educational toys to Ministry (similar as robots, aerospace, etc.).

5.2 CONCLUSION

There are numerous types of Rubik's cubes. Some reach minds in theoretical exploration and operation have been done. This document has totally introduced the origin and development of the Rubik's Cube, anatomized the characteristics Rubik's cube structure and performance characteristics, assaying Rubik's cube as a group, Rubik's cube encryption, Rubik's Cube Operations and reviewed the state of Rubik's Cube exploration, including scientific conceits, restoration algorithms and Characteristic operations. The external characteristics of the Rubik's Cube have it has been studied and applied in multidisciplinary fields, thus the principles of the internal structure of the Rubik's cube they must be explored at the same time. Exploration on the Rubik's cube in the field of mechanics is still in its immaturity. The new problems of the Rubik's cube medium should be studied, and a methodical proposition of the Rubik's Cube the medium must be formed. Some consummations of the exploration have a guiding significance in development of the Rubik's Cube Medium in Mechanical Engineering- operations. The topological proposition of the cube medium has not yet been studied further.

5.3 CURRENT RESEARCH STATUS

The Rubik's Cube was rated as the stylish toy of the public fair in Nuremberg in Germany in 1978, and his innovator Rubik was awarded "Stylish game in the world", Invention Award "Since also, Rubik's fissionability Cube has spread each over the world. At the same time, a transnational conference of representatives of mathematicians it happen in Helsinki. The Rubik's cube has attracted great attention with experts and experimenters. After that, the papers and have been published written on the Rubik's cube continuously, and the door to explore the mystifications of Rubik's cube has been opened. Latterly, the current examinations of the Rubik's cube are reviewed in colorful disciplines at home and abroad, including examinations of the scientific conceits of the Rubik's Cube, reduction algorithms, characteristic operations and problems of mechanisms. Eventually, the operations and perspectives of the Rubik's Cube in the field of mechanisms are banded.

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