ST. TERESA'S COLLEGE, ERNAKULAM

(AUTONOMOUS)

Affiliated to Mahatma Gandhi University, Kottayam.

CURRICULUM FOR MASTER'S PROGRAMME IN MATHEMATICS

Under Credit & Semester System

(2016 Admissions Onwards)

ST. TERESA'S COLLEGE (AUTONOMOUS), ERNAKULAM

DEPARTMENT OF MATHEMATICS

BOARD OF STUDIES IN MATHEMATICS

LIST OF MEMBERS

1. **Dr. A. Sunny Kuriakose, Professor and Dean, Department of Mathematics, FISAT, Ankamaly. (Chairperson).**
2. **Dr. Paul Isaac, Associate Professor, Department of Mathematics, Bharata Mata College, Thrikkakara.**
3. **Dr. T.P Johnson, Associate Professor, School of Engineering, CUSAT.**
4. **Dr. Varghese Jacob, Associate Professor, Department of Mathematics, Govt. College, Kottayam. (University Nominee)**
5. **Dr. Mary Metilda , Retd Principal, Maharajas College, Ernakulam. (Alumnae-member)**
6. **Nishi B.P, Sub Divisional Engineer(NS), Office of Principal General Manager Telecom, BSNL Bhavan, Ernakulam .(Expert from industry**)

ST. TERESA'S COLLEGE, ERNAKULAM

MINUTES OF THE BOARD OF STUDIES MEETING HELD ON 08/12/15

The third meeting of the Board of Studies (Mathematics UG) was held at 2 pm on 08/12/2015, at the Department of Mathematics, St. Teresa's College, Ernakulam

The Chairman of B.O.S, Dr. Sunny Kuriakose presided over the meeting. The meeting began with a silent prayer. The Agenda already circulated was taken item wise. Based on the discussions, the following resolutions were taken.

ITEM.11/BOS/MAT/UG/08.12: Resolved to approve the minutes of the previous meeting held on 24/07/2015.

ITEM.12/BOS/MAT/UG/08.12: Resolved to approve the syllabus of the M.Sc. Mathematics course to be started in the next academic year with the following modifications:

- Remove the paper Real Analysis from IInd semester and include it in the Ist semester.
- Remove the paper Measure Theory and Intergration from Ist semester and include it in the IInd semester.

ITEM.13/BOS/MAT/UG/08.12: Resolved to approve the panel of examiners for Second and Fourth Semester Examinations, March 2016.The approved panel of examiners are:-

1. Dr.Bloomy Joseph Asst.Professor, Maharaja's College (Ernakulam)Ph: 9895308192
   Email: bloomykoshy@gmail.com

2. Dr. Sabu M.C., Assistant Professor, St. Albert's College, Ernakulam.Ph: 9447603122
   Email: saboochacko@gmail.com

3. Bernard K.A, Assistant Professor, St. Albert's College, Ernakulam, Ph:9995117996
   Email bernardantony_kochin@gmail.com

4. Dr. Jeenu Kurian, Assistant Professor, S.H. College, Thevara Ph.9633524275
   Email: jeenukurian@shcollege.ac.in

5. Murali , Asst.Professor, Maharaja's College ,Ernakulam, Ph:9447719414
   Email:2007murali@gmail.com

6. Smt. Bridgit Jeeji C. J., Assistant Professor, Aquinas College, Cochin.Ph: 9388095878
   Email:jeejimartin@gmail.com

7. .Dr. Shery Fernandez, Assistant Professor, St. Albert's College, Ernakulam., Ph:9846762450
 Email:sheryfernandez@yahoo.co.in

8. Smt. Valentine D'cruz, Associate Professor, St. Paul's College, Kalamassery.Ph:9497686501 Email: dcruzvalentine@yahoo.in

9. Smt.Jeema Jose, Assistant Professor, St. Albert's College, Ernakulam.Ph:9846009511 Email:jeema.alberts@gmail.com

10. Smt. Pramada Ramachandran, Assistant Professor,St. Paul's College, Kalamassery. Ph.No:- 9446229506 Email : pramadars@yahoo.com

ITEM.14/BOS/MAT/UG/08.12:

- Include books in the syllabus of Linear Algebra that focuses on its applications.
- Change the text book suggested for Graph Theory.

ITEM.15/BOS/MAT/UG/08.12: Recommended to

- Conduct invited lectures for each paper at least twice in a semester.
- Provide opportunity for PG students to take classes for UG students.

ITEM.16/BOS/MAT/UG/08.12

- Resolved to enhance extension activities by conducting social surveys, providing online tuitions and free tuition classes for students of nearby slums.

The meeting came to an end at 3.30 p.m. and was followed by light refreshments.

FACULTY OF THE DEPARTMENT WHO HAVE CONTRIBUTED TOWARDS CURRICULUM AND SYLLABUS IN MATHEMATICS:

- Smt. Teresa Felitia P.A, Associate Professor & Head ,Dept. Of Mathematics, St. Teresa's College, Ernakulam.
- Smt. Susan Mathew Panakkal, Assistant Professor, Dept. Of Mathematics, St. Teresa's College, Ernakulam.
- Smt. Ursala Paul, Assistant Professor, Dept. Of Mathematics, St. Teresa's College, Ernakulam
- Smt. Elizabeth Reshma M.T, Assistant Professor, Dept. Of Mathematics, St. Teresa's College, Ernakulam
- Smt. Neenu Susan Paul ,Assistant Professor, Dept. Of Mathematics, St. Teresa's College, Ernakulam
- Ms. Anju Antony, Guest Lecturer, Dept. Of Mathematics, St. Teresa's College, Ernakulam.
- Ms.Shilpa M, Guest Lecturer, Dept. Of Mathematics, St. Teresa's College, Ernakulam.

ACKNOWLEDGEMENT

The present time is experiencing unprecedented progress in the field of science and technology in which mathematics is playing a vital role; and so the curriculum and syllabi of any academic programme has to be systematically framed so as to make them more relevant and significant.

I wish to express my sincere thanks to Dr. N. J. Rao,Visiting Professor, International Institute of Information Technology, Bangalore and Dr. Rajan Gurukal, Former Vice-Chancellor, M.G. University, currently Visiting Professor, Centre for Contemporary Studies, Indian Institute of Science, for their selfless and timely service and for giving us all the help and guidance we needed . I also acknowledge my thanks to Dr. Achuthshankar S. Nair, Professor & Head, Department of Computational Biology and Bio Informatics, University of Kerala, for his invaluable suggestions.

Our Director Dr Sr Vinitha and our Principal Dr. Sajimol Augustine have always rendered motivation and help in all our ventures and were the driving force behind this new curriculum. On behalf of Mathematics department, St Teresa's college , I am happy to express my sense of gratitude to them.

Head of the Department

Smt. Teresa Felitia P.A

Foreword

Education is the key to achieve sustainable national development which will uplift society. Today the educational system is in a phase of transition and a paradigm shift is the need of the hour. The challenge before us is to incorporate necessary changes in the prevalent educational system, and this requires changes in the course curriculum both for under graduation and post-graduation programmes. The strategy adopted for the programmes in the areas of women development involves empowering women through education and giving greater emphasis on vocational training and employability so as to enable them to enter the mainstream of economic development as equal partners. To attain this objective, St. Teresa's College is committed to impart quality education to students by providing job-oriented and research-oriented courses in addition to the existing traditional ones. It is our deep desire that students imbibe knowledge, inculcate a culture of learning and develop the capability to compete for jobs in a global scenario.

In response to the growing need for programmes that will ensure employability of women, the College has initiated both traditional as well as innovative Postgraduate programmes this year. While programmes such as M.Sc. Mathematics have ever been in demand, the College, attuned to the needs of the hour, has also introduced M.Sc. Nutrition and Dietetics which will offer solutions to mitigate the increasing number of degenerative diseases, with a focus on preventive medicines. M.Sc. in Fashion Technology will sensitize students to the concept of 'green' fashion while keeping in tune with the latest global trends. The course curriculum has been structured keeping in mind the demands of the time and incorporating the latest in the specific areas of study.

I would like to take this opportunity to thank Dr. Celine. E (Dr. Sr. Vinitha), Director of the College, who has taken the initiative to introduce career-oriented programmes. I specially thank all the faculty members of the Departments offering these new programmes, who have given their time and energy in building the curriculum for the same. I wish to place on record my gratitude to Ms. Teresa Felitia P.A., Head, Department of Mathematics, for her sincere efforts in overseeing the structuring of the syllabi. With sincere thanks, I acknowledge the support extended by the members of the Governing Council, Dr. Latha Nair, Associate Professor, Department of English, Dr. Kala M.S., Associate Professor, Department of Physics and Dr. Alphonsa Vijaya Joseph, Associate Professor, Department of Botany in framing the overall structure of the courses. We now need to take this endeavour forward as the next step in our journey of deepening, strengthening and spreading our work through engagement, collaboration and partnerships. I wish and hope that our institution will continue to serve the noble purpose for the years to come with glory.

Dr.Sajimol Augustine
Principal

CONTENTS

PREAMBLE

The aim of the post graduate education is to provide high quality education as well as a supportive learning environment for the students to reach their full academic potential. The higher education has to inculcate in students the spirit of hard work and research aptitude to pursue further studies in the nationally/internationally reputed institutions as well as prepare them for a wider range of career opportunities in industry and commerce.

Board of Studies in Mathematics has designed the curriculum for M.Sc. Mathematics so as to monitor, review and enhance educational provision which ensures the Post Graduate Education remains intellectually demanding and relevant to current needs of Mathematics graduates. The thrust is given in fostering a friendly and stimulating learning environment which will motivate the students to reach high standards, enable them to acquire real insight into Mathematics and become self-confident, committed and adaptable graduates. With this in mind, we aim to provide a firm foundation in every aspect of Mathematics and to develop analytical, experimental, computational logical and reasoning skills of students.

The Board of Studies acknowledges and appreciates the good effort put in by the faculty members of Mathematics Department to frame the syllabus for M.Sc. Programme in Mathematics in the institution which will be implemented for the admissions from 2016 onwards.

GRADUATE ATTRIBUTES

The Department of Mathematics is committed to provide an enriched educational experience to develop the knowledge, skills and attributes of students to equip them for life in a complex and rapidly changing world.

On completion of the M.Sc Programme in Mathematics, our students should be able to demonstrate the graduate attributes listed below

- *Professionalism, employability and enterprise*
  - Proficiency in problem solving, creativity, numeracy and self-management.
  - Confidence in accepting professional challenges, act with integrity, set themselves high standards.
  - Ability to work independently and along a team with professional integrity.
- *Learning and research skills*
  - Acquire skills of logical and analytical reasoning.
  - Develop a critical attitude towards knowledge.
  - Equipped to seek knowledge and to continue learning throughout their lives.
  - Develop intellectual curiosity, effective learning and research abilities.
  - Commitment to the pursuit of truth and academic freedom.
- *Intellectual depth, breadth and adaptability*
  - Proficiency in curricular, co-curricular and extracurricular activities that deepen and broaden knowledge
  - Develop skills of analysis, application, synthesis, evaluation and criticality.
- *Respect for others*
  - Develop self-awareness, empathy, cultural awareness and mutual respect.
  - Ability to work in a wide range of cultural settings and inculcate respect for themselves and others and will be courteous.
- *Social responsibility*
  - Knowledge in ethical behaviour, sustainability and personal contribution.
  - Awareness in the environmental, social and cultural value system.

OBJECTIVES

The syllabi are framed in such a way that it provides a more complete and logic frame work in almost all areas of Mathematics.

By the end of the first year, the students should have

1) attained a secure foundation in core subjects like Linear Algebra, Real Analysis, Complex Analysis, Topology , Measure Theory and Discrete Mathematics
2) developed the ability to write rigorous mathematical proofs using the fundamental tools in Mathematics.

By the end of the second year, the students should have

1) introduced to powerful tools for tackling a wide range of topics in Calculus, Theory of Equations and Numerical methods.
2) familiarised with additional relevant mathematical techniques and other relevant subjects.
3) familiarised with modern tools of applied mathematics.
4) updated with project presentations, seminars etc which will form a base for research in future.

STRUCTURE OF MASTER'S PROGRAMME IN MATHEMATICS

Theory Courses

There are twenty theory courses spread equally in allfour semesters in the M.Sc. Programme. Distribution of theory courses is as follows: There are sixteencompulsory courses common to all students. Semester I, Semester II and Semester III will have **five** core courses each. Semester IV will have **one** core course and **Four** elective courses. Total credits for the Master's programme in Mathematics is 80.

Project

The project of the PG programmme should be very relevant and innovative in nature. The type of project can be decided by the student and the guide (a faculty of the department or other department/college/university/institution). The project work should be taken up seriously by the student and the guide. The project should be aimed to motivate the inquisitive and research aptitude of the students. The students may be encouraged to present the results of the project in seminars/symposia. The conduct of the project may be started at the beginning of Semester III, with its evaluation scheduled at the end of Semester IV. The project is evaluated by one external and one internal examiner.

Viva Voce

A viva voce examination will be conducted by one external examiner along with the internal examiner at the time of evaluation of the project. The components of viva consists of subject of special interest, fundamental Mathematics, topics covering all semesters and awareness of current and advanced topics with separate marks.

Course Code

The 16 core courses in the programmme are coded according to the following criteria. The first two letters of the code indicates the name of programme, ie. MT stands for Mathematics. Next digit is to indicate the semester. i.e., MT1 (Mathematics, 1st semester) followed by the letter C or E indicating whether the course is core course or elective course as the case may be. Next digits indicate course number. The letter/letters T/ PR/V follows it and is used to indicate theory/ project/viva. The last letter will be M which indicates whether the programme is for masters.

DISTRIBUTION OF COURSES AND CREDITS

SEM	Name of the course with course code	No.of Hrs/ week	No. of credits	Total Hrs/ SEM.	Exam Duration Hrs	Total marks	
						Sessional	Final
I	**MT1C01TM** Linear Algebra	5	4	90	3	25	75
	MT 1C02TM Basic Topology	5	4	90	3	25	75
	MT1C03TM Real Analysis	5	4	90	3	25	75
	MT 1C04TM Graph Theory	5	4	90	3	25	75
	MT1C05TM Complex Analysis	5	4	90	3	25	75
II	**MT2C06TM** Abstract Algebra	5	4	90	3	25	75
	MT2C07TM Advanced Topology	5	4	90	3	25	75
	MT2C08TM Advanced Complex Analysis	5	4	90	3	25	75
	MT2C09TM Partial Differential Equations	5	4	90	3	25	75
	MT2C10TM Measure Theory and Integration	5	4	90	3	25	75
III	**MT3C11TM** Multivariate Calculus and Integral Transforms	5	4	90	3	25	75
	MT3C12TM Functional Analysis	5	4	90	3	25	75
	MT3C13TM Differential Geometry	5	4	90	3	25	75
	MT3C14TM Number Theory and Cryptography	5	4	90	3	25	75
	MT3C15TM Optimization Techniques	5	4	90	3	25	75

IV	**MT4C16TM** Spectral Theory	5	3	90	3	25	75
	ELECTIVES (Four are to be selected)						
	MT4E17TM Analytic Number Theory	5	3	90	3	25	75
	MT4E18TM Combinatorics	5	3	90	3	25	75
	MT4E19TM Mathematical Economics	5	3	90	3	25	75
	MT4E20TM Operations Research	5	3	90	3	25	75
	MT4E21TM Probability Theory	5	3	90	3	25	75
	MT4E22TM Commutative Algebra	5	3	90	3	25	75
	MT4C1PRM Project/Dissertation/	NIL	3	NIL	45 minute	25	75
	MT4C01VM Viva Voce	Nil	2	Nil		Nil	100

Distribution of Credits

The total credit for the programme is fixed at 80.The distribution of credit points in each semester and allocation of the number of credit for theory courses, project and viva is as follows. The credit of theory courses is 4 per course in the first, second and third semesters. The courses in the fourth semester will have 3 credits .The project and viva voce will have a credit of 3 and 2 respectively. The distribution of credit is shown below.

Semester	Courses	Credit	Total Credit
I	5 Theory Courses	$5\times 4 = 20$	20
II	5 Theory Courses	$5\times 4 = 20$	20
III	5 Theory Courses	$5\times 4 = 20$	20
IV	5 Theory Courses 1 Project / Dissertation 1 Viva- Voce	$5\times3 = 15$ $1\times 3 = 3$ $1 \times 2 = 2$	20
Total Credit of the M.Sc. Programme			**80**

EVALUATION

The evaluation of each course shall contain two parts such as Sessional Assessment and Final Assessment. The ratio between Sessional and Final shall be 1:3. The mark distribution for Sessional and Final is 25 and 75. The Sessional and Final assessment shall be made using Mark based Grading system based on 7-point scale.

a) SESSIONAL ASSESSMENT

The Sessional evaluation is to be done by continuous assessments of the following components. Pass minimum for sessional assessment for each course is 10 marks. The components of the sessional for theory and practical and their mark distributions are given in the table below.

Component	*Marks*
Attendance	5
Assignments	5
Seminar	5
Test Papers (Average of 2)	10
Total	**25**

Attendance, Assignment and Seminar

Monitoring of attendance is very important in the credit and semester system. All the teachers handling the respective courses are to document the attendance in each semester. Students with attendance less than 75% in a course are not eligible to attend external examination of that course.

Condonation of shortage of attendance to a maximum of 10 days in a semester subject to a maximum of two times during the whole period of post graduate programme may be granted .If a student represents her institution, University, State or Nation in Sports, NCC, NSS or Cultural or any other officially sponsored activities such as college union / university union activities, she shall be eligible to claim the attendance for the actual number of days participated subject to a maximum of 10 days in a Semester based on the specific recommendations of the Head of the Department and Principal of the College.

The performance of students in the seminar and assignment should also be documented.

Their mark distributions are given in the table below

Attendance		**Assignments**		**Seminar**	
% of Attendance	*Marks*	*Components*	*Marks*	*Components*	*Marks*
≥ 90%	5	Punctuality	2	Content	2
< 90% and ≥ 85%	4				
< 85% and ≥ 80%	3	Content	3	Presentation	3
< 80% and ≥ 75%	2				
< 75%	0				

<u>Test Paper</u>

- Average mark of two sessional examinations shall be taken.

<u>Project Evaluation</u>

The sessional evaluation of the project is done by the supervising guide of the department or the member of the faculty decided by the head of the department. The project work may be started at the beginning of the Semester III. The supervising guide should keenly and sincerely observe the performance of the student during the course of project work. The supervising guide is expected to inculcate in student(s), the research aptitude and aspiration to learn and aim high in the realm of research and development. A maximum of three students may be allowed to perform one project work if the volume of the work demands it.

Project evaluation begins with (i) the selection of problem (ii) literature survey (iii) work plan (iv) experimental / theoretical setup/data collection(v) characterization techniques/computation/analysis (vi) use of modern software for data analysis/experiments and (vi) preparation of dissertation. The project internal marks are to be submitted at the end of Semester IV.

The internal evaluation is to done as per the following general criteria given in below.

Component	Marks
Literature Survey	5
Theoretical setup/Data Collection	10
Result and Discussion	5
Presentation	5
Total	25

General Instructions for sessional assessment

(i) One teacher appointed by the Head of the Department will act as a coordinator for consolidating score sheet for internal evaluation in the department in the format supplied by the controller of the examination. The consolidated score sheets are to be published in the department notice board, one week before the closing of the classes for final examinations. The score sheet should be signed by the coordinator and counter signed by the Head of the Department and the college Principal.

(ii) The consolidated score sheets in specific format are to be kept in the college for future references. The consolidated marks in each course should be uploaded to the Institution Portal at the end of each semester as directed by the Controller of Examination.

(iii) A candidate who fails to register for the examination in a particular semester is not eligible to continue in the subsequent semester.

(iv) Grievance Redressal Mechanism for Internal evaluation: There will be provision for grievance redress at four levels, viz,

a) at the level of teacher concerned, at the level of departmental committee consisting of Head of the Department,

b) Coordinator and teacher concerned,

c) at the level of college committee consisting of the Principal, Controller of Examination and Head of the Department

College level complaints should be filed within one week of the publication of results

b.FINAL ASSESSMENT

The final examination of all semesters shall be conducted by the institution on the close of each semester. Pass minimum for final assessment of each course is 30 marks. For reappearance/ improvement, students may appear along with the next batch.

Question Paper Pattern for Theory Courses

All the theory question papers are of three hour duration. All question papers will have three parts. Total marks is 75.

Part A: Questions from this part are very short answer type. Five questions have to be answered from among seven questions. Each question will have 3 marks and the Part A will have total marks of 15.

Part B: Six problems out of nine given have to be answered. Each question has 5 marks making the Part B to have total marks of 30.

Part C: Part C will have four questions. One question must be asked from each module. Two questions have to be answered out of four questions. Each question will have 15 marks making the total marks 30 in Part C.

Questions from all the modules of the syllabus shall be included in Parts A, B and C of the question paper. Not more than 2 questions can be included in Part A from each module whereas in part B, a minimum of two questions has to be there from each module. In Part C, one question from each module is to be included.

Maximum marks allocated to a module is 33

Directions for question setters

- **(i)** Follow the text book specified in the syllabus as far as possible.
- **(ii)** The question paper should cover uniformly the entire syllabus. For that the pattern of question paper mentioned above must be strictly followed.
- **(iii)** Set Part A questions to be answered in six minutes each, Part B questions in twelve minutes each and Part C questions in thirty five minutes each.

Weightage to objectives and difficulty levels in the question paper should be as given in the Table below.

Weightage to Objectives		*Weightage to Difficulty Levels*	
Objective	%	Level of Difficulty	%
Information	20	Easy	30
Understanding	60	Average	50
Application	20	Difficult	20

Project and Viva Voce Examinations

Project Evaluation: The project is evaluated by one external and one internal examiner deputed from the board of practical examination. The dissertation of the project is examined along with the oral presentation of the project by the candidate. The examiners should ascertain that the project and report are genuine. Innovative projects or the results/findings of the project presented in national seminars may be given maximum advantage. The different marks for assessment of different components of project are as shown in the following table.

Component	**Weights**
Quality of project under study	10
Theses /Presentation of the project	10
Theoretical setup/Data Collection	20
Result and Dissertation layout	10
Total marks for Dissertation	50
Oral presentation and Viva on Project	25
Total marks for final assessment of project	75

Viva Voce Examination: Viva voce examination is conducted only in the final examination by the internal and the external examiner. The viva voce examination is given a credit two. The marks for different components should be awarded in the following format shown below.

Type of Questions	Percentage	Weightage to Difficulty Level	
B.Sc/ + 2 level	20	Level of Difficulty	%
M.Sc. Syllabus Based	40	Easy	30
Subject of Interest	30	Average	50
Advanced Level	10	Difficult	20

COMPUTATION OF CCPA

Grade and Grade Point is given to each course based on the percentage of marks obtained as follows:

Percentage of Marks	**Grade**	**Grade Point**
90 and above	A+ - Outstanding	10
80-89	A - Excellent	9
70-79	B - Very Good	8
60-69	C - Good	7
50-59	D - Satisfactory	6
40-49	E - Adequate	5
Below 40	F - Failure	4

Note: Decimal are to be rounded to the next whole number

CREDIT POINT AND CREDIT POINT AVERAGE

Credit Point (CP) of a course is calculated using the formula ***CP = C* x *GP,***

*Where **C** = Credit for the course; **GP** = Grade point*

Semester Credit Point Average (SCPA) is calculated as **SCPA**= $\frac{\boldsymbol{TotalCreditPoints}\ (\boldsymbol{TCP})}{\boldsymbol{TotalCredits}(\boldsymbol{TC})}$

Grades for the different semesters / programme are given based on the corresponding SCPA on a 7-point scale as shown below:

SCPA	Grade
Above 9	A+ - Outstanding
Above 8, but below or equal to 9	A - Excellent
Above 7, but below or equal to 8	B -Very Good
Above 6, but below or equal to 7	C - Good
Above 5, but below or equal to 6	D - Satisfactory
Above 4, but below or equal to 5	E - Adequate
4 or below	F - Failure

Cumulative Credit Point Average for the programme is calculated as follows:

$$\textbf{CCPA} = \frac{(TCP)_1 + \ldots\ldots + (TCP)_4}{TC_1 + \ldots + T_4}$$

where **TCP_1…….., TCP_4**are the **Total Credit Points** in each semester and **TC_1……..,** **TC_4**are the **Total Credits** in each semester

Note: A separate minimum of 4**0% marks** each for Sessionals and Finals (for both theory and practical) is required for pass for a course. For a pass in a programme, a separate minimum of Grade E is required for all the individual courses. If a candidate secures **F**

Grade for any one of the courses offered in a Semester/Programme only **F** grade will be awarded for that Semester/Programme until he/she improves this to **E** grade or above within the permitted period.

SYLLABUS

MASTER OF SCIENCE (M.Sc.)

IN MATHEMATICS

SEMESTER I

MT1C01TM - LINEAR ALGEBRA

Total Credits :4

Total Lecture Hours: 90

Aims

The course aims at giving more awareness of the vector spaces, linear transformations and its relation with matrices and dual spaces. It then follows by commutative rings, determinant functions and additional properties of determinants. Elementary canonical forms have also been introduced. It also provides an insight into triangulations, diagolizations and direct sum decompositions.

Course Overview and Context

The course starts with an introduction to vector spaces and related concepts with various examples. It then follows by linear transformations, representation of linear transformations by matrices and dual spaces. The course also includes detailed study of determinants and properties of determinants. An introduction to elementary canonical forms, simultaneous triangulations and diagonalisations have also been inlcuded.

Syllabus Content

Module 1 **(15 hours)**

Vector spaces, subspaces, basis and dimension (Chapter 2, 2.1, 2.2, 2.3 of the text) (Proof of theorems excluded) **C**o-ordinates, summary of row-equivalence (Chapter 2-2.4 & 2.5 of the text)

Module 2 **(30 hours)**

Linear transformations, the algebra of linear transformations, isomorphism, representation of transformations by matrices, linear functionals, double dual, transpose of a linear transformation.(Chapter 3 - 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 & 3.7 of the text)

Module 3 **(18 hours)**

Determinants: Commutative Rings, Determinant functions, Permutation and uniqueness of determinants, Additional properties of determinants. (Chapter 5 - 5.1, 5.2, 5.3 & 5.4 of the text)

Module 4 **(27 hours)**

Introduction to elementary canonical forms, characteristic values, annihilatory polynomials, invariant subspaces, simultaneous triangulations, simultaneous diagonalisation, direct sum decompositions, invariant direct sums (Chapter 6 - 6.1, 6.2, 6.3, 6.4, 6.5 & 6.6 of the text)

Learning Resources

Text Book

1. Kenneth Hoffman / Ray Kunze (Second Edition), *Linear Algebra*, Prentice-Hall of India Pvt. Ltd., New Delhi, 1992.

References:

1. Klaus Jonich. Linear Algebra, Springer Verlag.

3. Paul R. Halmos, Linear Algebra Problem Book, The Mathematical Association of America.

4. S. Lang, Algebra, 3rd edition, Addison-Wesley, 1993.

5. K.B. Datta, Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., NewDelhi, 2000.

6. S. Kumaresan, Linear Algebra A Geometrical Approach, Prentice Hall of India, 2000.

Competencies of the Course

C1. Refresh Vector spaces, subspaces, basis, dimension, linear transformations together with introducing linear functional and double dual

C2. Studying in detail about Determinants of square matrices, determinant being viewed as an alternating n-linear function of the rows of a matrix.

C3. Beautiful exposition of elementary canonical forms.

C4. Describe simultaneous triangulations and simultaneous diagolisations.

MT1C01TM - LINEAR ALGEBRA

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	1	3	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FIRST SEMESTER M.Sc. MATHEMATICS EXAMINATION

MT1C01TM-LINEAR ALGEBRA

Time : 3 hours Maximum marks : 75

PART A

(Answer any **five** questions. Each question carries **3** marks)

1. Show that the vectors u = (1,0,-1), v = (1,2,1) and w = (0,-3,2) form a basis for R^3.
2. Find three vectors in R^3 which are linearly independent and are such that any two of them are linearly independent.
3. Let F be a field and T be a linear operator on F^2 defined by $T(x_1,x_2)=(x_1+x_2,x_1)$. Find T^{-1}.
4. Let K be a commutative ring with identity and let n be a positive integer. Show that there exists at least one determinant function on $K^{n \times n}$.
5. Let V be an n dimensional vector space over F. Find the characteristic polynomial of the identity operator and zero operator.
6. Find the co-ordinates of the vector (2,1,-6) of R^3 relative to the basis (1,1,2), (3,-1,0), (2,0,-1).
7. Let K be a commutative ring with identity. Show that the determinant function on 2 × 2 matrices A over K is alternating and 2-linear as function of columns of A.

(5 x 3 = 15)

PART B

(Answer any **six** questions. Each question carries **5** marks)

8. State and prove Cayley Hamilton theorem for linear operators.
9. If u,v,w are linearly independent, then show that u+v, u-v, u-2v+w are also linearly independent.

10. Let T be a linear transformation from V into W. Show that T is non-singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.
11. Let R be a non-zero row reduced echelon matrix. Prove that the non-zero vectors of R form a basis for the row space of R.
12. Let V be a finite dimensional vector space over the field F and let W be a subspace of V. Show that dim W + dim W^0 = dim V.
13. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Show that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.
14. Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space over F. Show that the space L(V,W) is finite dimensional and has dimension mn.
15. (I) Let K be a commutative ring with identity and let A and B be n by n matrices. Prove that det(AB) = (det A) (det B).

    (II) Find a 3×3 matrix for which the minimal polynomial is x^2.
16. (I) Let T be a linear operator on an n dimensional vector space V. Show that the characteristic and minimal polynomials for T have the same roots, except for multiplication.

    (II) Prove that the space of all m by n matrices over the field F has dimension mn, by exhibiting a basis for this space.

(6 x 5 = 30)

PART C

(Answer any **two** questions. Each question carries **15** marks)

17. (I) Find the subspace annihilated by the following functional x^4 :

$f(x_1, x_2, x_3, x_4) = x_1 + 2x_2 + 2x_3 + x_4$

$g(x_1, x_2, x_3, x_4) = 2x_2 + x_4$

$h(x_1, x_2, x_3, x_4) = -2x_1 - 4x_3 + 3x_4$

(II) Let T: V→ W be linear where V and W are vector space over F.

Show that Rank (T^t) = Rank(T).

18. (I) If B = {(1,-1,3), (0,1,-1), (0,3,-2)} be a basis for $V_3(R)$, find its dual basis B*.

    (II) Find the matrix of a linear transformation T on $V_3(R)$ defined as :

    T(a,b,c) = (2b +c, a – 4b, 3c) with respect to the ordered basis{(1,1,1), (1,1,0),(1,0,0)}

19. I) Differentiate between simultaneous triangulation and simultaneous diagonalisation with examples.

    II) Explain annihilatory polynomial and characteristic polynomial.

20. I) Let V be a finite dimensional vector space over the field F and let T be a linear opeator on V . Prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x-c_1) \dots (x-c_k)$, where $c_1, c_2, \dots, c_k$ are distinct elements of F.

    II) Show that every matrix A such that $A^2 = A$ is similar to a diagonal matrix.

(2 x 15 = 30)

SEMESTER I
MT1C02TM -BASIC TOPOLOGY

Total Credits :4

Total Lecture Hours: 90

Aims

The course aims to introduce basic topology which is a branch of mathematics that studies the shape of mathematical objects without caring about their size and other distance related properties. These notions will be made precise using the concepts of continuity, compactness and connectedness. The course also aims at providing the ability to use and recognize the importance of separation axioms.

Course Overview and Context

The course starts with the introduction of topological space and various examples of topological spaces together with some basic concepts relating to it. It then follows by continuity of topological spaces and some properties like compactness and connectedness of topological spaces. Separation axioms are also described.

Syllabus Content

Module 1 **(24 hours)**

Definition of a topological space – examples of topological spaces, bases and sub bases – sub spaces.Basic concepts: closed sets and closure – neighborhood, interior and accumulation points (Chapter 4 Section – 1, 2, 3, 4 - Chapter 5 Section -. 1 and 2 of the text.5.2.11 & 5.2.12 excluded.)

Module 2 **(22 hours)**

Continuity and related concepts: making functions continuous, quotient spaces.Spaces with special properties: Smallness condition on a space (Chapter 5. Section. 3 and 4 of the text, 5.3.2(4) excluded)(Chapter 6 Sec. 1 of the text)

Module 3 **(22 hours)**

Connectedness: Local connectedness and paths(Chapter 6 Section. 2 & 3 of the text)

Module 4 **(22 hours)**

Separation axioms: Hierarchy of separation axioms – compactness and separation axioms (Chapter – 7 Section 1 & 2 of the text) (2.13 to 2.16 of section.2 excluded)

Learning Resources

Text Book

K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd,1984

References

1. Munkres J.R, Topology-A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. J.L Kelley, General Topology, Van Nostrand, Reinhold Co., New York, 1995.
3. Stephen Willard, General Topology, Addison-Wesley.
4. Dugundji, Topology, Universal Book Stall, New Delhi.
5. George F Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, 1963.

Competencies of the course

C1. Introducing topological spaces and its examples.

C2. Define continuity of topological spaces and quotient spaces.

C3. Describing properties like compactness, connectedness and local connectedness.

C4.Defining separation axioms.

MT1C02TM -BASIC TOPOLOGY

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FIRST SEMESTER M.Sc. MATHEMATICS EXAMINATION

MT1C02TM -BASIC TOPOLOGY

Time : 3 hours Maximum marks : 75

PART A

(Answer any **five** questions. Each question carries **three** marks)

1. Show that the property of being discrete space is divisible.
2. Prove or disprove : Separability preserves under continuous map.
3. Prove that the topological product of any finite number of connected spaces is connected.
4. Show that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.
5. Show that every closed and bounded interval of the real line is compact.
6. Define a completely regular space. Show that every completely regular space is regular.
7. Show that composition of continuous functions is continuous.

(5 x 3 = 15)

PART B

(Answer any **six** questions. Each question carries **five** marks)

8. Show that every closed surjective map is a quotient map.
9. Prove that every regular, Lindeloff space is normal.
10. State and prove Lebesgue Covering Lemma.
11. Prove that a subset of the real **R** is connected if and only if it is an interval.
12. Explain the weak topology on X. Verify the existence of such topology determined by collection of functions.

13. Is R with semi-open interval topology is second countable? Verify.
14. Prove that X is second countable if and only if it has a countable sub-base.
15. Prove that a set is closed if and only if it contains its boundary and open if and only if it is disjoint from its boundary.
16. Show that the interior of a set in a topological space is the complement of the closure of the complement of the set.

(6 x 5 = 30)

PART C

(Answer any **two** questions. Each question carries **fifteen** marks)

17. I) For a topological space X, show that the following are equivalent :
    (a) X is locally connected.
    (b) Components of open subsets of X are open in X.
    (c) X has a base consisting of connected subsets.
    (d) For every x ϵ X and every neighborhood N of x there exists a connected open neighborhood M of x such that M is contained in N.

    II) Prove that a discrete space is countable if and only if the underlying set is countable.
18. I) Show that the product topology and the usual topology induced by the Euclidean metric on R^n are same.

    II) f : X → Y, X and Y are topological spaces. Show that f is continuous if and only if inverse of f is an open map.
19. I) Open ball in a metric space is open set. Prove.

    II) Prove or disprove : A metric space X is second countable if it contains a countable dense subset.
20. I) Prove that if a space is second countable, then every open cover of it has a countable sub cover.

    II) Prove that metrisabiltiy is a hereditary property.

(2 x 15 = 30)

SEMESTER I
MT1C03TM - REAL ANALYSIS

Total Credits :4

Total Lecture Hours: 90

Aims

The objective of this course is to develop the ability to write rigorous mathematical proofs for basic theorems in multivariable calculus involving the fundamental tools such as continuity and differentiability which is essential for any student majoring in mathematics. This course also aims at providing a thorough knowledge about the convergence of functions.

Course Overview and Context

The course begins with monotonic functions, functions of bounded variations and its properties. Then it follows by the Riemann Steiljes integral and its properties. Uniform convergence of a sequence and series of functions have also been described with examples. Some special functions such as exponential function, logarithmic function etc have also been included.

Syllabus Content

Module 1 **(20 hours)**

Functions of bounded variation and rectifiable curves : Introduction, properties of monotonic functions, functions of bounded variation, total variation, additive property of total variation, total variation on (a, x) as a functions of x, functions of bounded variation expressed as the difference of increasing functions, continuous functions of bounded variation, curves and paths, rectifiable path and arc length, additive and

continuity properties of arc length, equivalence of paths, change of parameter.
(Chapter 6, Section: 6.1 - 6.12. of Text 1)

Module 2: The Riemann-Stieljes Integral (25 hours)

Definition and existence of the integral, properties of the integral, integration and differentiation, integration of vector valued functions.
(Chapter 6 - Section 6.1 to 6.25 of Text 2)

Module 3: Sequence and Series of Functions (25 hours)

Discussion of main problem, uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, the Stone-Weierstrass theorem (without proof). (Chapter 7 Section. 7.7 to 7.18 of Text 2)

Module 4: Some Special Functions (20 hours)

Power series, the exponential and logarithmic functions, the trigonometric functions, the algebraic completeness of complex field, Fourier series. (Chapter 8 - Section 8.1 to 8.16 of Text 2)

Learning Resources

Text Book

1. Tom Apostol, Mathematical Analysis (second edition), Narosa Publishing House
2. Walter Rudin, Principles of Mathematical Analysis (Third edition), International Student Edition

References:-

1. Royden H.L, Real Analysis, 2nd edition, Macmillan, New York.
2. Bartle R.G, The Elements of Real Analysis, John Wiley and Sons.
3. S.C. Malik, Savitha Arora, Mathematical Analysis, New Age International Ltd.
4. Edwin Hewitt, Karl Stromberg, Real and Abstract Analysis, Springer International,.

Competencies of the course

(C1) : Introduce functions of bounded variation and rectifiable curves.

(C2) : Define Riemann-Stieljes Integral and its properties.

(C3) : Define Uniform convergence of sequence and series of functions.

(C3) :Define some special functions like exponential function and logarithmic function.

MT1C03TM - REAL ANALYSIS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	1	3	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FIRST SEMESTER M SC MATHEMATICS EXAMINATION

MT1C03TM REAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A

(Answer any **five** questions. Each question carries **three** marks)

1. Prove that total variation of a function over an interval is zero if and only if it is a constant.
2. Show, by an example, that a convergent series of continuous functions may have a discontinuous sum.
3. Obtain Weierstrass test for uniform convergence.
4. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, prove that $\{f_n + g_n\}$ also converges uniformly of E.
5. Show that a function may have a finite directional derivative in every direction at some point, but may fail to be continuous at that point.
6. Obtain Cauchy criterion for uniform convergence of sequence of functions.
7. Define equicontinuous family of functions and give an example of a family of functions which is not equicontinuous.

(5 x 3 = 15)

PART B

(Answer any **six** questions. Each question carries **five** marks)

8. Find and classify the extreme values of $x^2 + y^2 + x + y + xy$.
9. Obtain necessary conditions for the function $f = u + iv$ to have derivative at z_0. Show also that they are not sufficient.

10. State and prove the Mean value theorem for vector valued functions.
11. Establish the existence of a real continuous function on the real line which is nowhere differentiable.
12. Explain uniformly convergent sequence and point wise convergent sequence and bring out the difference between them.
13. State and prove fundamental theorem of calculus.
14. Show by an example that continuous function need not be of bounded variation.
15. Prove that if $\{f_n\}$ is a sequence of continuous functions on E and if $f_n$converges to f uniformly on E, then f is continuous on E.
16. Characterise all rectifiable curves.

(6 x 5 = 30)

PART C

(Answer any **two** questions. Each question carries **fifteen** marks)

17. State and prove inverse function theorem and implicit function theorem.
18. I) State and prove a set of necessary and sufficient conditions for the sequence $\{f_n\}$ to converge uniformly on a set E.

    II) If $\{f_n\}$ and $\{g_n\}$ are two sequences of bounded functions that converge uniformly on E, then prove that $\{f_n g_n\}$ is also uniformly convergent on E.
19. I) Prove that even if $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set E, there need not exist a subsequence which converge point wise on E.

    II) Define the Jacobian matrix of a function at a point. Give an example.
20. I) State and prove Taylor's formula for functions from R^n to R^1.

    II) Let S be an open connected subset of R^n and let f : S → R^mbe differentiable at each point of S. If f '(c) = 0 for each c in S, then prove that f is constant on S.

(2 x 15 = 30)

SEMESTER I
MT 1C04TM GRAPH THEORY

Total Credits :4

Total Lecture Hours: 90

Aim

The aim of this course is to introduce graph theory together with very few applications. The book also stressed the importance of efficient methods of solving problems.

Course Overview and Context

The course starts with introducing graphs and characteristics of graphs. Also contains various definitions required to know in connection with graphs. It then follows by the connectivity of a graph and its application in the construction of reliable communication networks. Euler tours and Hamiltonian cycles and its applications have also been included.

Module 1 **(30 hours)**

Graphs and subgraphs : Graphs and simple graphs, Graph Isomorphism, The Incidency matrix and adjacency matrices, subgraphs, vertex degrees, Paths and connection, Cycles, Sperner's lemma ,Trees, cut edges and bonds, cut vertices, cayley's formula(Chapter 1 – Sections 1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.9 of text 1, Chapter 2 – Sections 2.1,2.2,2.3,2.4)

Module 2 **(20 hours)**

Connectivity, blocks, construction of reliable communication networks, Euler tours, Hamiltonian cycles, Applications (Chapter 3 – Sections 3.1, 3.2, 3.3, Chapter 4 –

Sections 4.1,4.2,4.3,4.4)

Module 3 **(20 hours)**

Matchings, Matchings and coverings in bipartite graphs, Perfect matchings, Edge chromatic number, Independent sets (Chapter 5 – Sections 5.1, 5.2, 5.3, Chapter 6 – Section 6.1, Chapter 7 – Section 7.1)

Module 4 **(20 hours)**

Chromatic number, girth and chromatic number, Plane and Planar graphs, Dual graphs, Euler's formula, Kuratowski's theorem (Chapter 8 – Sections 8.1, 8.5, Chapter 9 – Sections 9.1, 9.2, 9.3, 9.5 of text 1)

Learning Resources

Text Book

1. J.A. Bondy & U.S.R. Murthy, Graph Theory with Applications.

References:

1. John Clark and Derek Allan Holton, A First Look at Graph Theory, Allied Publishers.

2. Douglas B West, Introduction to Graph Theory, Prentice Hall of India

3. F.Harary, Graph Theory, Addison-Wesley, 1969

Competencies of the course

(C1) Introduce graphs and subgraphs.

(C2) State Sperner's lemma.

(C3) Introduce connectivity and Euler tour with applications.

(C4) Introduce dual graphs.

(C5) State Euler's formula and Kuratowski's theorem.

MT 1C04TM GRAPH THEORY

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	1	3	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FIRST SEMESTER M.Sc. MATHEMATICS EXAMINATION

MT 1C04TM-GRAPH THEORY

Time : 3 hours Maximum marks : 75

PART A

(Answer any **five** questions. Each question carries **three** marks)

1. Define graph isomorphism with an example.
2. Show that there are eleven nonisomorphic simple graphs on four vertices.
3. Give the incidence and adjacency matrices of $K_{2,3}$
4. Prove that in any graph, the number of vertices of odd degree is even.
5. Prove that a connected graph is a tree if and only if every edge is a cut edge.
6. Define edge chromatic number and give an example of a graph that is 4-edge chromatic.
7. Define a clique and an independent set with an example.

(5 x 3 = 15)

PART B

(Answer any **six** questions. Each question carries **five** marks)

8. Prove that in a bipartite graph G with $\delta>0$, the number of vertices in a maximum independent set is equal to the number of edges in a minimum edge covering.
9. Prove that in a critical graph, no vertex cut is a clique.
10. Prove that for any positive integer k, there exists a k-chromatic graph containing no triangle.
11. Prove or disprove : K_5 is nonplanar.
12. Show that every induced subgraph of a complete graph is complete and every subgraph of a bipartite graph is bipartite.

13. Prove that a graph is bipartite if and only if it contains no odd cycle.
14. Prove that if G is a connected plane graph, then $\upsilon - \varepsilon + \varphi = 2$.
15. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented by each vertex of degree atleast two.
16. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

(6 x 5 = 30)

PART C

(Answer any **two** questions. Each question carries **fifteen** marks)

17. State and prove Sperner's lemma.
18. Prove :
    1. If e is a link of G, then $\tau(G) = \tau(G-e) + \tau(G.e)$.
    2. $\tau(K_n) = n^{n-2}$.
19. Prove that a non-empty connected graph is Eulerian if and only if it has no vertices of odd degree. Also prove that a connected graph has an Euler trail if and only if it has atmost 2 vertices of odd degree.
20. Prove :

    I) A graph G with $\upsilon \geq 3$ is 2-connected if and only if any two vertices of G are connected by atleast two internally disjoint paths.

    II) If G is 2-connected, then any two vertices of G lie on a common cycle.

(2 x 15 = 30)

SEMESTER I
MT1C05TM -COMPLEX ANALYSIS

Total Credits :4

Total Lecture Hours: 90

Aim

This course aims to provide an understanding of the basic concepts of complex analysis including nice properties enjoyed by derivatives and integrals of functions of a complex variable. It also provides an insight into the fact that how complex analysis can be used to evaluate real integrals.

Course Overview and Context

The course starts with analytic functions and conformal mapping. It then follows by the famous Cauchy's theorem and Cauchy's integral formula. It also describes about the singularities of a complex valued function. Calculus of residues are also included.

Syllabus Contents

Module 1 **(20 hours)**

Analytic functions as mappings.Conformality: arcs and closed curves, analytic functions in regions, conformal mapping, length and area.Linear transformations: linear group, the cross ratio, symmetry, oriented circles, family of circles. Elementary conformal mappings: the use of level curves, a survey of elementary mappings, elementary Riemann surfaces.

(Chapter 3 – sections 2, 3 and 4. of the text)

Module 2 **(20 hours)**

Complex Integration Fundamental theorem: line integrals, rectifiable arcs, line integrals as functions of arcs, Cauchy's theorem for a rectangle, Cauchy's theorem in a disk ,Cauchy's integral formula: the index of a point with respect to a cloud curve, the integral formula, higher derivatives.

(Chapter 4 – Sections 1 and 2. of the text.)

Module 3 **(25 hours)**

Local properties of analytical functions: removable singularities, Taylor's theorem, zeroes and poles, the local mapping, the maximum principle.The general form of Cauchy's theorem: chains and cycles, simple connectivity, homology, general statement of Cauchy's theorem, proof of Cauchy's theorem, locally exact differentiation, multiply connected regions.(Chapter 4 – Sections 3 and 4. of the text)

Module 4 **(25 hours)**

Calculus of Residues: the residue theorem, the argument principle, evaluation of definite integrals.Harmonic functions: definition and basic properties, the mean value property,Poisson's formula, Schwarz theorem, the reflection principle.

(Chapter 4 – Sections 5 and 6 of the text)

Learning Resources

Text Book

1. Lars V. Ahlfors, Complex Analysis, Third edition, McGraw Hill Internationals

References:

1. Chaudhary. B, The elements of Complex Analysis, Wiley Eastern.

2. Cartan. H (1973), Elementary theory of Analytic functions of one or several variable,
   Addison Wesley.
3. Conway .J.B, Functions of one Complex variable, Narosa publishing.
4. Lang. S, Complex Analysis, Springer.
5. H.A. Priestly, Introduction to Complex Analysis, Clarendon press, Oxford, 1990.

Competencies of the course

C1. Describing about conformal mappings

C2. Give awareness about the use of complex analysis to evaluate real integrals through the famous Cauchy's theorem

C3. Explain various singularities of complex functions

C4. Describe calculus of residues

MT1C05TM -COMPLEX ANALYSIS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FIRST SEMESTER M.Sc. MATHEMATICS EXAMINATION

MT1C05TM -COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A

(Answer any **five** questions. Each question carries **three** marks)

1. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2? Justify your answer.
2. Give a single valued analytic branch of $\sqrt{z}$ and establish the analyticity.
3. Define a cnformal mapping and give an example.
4. If $T_1(z) = (z+2)/(z+3)$, $T_2(z) = z/(z+1)$, find $T_1^{-1}T_2(z)$.
5. Prove that a function which is analytic and bounded in the whole plane must reduce to a constant.
6. State and prove the argument principle.
7. Define a chain and a cycle in a region Ω and give an example of a cycle which is homologous to zero in that region

(5 x 3 = 15)

PART B

(Answer any **six** questions. Each question carries **five** marks)

8. Find a conformal mapping of the complement of a line segment with end points ±1 onto the right half plane.
9. Describe the Riemann surface associated with the function $w = z^2$.
10. Define n(r,a), index of a point a with respect to a piecewise differentiable curve r. Prove that the index n(r,a) is zero in the unbounded region determined by r.
11. State and prove Morera's theorem.
12. Define an isolated singularity of an analytic function and say when it is an essential

singularity.

13. State and prove the maximum principle for analytic functions.
14. Prove that a region Ω is simply connected if and only if n(r,a) = 0 for every cycle r in Ω and all points a which do not belong to Ω.
15. If the functions $f_n(z)$ are analytic and not equal to 0 in a region Ω and if $f_n(z)$ converges to f(z), uniformly on every compact subset of Ω, then prove that f(z) is either identically zero or never equal to zero in Ω. Give an example to show that the first possibility can arise.
16. Prove that if f(z) is analytic at z_0 with $f'(z_0) \neq 0$, it maps a neighborhood of z_0 conformally and topologically onto a region.

(6 x 5 = 30)

PART C

(Answer any **two** questions. Each question carries **fifteen** marks)

17. I) Prove that the arithmetic mean of a harmonic function over concentric circles |z| = r is a linear function of log r and is constant if the function is harmonic in a disk.

    II) Given $f_n(z)$ is analytic in the region Ω_n and the sequence $\{f_n(z)\}$ converges to f(z) in a region Ω, uniformly on every compact subset of Ω. Prove that f(z) is analytic in Ω.
18. I) Prove that a region is obtained from a simply conncected region by removing m points has the connectivity m+1 and find a homology basis.

    II) State and prove the Residue theorem.
19. I) Prove that an analytic function which is not identically zero in a region has no zeros of infinite order .

    II) Show that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.
20. I) Show that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant.

    II) Define the cross ration (z_1,z_2,z_3,z_4). Prove that the cross ratio (z_1,z_2,z_3,z_4) is real if and only if the four points lie on a circle or a straight line.

(2 x 15 = 30)

Semester II

MT2C06TM -ABSTRACT ALGEBRA

Total Credits :4

Total Lecture Hours: 90

Aim

Algebra is a basic pillar of modern Mathematics**.**The objective of the course is to provide an introduction to algebraic structures and abstract reasoning. Abstract algebra gives to student a good mathematical maturity and enables to build mathematical thinking and skill.

Course overview and Context

The course begins with the discussion of polynomial rings, their applications and various methodsof factorization of polynomials over a field. This course also gives an introduction to extension fields and discusses about the different types of extension fields. Various other topics like Splitting field Sylow theorems are also included.

Syllabus Content

Module 1: **(25 hours)**

Direct products and finitely generated Abelian groups, fundamental theorem(without proof), Applications Rings of polynomials, factorisation of polynomials over a field.(Part II – Section 11) & (Part IV – Sections 22 & 23)

Module 2: **(25 hours)**

Introduction to extension fields, algebraic extensions, Geometricconstructions. Finite fields (Part VI – Section 29, 31 – 31.1 to 31.18, 32, 33)

Module 3: **(20 hours)**

Sylow's theorems (without proof), Applications of sylow theory

Automorphism of fields, the isomorphism extension theorem (proof of the theorem excluded)(Part VII Sections 36 & 37)

(Part X – Sections 48 & 49, (49.1 to 49.5)

Module 4: **(20 hours)**

Splitting fields, separable extensions, Galois theory(Part X – Sections 50, 51, 53 -53.1 to 53.6)

Learning Resources

Text Book

John B. Fraleigh, A First Course in Abstract Algebra, 7th edition,Pearson Education

References

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
2. Hungerford, Algebra, Springer
3. M. Artin, Algebra, Prentice -Hall of India, 1991
4. N. Jacobson, Basic Algebra Vol. I, Hindustan Publishing Corporation
5. P.B. Bhattacharya, S.K. Jain, S.R. Nagapaul, Basic Abstract Algebra, 2ndedition, Cambridge University Press, Indian Edition, 1997.

Competencies of the course:

C1. Describe the various concepts mentioned in the syllabus

C2. Introduce Polynomial rings

C3. Identify the Applications of Polynomial Rings

C4. Introduce Extension Fields

C5. Identify different extension fields.

C6. Explain Sylow Theorems

C7. Identify the applications of Sylow theorems

C8. Describe Automorphism of fields

MT2C06TM -ABSTRACT ALGEBRA

	Part A	**Part B**	**Part C**
Module I	1	3	1
Module II	2	2	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

Model Question Paper

Second Semester M.Sc. Degree Examination

MT2C06TM -Abstract Algebra

Time:3 hrs **Max. Marks: 75**

Part A

Answer any five questions. Each question carries 3 marks.

1. Prove that $a \in F$ is a zero of $f(x) \in F[x]$ if and only if $(x-a)$ is a factor of $f(x)$ in $F[x]$
2. Find all elements of $Z_2 \times Z_4$. Find the order of each of their elements. Is the group cyclic?
3. Explain with examples (i)Simple Extension (ii)Algebraic Extension (iii)Finite Extension-
4. Prove that a non-zero polynomial $f(x) \in F[x]$ of degree n can have at most n zeros in a field F.
5. Prove that the field C of complex numbers is an algebraically closed field
6. If $F \leq E \leq K$ where K is a finite extension field of the field F then prove that $\{K:F\} = \{K:E\}\{E:F\}$
7. Find the splitting field of x^4+x^2+1 over Q

(5 x 3 = 15)

Part B

Answer any six questions. Each question carries 5 marks.

8. Prove that for a prime number p, every group of order p^2 is abelian.
9. Prove that the group $Z_m X\ Z_n$ is isomorphic to Z_{mn}.if and only if m and n are relatively prime.
10. Prove that every field of characteristic zero is perfect.
11. An ideal $<p(x)> \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible over F.
12. State and prove the necessary and sufficient condition for $\alpha \in E$, where E is an extension field of F , to be transcendental over F.
13. Prove that a finite extension field E of a field F is an algebraic extension of F.

14. If F is a finite field of characteristic p, then prove that the map σ_p : F $\rightarrow$ F defined by $a\sigma_p = a^p$ for a $\in$ F is an automorphism of F and $F(\sigma_P) \approx Z_P$

15. Let f(x) be a polynomial with real coefficients such that f(a+ib) = 0. Then prove that f(a-ib) = 0

16. If K is a finite extension of E and E is a finite extension of F ie, F $\leq$ E $\leq$ K. Then prove that K is seperable over F if and only if K is seperable over E and E is seperable over F

(6 x 5 = 30)

Part C

Answer any two questions. Each question carries 15 marks.

17. a) State fundamental theorem of finitely generated abelian groups.

    b)State and prove Eisenstein criterion for irreducibility of polynomials

    c) State and prove Cauchy's theorem on the order of a subgroup.

18. a) Construct a field of four elements.

    b) Let E be an algebraic extension of a field F. Then prove that there exist a finite number of elements $\alpha_1, \alpha_2, ...\alpha_n$ in E such that E=F($\alpha_1, \alpha_2, ...\alpha_n$) if and only if E is a finite dimensional vector space over F.

19. a) State the Sylow theorems. Use them to show that no group of order 15 is simple.

    b) If F is a field and α, β are algebraic over F with degree(α, F)= n. Then prove that the map$\psi_{\alpha,\beta}$:F$(\alpha) \rightarrow F(\beta)$ defined by $(c_0 + c_1\alpha + ... + c_{n-1}\alpha^{n-1})\ \psi_{\alpha,\beta}$= $(c_0 + c_1\beta + ... + c_{n-1}\beta^{n-1})$

    for $c_i \in F$ is an isomorphism ofF(α) onto F(β) if and only if α and β are conjugate over F.

20. a)Prove that a field E, where F $\leq$ E $\leq$ $\overline{\text{F}}$ is a splitting field over F if and only if every automorphism of $\overline{\text{F}}$ leaving F fixed maps E onto itself and thus induces an automorphism of E leaving F fixed.

    b) Find the degree of Q$(\sqrt{2}, \sqrt{3})$ over Q.

(2 x 15 = 30)

Semester II

MT2C07TM -ADVANCED TOPOLOGY

Total Credits :4

Total Lecture Hours: 90

Aim

Topology is one of the major branches of modern Mathematics and this aim of this course is to provide a firm foundation in topology to enable the student to continue more advanced study in this area.

Course overview and Context

This course introduces the concepts like different characterisations of Normality, Product Topology, Embedding and Metrisation. This course also discusses about Nets and Filters, their convergence etc. A study of different types of compactness is also done.

Syllabus Content

Module 1: **(25 hours)**

UrysohnCharacterisation of Normality – TietzeCharacterisation ofNormality.(Chapter 7 Section-.3 and 4 of the text.)(Proof of 3.4, 4.4, and 4.5 excluded)Products and co-products: Cartesian products of families of sets – Product Topology – Productive properties.(Chapter 8 Section. 1, 2 & 3 of the text) (proof of 1.6 &1.7 excluded)

Module 2: **(15 hours)**

Embedding and Metrisation – Evaluation Functions in to Products,Embedding Lemma and Tychnoff Embedding, TheUrysohnMetrisationTheorem.(Chapter 9. Sec. 1, 2 & 3 of the text)

Module 3: **(25 hours)**

Nets and Filters: Definition and Convergence of Nets, Topology andConvergence of Nets, Filters and their Convergence, Ultra filters and Compactness.

(Chapter – 10 Sections -1, 2, 3 & 4 of the text)

Module 4: **(25 hours)**

Compactness: Variations of compactness – local compactness –compactification.Chapter 11. Section 1 (Proof of theorem 1.4 & 1.12 excluded),Section 3 Section 4(from 4.1 to 4.7) of the text

Learning Resources

Text Book:

K.D. Joshi, Introduction to General Topology, Wiley EasternLtd.

References:-

1. Munkres J.R, Topology-A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. J.L Kelley, General Topology, Van Nostrand, Reinhold Co., New York, 1995.
3. Stephen Willard, General Topology, Addison-Wesley.
4. Dugundji, Topology, Universal Book Stall, New Delhi.
5. George F Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, 1963.

Competencies of the course:

C1. Understand Product Topology and its properties.

C2. Explain Product Topology and Productive properties.

C3. Introduce Embedding and Metrisation

C4. Explain Compactness

C5. Analyse different variations of compactness

C6. Explain Nets, Filters and their convergence

C7. Describe Ultra filters and their Compactness

C8. Introduce variations of compactness

MT2C07TM -ADVANCED TOPOLOGY

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	1	3	1
Module III	2	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

Model Question Paper

Second Semester M.Sc. Degree Examination

MT2C07TM -Advanced Topology

Time:3 hrs **Max. Marks: 75**

Part A

Answer any five questions. Each question carries 3 marks.

1. Prove that projection functions are open.
2. Prove that a Topological product is T_0 if and only if each coordinate space is T_0
3. State and prove Urysohn Metrisation Theorem.
4. Define Filters and their convergence with example. Also explain ultra filters.
5. Define subnet of a net., show that if a net converges, so does any subset.
6. Prove that a metric space is compact iff it is countably compact.
7. Define (i) Sequential Compactness (ii) Countable Compactness (iii) Local Compactness

(5 x 3 = 15)

Part B

Answer any six questions. Each question carries 5 marks.

8. State and prove Tietze Extension Theorem
9. Prove that a product of Topological spaces is connected if and only if each coordinate space is connected
10. Prove that a topological space is a Tychonoff space iff it is embeddable into a cube.
11. Obtain necessary and sufficient conditions for a topological space to be embeddable in the Hilbrt cube
12. Prove that a Topological space is Hausdorff iff limits of all nets in it are unique.
13. Prove that every filter is contained in an ultra filter
14. Prove that a topological space is Hausdorff if and only if no filter on it converges to more than one point in it.

15. Prove that a topological space X is compact iff there exist a closed subbase $\mathcal{C}$ for X such that every subfamily of $\mathcal{C}$ having the finite intersection property has a non – empty intersection.
16. Prove that every countably compact metric space is second countable.

(6 x 5 = 30)

Part C

Answer any two questions. Each question carries 15 marks.

17. Prove that Metrisability is a Countably Productive Property
18. State and prove Embedding Lemma
19. Prove that a point $x \in X$ is a cluster point of the net $S: D \rightarrow X$ iff there exist a subnet of S converging to x in X.
20. (i)Prove that sequential compactness is a countably productive property.

    (ii)Prove that every continuous real valued function on a countably compact space is bounded and attains its extrema.

(2 x 15 = 30)

Semester II

MT2C08TM -ADVANCED COMPLEX ANALYSIS

Total Credits :4

Total Lecture Hours: 90

Aim

Complex analysis is a useful tool for solving a wide variety of problems inmany branches of mathematics and engineering science — the analysis of ac electrical circuits, the solution of linear differential equations with constant coefficients, the representation of wave forms, and so on.The aim of this course is to study the theoretical foundations of complex variable theory and to develop skills in the application of this theory to particular problems.

Course overview and Context

This course starts with a brief discussion on the theory of power series, power series expansions of functions and infinite products. A study of some special functions like Gamma Function, Riemann Zeta Function is done in the course. This course also includes Harmonic functions, Dirichlet Problem, Elliptic functions and Weirstrass theory.

Syllabus Content

Module 1: **(25 hours)**

Elementary theory of power series: sequences, series, uniform convergence,power series, Abel's limit theorem. Power series expansions: Weierstrass' theorem, the Taylor's series, the Laurent's series Partial fractions and factorisation: partial fractions, infinite products, canonical products, the gamma functions.(Chapter 2, Section 2 - Chapter 5, Sections 1, 2.1 to 2.4 of the text)

Module 2: **(25 hours)**

Entire functions: Jenson's formula, Hadamard's theorem (without proof)the Riemann zeta function: the product development, extension of x to thewhole plane, the functional equation,

the zeroes of zeta function.Normal families: Equi continuity, normality and compactness, Arzela's theorem (without proof)
(Chapter 5 - Sections 3, 4, 5.1,5.2, and 5.3 of the text)

Module 3: (20 hours)

The Riemann mapping theorem: statement and proof, boundary behavior, use of reflection principle, analytic arcs. Conformal mappings of polygons: the behavior of an angle, the Schwarz- ChristoffelformulaI (Statement only).A closer look at harmonic functions: functions with mean value property, Harnack's principle.The Dirichlet problem: sub harmonic functions, solution of Dirichlet problem(statement only)(Chapter 6 Section 1, 2.1, 2.2, 3, 4.1 & 4.2 of the text)

Module 4: (20 hours)

Elliptic functions: simply periodic functions, representation of exponentials,the Fourier development, functions of finite order Doubly periodic functions: The period module, unimodular transformations, the canonical basis, general properties of elliptic functions. The Weirstrass theory: the Weierstrass function, the functions x (y) and s (y),the differential equation. Analytic continuation: the Weierstrass theorem, Germs and Sheaves, sections and Riemann surfaces, analytic continuation along arcs, homotopic curves.(Chapter 7 Sections 1, 2, 3.1, 3.2, 3.3Chapter 8 Sections 1.1 to 1.5 of the text)

Learning Resources

Text Book: Lars V. Ahlfors, Complex Analysis, Third edition, McGraw Hill Internationals

References:

1. Chaudhary. B, The elements of Complex Analysis, Wiley Eastern.
2. Cartan. H (1973), Elementary theory of Analytic functions of one or several variable, Addison Wesley.

3. Conway .J.B, Functions of one Complex variable, Narosa publishing.
4. Lang. S, Complex Analysis, Springer.
5. H.A. Priestly, Introduction to Complex Analysis, Clarendon press, Oxford, 1990

Competencies of the course:

C1. Explain the theory of Power Series
C2. Introduce Partial fractions and factorization and infinite products.
C3. Define Gamma Function and Reimann Zeta Function.
C4. Describe Normal Families.
C5. Explain the Riemann mapping theorem.
C6. Describe the Dirichlet Problem.
C7. Define Elliptic Functions.
C8. Introduce Weirstrass theory.
C9. Explain Analytic Continuation.

MT2C08TM -ADVANCED COMPLEX ANALYSIS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

Model Question Paper

Second Semester M.Sc. Degree Examination

MT2C08TM –Advanced Complex Analysis

Time:3 hrs **Max. Marks:75**

Part A

Answer any five questions. Each question carries 3 marks.

1. Prove the formula $\Gamma z \quad \Gamma(1 - z) = \dfrac{\pi}{\sin \pi z}$.
2. Express $\sin \pi z$ in the form of canonical product.
3. Prove that for $\sigma = \mathrm{Re}\ s > 1$, $\dfrac{1}{\zeta(s)} = \prod_{n=1}^{\infty}(1 - p_n^{-s})$, where $p_1, p_2, \ldots$ is an ascending sequence of primes.
4. Prove that ζ function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at s=1 with residue one.
5. If v_1 and v_2 are subharmonic then v = max $\{v_1, v2\}$ is subharmonic.
6. Show that a continuous function u(z) which satisfies the mean value property is necessarily harmonic.
7. Prove that an elliptic function without poles is a constant.

(5 x 3 = 15)

Part B

Answer any six questions. Each question carries 5 marks.

8. State and prove Legendre's duplication formula.
9. Prove that the infinite product $\prod_{1}^{\infty}(1 + a_n)$, with $(1+a_n) \neq 0$ converges simultaneously with the series $\sum_{1}^{\infty} \log(1 + a_n)$ whose terms represent the values of the principal branch of logarithm.
10. Prove the functional equation $\varsigma(s) = 2^s (\pi)^{s-1} \sin \dfrac{\pi s}{2} \Gamma(1-s)\ \varsigma(1-s)$

11. A family $\Im$ is normal if and only if its closure $\overline{\Im}$ with respect to the distance function ρ is compact.
12. State and prove Harnack's Principle.
13. Prove that a continuous function v(z) is subharmonic in a region Ω if and only if it satisfies the inequality $v(z_0) \leq \frac{1}{2\pi}\int_0^{2\pi} v(z_0 + re^{\theta})\, d\theta$ for every disc $|z - z_0| \leq r$ contained in Ω.
14. Prove that any two basis of the same module are connected by a unimodular transformation.
15. Prove that $[\wp'(z)]^2 = 4[\wp(z)]^3 - g_2\wp(z) - g_3$ where
16. Prove the Legendre's relation $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$

(6 x 5 = 30)

Part C

Answer any two questions. Each question carries 15 marks.

17. a)State and prove Weierstrass Theorem

    b)Define entire functions and meromorphic functions. Give examples.Prove that every function which is meromorphic in the whole plane is the quotient of two entire functions.
18. a)State and Prove Jensen's Formula.

    b)Prove that a family $\Im$ is totally bounded if and only if to every compact set $E \subset \Omega$ and every $\varepsilon > 0$, it is possible to find $f_1, f_2, f_3, \ldots \in \Im$ such that every $f \in \Im$ satisfies $d(f, f_j) < \varepsilon$ on E for some f_j.
19. State and Prove the Riemann Mapping Theorem.
20. Prove that (i) A non- constant elliptic function has equally many poles as it has zeros.

    (ii)The zeros $a_1, a_2, \ldots, a_n$ and poles $b_1, b_2, \ldots, b_n$ of an elliptic function satisfy $a1+a2+\ldots+an = b_1+b_2+..+b_n \pmod{M}$

(2 x 15 = 30)

Semester II

MT2C09TM -PARTIAL DIFFERENTIAL EQUATIONS

Total Credits :4

Total Lecture Hours: 90

Aim:

The study of Partial Differential Equations started in the 18^{th} century. Partial differential equations can be used to describe a wide variety of phenomena such as sound, heat , electrostatistics, electrodynamics and fluid mechanics. The aim of this course is to teach how to find the solution of PDE's and interpret the resulting solutions.

Course overview and Context

This course discusses about solution of first order and second order partial differential equations and different methods of solving them. The course also includes the solution of Linearhyperbolic equations. A brief discussion on solutions of Laplace equation and Families of equipotential surfaces is also done.

Syllabus Content

Module:-1. **(25 hours)**

Methods of solutions of dx/p = dy/Q = dz/R. Orthogonal trajectories of asystem of curves on a surface. Pfaffian differential forms and equations.Solution of Pfaffian differential equations in three variables Partial differentialequations. Orgins of first order partial differential equation .Cauchy's problem forfirst order equation.Linear equations of first order.Integral surfaces passingthrough a given curve.Surfaces orthogonal to a given system of surfaces.
(Sections 1.3 to 1.6 & 2.1 to 2.6 of the text)

Module:-2. **(25 hours)**

Nonlinear partial differential equation of the first order .Cauchy'smethod of characteristics. Compatible systems of first order equations .CharpitsMethod.Special types of first order equations.Solutions satisfying given conditions. Jacobi's method.(Section 2.7 to 2.13 of the text)

Module:-3 **(20 hours)**

The origin of second order equations.Linear partial differential equations with constant coefficients. Equations with variable coefficients., Characteristic curves ofsecond order equations .(Section 3.1, 3.4, 3.5, 3.6 of the text)

Module:-4. **(20 hours)**

The solution of linear Hyperbolic equations. Separation of variables.Non linear equations of the second order .Elementary solutions of Laplace equation.Families of equipotential surfaces.Boundary value problems.(Section 3.8, 3.9 ,3.11 ,4.2, 4.3,4.4 of the text)

Learning Resources

Text Book:-Ian Sneddon, Elements of partial differential equations, McGraw Hill Book Company**.**

References:-

1 .Phoolan Prasad and Renuka Ravindran, Partial differential Equations, New Age International (p) Limited

2 K SankaraRao, Introduction to Partial Differential Equations, Prentice-Hall of India

3 E.T Copson , Partial differential equations , S .Chand & Co

Competencies of the course:

C1. Describe the method of solution of the differential equation dx/P = dy/Q = dz/R.

C2. Introduce Orthogonal trajectories

C3. Introduce Pfaffian differential equations and their solutions.

C4. Describe the Origin of Partial Differential Equations.

C5. Explain Cauchy's problem for first order equation.

C6. Introduce Linear equations of first order .

C7. Obtain the solution of Nonlinear partial differential equations

C8. Understand Charpit's method.

C9. Explain Jacobi's method of solution.

C10. Describe origin of second order equations.

C11. Discuss the solution of equations with variable coefficients.

C12. Obtain the solution of linear Hyperbolic equations.

C13. Describe elementary solutions of Laplace equation.

MT2C09TM -PARTIAL DIFFERENTIAL EQUATIONS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

Model Question Paper

Second Semester M.Sc. Degree Examination

MT2C09TM -Partial Differential Equations

Time:3 hrs **Max. Marks: 75**

Part A

Answer any five questions. Each question carries 3 marks.

1. Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$
2. Eliminate the arbitrary function f from the equation $f(x^2+y^2+z^2, z^2-2xy)=0$.
3. Show that the equations $xp-yq = x$, $x^2p+q = xz$ are compatible
4. Find a complete integral of $p^2q^2+x^2y^2 = x^2q^2(x^2+y^2)$.
5. If $z = f(x^2-y)+g(x^2+y)$ where the functions f and g are arbitrary , prove that

   $$\frac{\partial^2 z}{\partial x^2} - \frac{1}{x}\frac{\partial z}{\partial x} = 4x^2 \frac{\partial^2 z}{\partial y^2}$$
6. Classify the equation $U_{xx}+U_{yy} = U_z$
7. By separating the variables show that the solution of the equation $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0$ is of the form $\psi = e^{\pm i\nu x \pm \lambda z \pm \mu y}$

(5 x 3 = 15)

Part B

Answer any six questions. Each question carries 5 marks.

8. Verify that the differential equation $(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz = 0$ is integrable and find its primitive
9. Find the equation of the system of surfaces which cut orthogonally the cones of the system $x^2+y^2+z^2 = cxy$
10. Find the solution of the equation $z = \frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ which passes through the x- axis.
11. Prove that the necessary and sufficient condition that a surface be an integral surface of a partial differential equation is that at each point its tangent should touch the elementary cone of the equation.

12. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$
13. Find a particular integral of the equation $(D^2 - D')z = 2y - x^2$
14. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form
15. Solve the wave equation r = t by Monge's method
16. By separating the variables , find the solution of one dimensional diffusion equation

    $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ which tends to 0 as $t \to \infty$.

    (6 x 5 = 30)

Part C

Answer any two questions. Each question carries 15 marks.

17. a) Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersections with the family of planes parallel to z =0.

    b) Show that a necessary and sufficient condition that there exist between two functions u(x, y) and v(x, y) a relation F(u,v) = 0 not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$
18. a) Solve using Jacobi's method $2(z+xp+yq) = yp^2$.

    b)Find a complete integral of the partial differential equation $(p^2+q^2)x = pz$ and deduce the solution which passes through the cuve x= 0, $z^2= 4y$.
19. a)Solve the equation $r-s+2q-z = x^2y^2$

    b) Reduce the equation $y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it.
20. a)Solve the Dirichlet Problem for a sphere

    b)Define the family of equipotential surface and obtain the condition for f(x, y, z) = c to be equipotential.

(2 x 15 = 30)

Semester II

MT2C10TM - MEASURE THEORY AND INTEGRATION

Total Credits : 4

Total Lecture Hours: 90

Aim: Riemann Integral has several limitations .The aim of this course is to develop a more abstract concept of measure, and use it to construct the more Lebesgue integral, and to nvestigate its properties.This course also intends to develop the ability of students to work with abstract ideas.

Course Overview And Context:

Measure theory includes topics like measure spaces, measurable functions, signed measures, outer measure,measurable sets. The theory of integration on measure spaces including the classical convergence theorems, various modes of convergence, product integration is also included. Applications with emphasis on Lebesgue measure on R and the Lebesgue integral is also discussed

Syllabus Content:

Module 1: **(20 hours)**

Lebesgue measure: introduction, outer measure, measurable sets andLebesgue measure, & non-measurable sets, measurable functions.(Chapter 3 - Sec. 1 to 5. of Text 1)

Module 2: **(23 hours)**

Lebesgue integral: the Riemann integral, The Lebesgue integral of a boundedfunction over a set of finite measures, the integral of a non-negative function,the general Lebesgue integral, differentiation of monotone functions(Chapter 4 - Sec. 1 – 4. of Text 1 Chapter 5 - Sec. 1. of Text 1)

Module 3: **(22 hours)**

Measure and integration: measure spaces, measurable functions, Integration,general convergence theorems, signed measures, the Radon-Nikodym theorem, outer measure and measurability, the extension theorem.(Chapter 11 - Sec. 1 to 6 of Text 1Chapter 12 - Sec. 1& 2 of Text 1)

Module 4: **(25 hours)**

Convergence: convergence in measure, almost uniform convergence, measurability in a product space, the product measure and Fubini's theorem.(Chapter 8 - Sec. 7.1 & 7.2 of Text 2 Chapter 10 - Sec. 10.1& 10.2 of Text 2)

Learning Resources

Text Books

Text 1: H.L. Royden, Real Analysis, Third edition, Prentice Hall of India Private Limited.

Text 2: G. de Barra, Measure Theory and Integration, New Age International (P) Linnilect Publishers

References:-

1. Halmos P.R, Measure Theory, D.van Nostrand Co.

2. P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd., New Delhi, 1986(Reprint 2000).

3. R.G. Bartle, The Elements of Integration, John Wiley & Sons, Inc New York, 1966.

4. Inder K Rana, An Introduction to Measure and Integration, Narosa Publishing House, 1997.

Competencies of the course:

C1. Introduce Lebesgue Measure

C2. Explain outer measure measurable sets and measurable functions.

C3. Describe differentiation of monotone functions.

C4. Obtain the Lebesgue integral of a bounded function over a set of finite measures.

C5. Introduce Measure spaces, Measurable functions

C6. Explain outer measure and measurability.

C7. Introducesigned measures.

C8. Define convergence in measure

C9. Describe almost uniform convergence.

C10. Explain measurability in a product space and the product measure

MT2C10TM - MEASURE THEORY AND INTEGRATION

	Part A	**Part B**	**Part C**
Module I	1	3	1
Module II	2	2	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

MSC MATHEMATICS THIRD SEMESTER

CORE COURSE MT2C10TM – MEASURE THEORY AND INTEGRATION

Time: 3hours **Max.Marks:75**

PART A

(Answer any 5 questions. Each Question Carries 3 marks)

1. Define the Lebesgue outer measure m*(A) of a subset A of ℝand use the definition to show that the outer measure of a countable set is zero
2. If *f*is a measurable function and if $f = g$ a.e, show that *g*is measurable.
3. State the bounded convergence theorem
4. If f is integrable show that |f| is integarble. Is the converse true? Justify.
5. If f is a nonnegative measurable function on a measure space (X, β, μ) show that $\int f d\mu = 0$ = 0 if and only if *f*= 0 a.e
6. Define a positive set with respect to a signed measure υ .Show that the union of a countable collection of positive sets is positive.
7. If $f_n \to f$ in measure show that there is a subsequence $\{f_{nk}\}$which converge to *f*a.e.

(5 x 3 = 15)

PART B

(Answer any 6 questions. Each Question Carries 5 marks)

8. If (X, S, μ) and (Y, τ, v) are σ -finite measure spaces, define the product measure $\mu \times \upsilon$on $S \times \tau$
9. Show that the interval of the form (a, ∞) is measurable and use it to show that every Borel set is measurable.
10. If *f*is measurable and *B*is a Borel set, show that $f^{1}(B)$ is measurable

11. State Fatou's' lemma. Give an example to show that we may have strict inequality in the Fatou's lemma.
12. If f and g are integrable over E, show that $f+g$ is integarble over E and $\int f+g=\int f+\int g$
13. Let f be an integrable function on a measure space (X,β,μ). Show that given $\varepsilon > 0$ there exists $\delta > 0$ such that for each measurable set E with $m(E)<\delta$, we have $\left|\int_E f\right|<\varepsilon$
14. If v is a signed measure in a measurable space (X,β) then show that there is a positive set A and a negative set B such that $A \cap B = \Phi$ and $X = A \cup B$.
15. If $f_n \to f$ in measure and if $g_n \to g$ in measure then show that $f_n + g_n \to f+g$ in measure.
16. By integrating $e^{-y}\sin 2xy$ with respect to x and y, show that $\int_0^\infty \frac{e^{-y}\sin 2xy}{y} = \frac{1}{4}\log 5$

(6 x 5 = 30)

PART C

(Answer any 2 questions. Each question carries 15 marks)

17. Show that the outer measure m*(A) of a subset A of R satisfies the following properties.
    a. $m^*(E) \geq 0$ for every $E \subset R$
    b. $m^*(I) = l(I)$ where I is any interval.
    c. $m^*(E) = m^*(E+x)$ for every $E \subset R$ and $x \in R$
    d. $m^*(\cup E_n) \leq \sum m^*(E_n)$ where $E_n \subset R$ for every n = 1,2,3,…
    e. If $A \subseteq B$ show that $m^*(A) \leq m^*(B)$ for every subset A and B of R

18. A. Let *f* be a bounded function defined on a measurable set *E* with $mE < \infty$. Show that *f* is measurable if and only if $\inf_{f \le \psi} \int_E \psi(x)dx = \sup_{\varphi \le f} \int_E \varphi(x)dx$

    B. State Fatou's lemma and use it to prove the Monotone Convergence theorem. Give an example to show that the Monotone Convergence theorem will not hold for a decreasing sequence of non negative measurable functions.

19. A. State and prove the Lebesgue convergence theorem for measurable functions

    B. Let *f* be an increasing real valued function on the interval [a,b]. Then show that *f* is differentiable almost everywhere, the derivative f' is measurable and $\int_a^b f'(x)dx \le f(b) - F(a)$

20. State and prove the Radon - Nikodym theorem.

(2 x 15 = 30)

Semester – III

MT3C11TM - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

Credit-4

Total Lecture hours- 90

Aims:

It discusses the calculus of functions of several variables. Differential calculus is unified and simplified with the aid of linear algebra. It includes chain rules for scalar and vector fields, and applications to partial differential equations and extremum problems. Integral calculus includes line integrals, multiple integrals, and surface integrals, with applications to vector analysis.

Course Overview And Context:

This course starts by introducing Weirstrass theorem and other forms of Fourier Series. It also includes Multivariable Differential Calculus which includes directional derivatives, application of complex valued functions, the matrix of a linear function etc. A brief introduction of implicit functions and extremum problems are explained. Then Integration of differential forms are discussed.

Syllabus Content:

Module 1: (20 hours)

The Weirstrass theorem, other forms of Fourier series, the Fourier integral theorem, the exponential form of the Fourier integral theorem, integral transforms and convolutions, the convolution theorem for Fourier transforms.

(Chapter 11 Sections 11.15 to 11.21 of Text 1)

Module 2: (20 hours.)

Multivariable Differential Calculus , The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of complex- valued functions, the matrix of a linear function, the Jacobian matrix, the chain rate matrix form of the chain rule.(Chapter 12 Sections. 12.1 to 12.10 of Text 1)

Module 3: (25 hours.)

Implicit functions and extremum problems, the mean value theorem for differentiable functions, a sufficient condition for differentiability, a sufficient condition for equality of mixed partial derivatives, functions with non-zero Jacobian determinant, the inverse function theorem (without proof), the implicit function theorem (without proof), extrema of real-valued functions of one variable, extrema of real- valued functions of several variables.Chapter 12 Sections-. 12.11 to 12.13. of Text 1 Chapter 13 Sections-. 13.1 to 13.6 of Text 1

Module 4: (25 hours.)

Integration of Differential Forms Integration, primitive mappings, partitions of unity, change of variables, differential forms, Stokes theorem (without proof) Chapter 10 Sections. 10.1 to 10.25, 10.33 of Text 2

Learning Resources

Textbooks:

Text 1: Tom APOSTOL, Mathematical Analysis, Second edition, NarosaPublishing House.
Text 2: WALTER RUDIN, Principles of Mathematical Analysis, Third edition –International Student Edition.

References:

1. Limaye Balmohan Vishnu, Multivariate Analysis, Springer.
2.Satish Shirali and Harikrishnan, Multivariable Analysis, Springer.

Competencies of the Course:

C_1 Explain Weirstrass Theorem

C_2 Describe other forms of Fourier Series

C_3 Introduce multivariable differential Calculus

C_4 Describe application of complex valued function

C_5 Explain the matrix of a linear transformation

C_6 Define implicit functions

C_7 Define Jacobian Determinant

C_8 Explains integrations of differential forms

C_9 Define Stoke's Theorem

MT3C11TM - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	2	2	1
Module IV	1	3	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

THIRD SEMESTER M.SC MATHEMATICS EXAMINATION

MT3C11TM - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

Time: 3hours **Max.Marks:75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Write the Fourier series generated by $f \in L([0,P)]$ with period p. Also write the formula for the Fourier coefficients.
2. Show by an example that a function can have a finite directional derivative $f'(c,u)$ for every u but may fail to be continous at c.
3. State and prove Weierstrass approximation theorem
4. Show that $B(p,q)$=$B(p,q) = \frac{\sqrt{p}\sqrt{q}}{\sqrt{p+q}}$
5. Let $f : R^2 \to R^3$ defined by the equation *f(x,y)= (sin x cos y, sin x,sin y, cosx cosy)*.Determine the Jacobian matrix $Df(x,y)$.
6. Let $r(t)= (a \cos t, b \sin t)$, $0 \le t \le 2\pi$, a,b >0.Find $\int_r xdy$ and $\int_r ydx$.
7. If f is differentiable c with total derivative T_c, show that $T_c(u)= f'(c,u)$.

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each question carries 5 marks)

8. For every $f \in \zeta(T^K)$, prove that L(f)= L'(f)
9. Let A be an open subset of R^n and assume that $f : A \to R^n$ has continous partial derivatives Df_i on A. If $J_f(x) \neq 0$ for all x in A, prove that f is an open mapping.
10. Suppose $w = \sum_1 b_1(x) dx_1$ is the standard representation of a k form w in an open set $E \subset R^n$ .Prove that if $w=0$ in E, then $b_1(x)=0$ for every increasing k index I and for every $x \in E$.

11. State and prove mean value theorem for differential calculus
12. Prove that if f is differentiable at c, then f is continous at c.
13. Show that $\log\left|\sin\frac{x}{2}\right| = -\log 2 - \sum_{n=1}^{\infty}\frac{\cos nx}{n}$ if $x \neq 2K\pi$, K is an integer.
14. Suppose T is a ζ^{-1} mapping of an open set $E \subset R^n$ into an open set $V \subset R^m$, ϕ is a K surface in E, and w is a k- form in V.Show that $\int_{TQ} w = \int_{\phi} w_T$.
15. Let $R = (-\infty, \infty)$. Assume that $f \in L^2(R)$ and $g \in L^2(R)$ .Show that the convolution integral $h(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt$ exists for each $x \in R$ and h is bounded on R.
16. Compute the gradient vector $\nabla f(x, y)$ at those points $(x,y) \in R^2$ where if exists for $f(x, y) = xy\sin\frac{1}{x^2 + y^2}$ if $(x, y) \neq (0,0), f(0,0) = 0$.

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. State and prove the convolution theorem for Fourier Transforms.
18. Prove that if both partial derivatives $D_r f$ and $D_k f$ exist in an n ball (c, δ) and if both are differential at c, then $D_{r,k}f(c)= D_{k,r}f(c)$.
19. State and prove the chain rule for differentiation.
20. a)If $x(r,\theta) = r\cos\theta, y(r,\theta) = r\sin\theta,$ show that $\frac{\partial(x, y)}{\partial(r,\theta)} = r$.

    b)If $x(r,\theta\phi) = r\cos\theta\sin\phi, y(r,\theta\phi) = r\sin\theta\sin\phi, z = r\cos\phi$,

    show that $\frac{\partial(x, y, z)}{\partial(r,\theta,\phi)} = -r^2\sin\phi.$

(2 x 15 = 30)

Semester – III

MT3C12TM - FUNCTIONAL ANALYSIS

Credit-4

Total Lecture hours- 90

Aims:

Functional analysis plays an increasing role in the applied sciences as well as in Mathematics itself. This intended to familiarize the reader with the basic concepts, principles and methods of functional analysis and its applications.

Course Overview And Context:

This course starts by introducing vector space, normed space and the properties of normed space. It explains about linear functionals and linear operators.It defines Hilbert space and further properties of inner product space.It gives a brief description about orthogoonal complements and direct sums.It also deal with adjoint operators and reflexive spaces.

Syllabus Content:

Module 1 (20 hours)

Vector Space, normed space. Banach space, further properties of normed spaces,finite dimensional normed spaces and subspaces, compactness and finite dimension,linear Operators, bounded and continuous linear operators.
(Chapter 2 - Sections 2.1 – 2.7 of the text)

Module 2 (20 hours)

Linear functionals, linear operators and functionals on finite dimensional spaces,normed spaces of operators. dual space, inner product space. Hilbert space, furtherproperties of inner product space.
(Chapter 2 - Section 2.8 to 2.10, chapter 3 - Sections 3.1 to 3.2 of the text)

Module 3 **(25 hours)**

Orthogonal complements and direct sums, orthonormal sets and sequences, seriesrelated to orthonormal sequences and sets, total orthonormal sets and sequences.representation of functionals on Hilbert spaces, Hilbert adjoint operators, Self adjoint,

(Chapter 3 - Sections 3.3 to 3.6, 3.8 to 3.10 of the text)

Module 4 **(25 hours)**

Zorn's lemma, Hahn- Banach theorem, Hahn- Banach theorem for complex vectorspaces and normed spaces, adjoint operators, reflexive spaces, category theorem (Statement only), uniform boundedness theorem

(Chapter 4 – Sections 4.1 to 4.3, 4.5 to 4.7 of the text)

Learning Resources

Textbooks:

Text Book: Erwin Kreyszig, Introductory Functional Analysis with applications,
John Wiley and sons, New York

References:

1. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York , 1963.
2. Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi: 1989
3. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt. Ltd, Madras, 1994
4. Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut: 1995-96
5. M. Thamban Nair, Functional Analysis, A First Course, Prentice – Hall of India Pvt. Ltd, . 2008
6. Walter Rudin, Functional Analysis, TMH Edition, 1974.

Competencies of the Course:

C1. Explains Vector space
C2. explains Normed space and its properties
C3. Describes Banach Space
C4. Introduce linear operators
C5. Describe Linear functionals
C6. Explain orthognal complements
C7. Explains Hilberts Spaces
C8. Explains Zorns Lemma
C9. Explains Hahn Banach Theorem
C10. Explains Uniform boundedness theorem

MT3C12TM - FUNCTIONAL ANALYSIS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

THIRDSEMESTER M.SC MATHEMATICS EXAMINATION

CORE COURSE MT3C12TM - FUNCTIONAL ANALYSIS

Time: 3hours **Max.Marks:75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Prove or disprove: The set N of natural numbers considered as a subset ofthe real line with the discrete metric is bounded.
2. Let d and d' be two metrics on a non empty set X. If d is stronger thand' then prove that every open subset of X with respect d' is also open withrespect to d.
3. Show that there are divergent sequences (x_n) in l^psuch that the sequence$(x_n(j))$ converges in the scalar field for each $j = 1,2.......$
4. Prove or disprove: All essentially bounded measurable functions are bounded.
5. If $X = c_\infty$ with the norm $\|\cdot\|_\infty$and $E = \{x \in X : |x(j)| \leq 1/j \ ; j = 1,2......\}$prove that E spans X but $E^o = \emptyset$.
6. Show that a linear map from a normed space X into a normed space maybe closed without it being continuous.
7. Show that the map $f : X \to K$ defined by $f(x) = \sum_{j=1}^{\infty} x(j)$ for $x = (x(1), x(2)....)) \in X$ is discontinuous, where X is with the norm $\|\cdot\|_2$.

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each question carries 5 marks)

8. Prove that among all the l^p, $1 \leq p \leq \infty$ spaces, only l^2 an inner product space.
9. Let X be an inner product space and $\{x_1, x_2\}$ an orthogonal set in X. Provethat $\|x_1 + x_2\|2 = \|x_1\|^2 + \|x_2\|^2$.

10. Let<...... >be an inner product on a linear space X. Let $x \in X$ Prove that< x, y >= 0 for all $y \in X$ if and only if $x = 0$.
11. Define a complete metric space and prove that the property of completenessof a metric space may not be shared by an equivalent metric.
12. State Korovkin's theorem and deduce that the set of polynomials is dense in$C[a; b]$ with the sup metric.
13. Let $x \in L^1([-\pi,\pi])$. Show that $\tilde{x}(n) \to 0$ as $n \to \infty$
14. Let X_1 be a closed subspace and X_2 be a finite dimensional subspace of anormed space X. Then prove that $X_1 + X_2$is closed in X.
15. Let X and Y be normed spaces and $F : X \to Y$ be a linear map. Show that$F$ is continuous if the zero space $Z(F)$ is closed in X and the linear map and $\tilde{F} : X/Z(F) \to Y$ defined by $\tilde{F}$ $(x + Z(F)) = F(x)$ is continuous.
16. Let $H = L^2([0; 1])$ be the Hilbert space with the inner product $< x, y >= \int_0^1 x(t)\,\overline{y(t)}dt$ for $x; y \in L^2([0, 1]$. Show that $\{1, \sqrt{2} \cos \pi t, \sqrt{2} \cos 2\pi t.....\}$forms an orthonormal basis for H.

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. (a) Let x be a continuous K- valued function on $[-\pi, \pi]$ such that $x(\pi) = x(-\pi)$. Prove that the sequence of arithmetic means of the partial sumsof the Fourier series of x converges uniformly on $[-\pi, \pi]$.

    (b) Let E be a measurable subset of R with finite measure. If $1 \leq p < r < \infty$ then prove that $L^r(E) \subset L^p(E)$ and the inclusion function from$L^r(E) \to L^p(E)$ is continuous.

    (c) Show by an example that this is not the case when $m(E) = \infty$
18. Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H. Prove that the followingconditions are equivalent.

(i) $\{u_\alpha\}$ is an orthonormal basis for H,

(ii) For every $x \in H$, the set $\{\{u_\alpha\} :< x , u_\alpha > \neq 0\}$ is countable and if $\{\{u_\alpha\} :< x , u_\alpha > \neq 0\} = \{u_1, u_2, ... \}$, then $x = \sum_n \langle x, u_n \rangle \; u_n$.

(iii) For every $x \in H$ we have $\|x\|^2 = \sum_n |\langle x, u_n \rangle|^2$ where $\{u_1, u_2, ...\} = \{\{u_\alpha\} :< x, \{u_\alpha\} > \neq 0\}$,

(iv) Span $\{u_\alpha\}$ is dense in H,

(v) If $x \in H$ and $< x, \{u_\alpha\} >= 0$ for all $\{u_\alpha\}$ then $x = 0$:

19. Let X be normed space over K, E be a non empty open convex subset of $X$and Y be a subspace of X such that $E \cap Y = \emptyset$. Prove that there exists an$f \in X'$ such that $f(x) = 0$ for every $x \in Y$ but $Re\, f(x) \neq 0$ for every $x \in Y$and deduce that if E_1 and E_2 are two non empty disjoint convex subsets of$X$, where E_1 is open in X, then there exists an $f \in X'$ and $c \in$R such that$Re\, f(x_1) < c < Re\, f(x_2)$ for all $x_1 \in E_1$ and $x_2 \in E_2$
20. (a) Let X be a Banach space, Y be a normed space and F be a subset of$BL(X; Y)$ such that for each $x \in X$, the set $\{F(x) : F \in F\}$ is boundedin Y . Prove that sup$\{\|F\|; F \in F\} < \infty$ and deduce that if (F_n) isa sequence in $BL(X; Y)$ such that the sequence $(F_n(x))$ converges inY for every $x \in X$ to say, $F(x)$ then prove that $\|F\| \leq \liminf_{n\to\infty} \leq$sup$\{\|F_n\|, n = 1,2.......\} < \infty$

    (b) Show by examples both the results in (a) need not work if X not aBanach space.

(2 x 15 = 30)

Semester – III

MT3C13TM - DIFFERENTIAL GEOMETRY

Credit-4

Total Lecture hours- 90

Aims:

Differential geometry is a mathematical discipline that uses the techniques of differential calculus , integral calculus, linear algebra and multilinear algebra to study problems in geometry. To provide the student with the concept and the understanding in Space curves, Geodesics, Intrinsic and Non-Intrinsic properties of a surfaces. Di§erential geometry is the study of curves and surfaces by means of calculus (both di§erential and integral). It is one of the oldest and most highly developed branches of mathematics, and remains central to modern pure mathematics as well as to much of theoretical physics. This course is a study of the curvature properties of curves and surfaces in two and three dimensions.

Course Overview And Context:

This course starts by introducing graphs and level sets. It also defines the Gauss map, geodesics and parallel transport. A brief introduction about Weingarten map are explained. Then curvature of surfaces and parametrized surfaces are also explained.

Syllabus Content:

Module 1 **(15 hours)**

Graphs and level sets, vector fields, the tangent space, surfaces, vector fields on surfaces, orientation.(Chapters 1 to 5 of the text)

Module 2: **(20 hours)**

The Gauss map, geodesics, Parallel transport, (Chapters 6, 7 & 8 of the text)

Module 3: **(25 hours)**

The Weingarten map, curvature of plane curves, Arc length and lineIntegrals (Chapters 9, 10 & 11 of the text)

Module 4: (30 hours)

Curvature of surfaces, Parametrized surfaces. (Chapters 12 & 14 of the text).

Learning Resources

Textbook:

John A. Thorpe, Elementary Topics in Differential Geometry

References:-

1. Serge Lang, Differential Manifolds
2. I.M. Siger, J.A Thorpe, Lecture notes on Elementary topology and Geometry, Springer – Verlag, 1967.
3. S. Sternberg, Lectures on Differential Geometry, Prentice-Hall, 1964.
4. M. DoCarmo, Differential Geometry of curves and surfaces.
5. Goursat, Mathematical Analysis, Vol – 1(last two chapters)

Competencies of the Course:

C_1 Helps to Calculate and work with principal, Gaussian and mean curvatures for surfaces in R3 and deduce general features of the surface from these functions

C_2 Describe Gauss map, Weingarten map

C_3 Define geodesics

C_4 Explains curvature of surfaces

C_5 Explains parameterized surfaces

MT3C13TM - DIFFERENTIAL GEOMETRY

	Part A	Part B	Part C
Module I	2	2	1
Module II	1	3	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

THIRD SEMESTER M.SC MATHEMATICS EXAMINATION

MT3C13TM - DIFFERENTIAL GEOMETRY

Time: 3 hours **Max.Marks:75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Show by example that the set of all vectors tangent to a level set at a point p may be R_p^{n+1} itself or may not be a subspace of R_p^{n+1}.
2. Describe the spherical image of the hyperbola $x_1^2 - x_2^2 = a^2, x_1 > 0$ under some orientations.
3. Let S be an n surface in R^{n+1} with orientation N.Let $p \in S$ and $V \in S_p$. Show that $\nabla_V N \in S_p$.
4. For the hyperboloid $x_2^2 + x_3^2 - x_1^2 = 1$ determine the normal curvature at (0,0,1) and the principal curvatures under a suitable orientation.
5. Let $\phi(\theta,\psi) = ((a + b\cos\psi)\cos\theta, (a + b\cos\psi)\sin\theta, b\sin\psi)$ .Determine the co ordinate vector fields along ϕ and show that the Gaussian curvature is $\dfrac{\cos\psi}{(a + b\cos\psi)b}$.
6. Show that the n- sphere $x_1^2 + x_2^2 + ... + x_{n+1}^2 = a^2$ is connected and describe the orientations.
7. Let C be a connected oriented plane curve and let $\beta : \mathrm{I} \to \mathrm{C}$ be a unit speed global parametrization of C. Then prove that β is either one-one or periodic.

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each question carries 5 marks)

8. Let S be an n surface at $\alpha : I \to S$ a parametrised curve. Let X and Y be smooth tangent vector fields along α .Prove the following i) $(X+Y)' = X' + Y'$ (ii) $(X..Y)' = X'.Y + X.Y'$

9. Let $\alpha : I \to R^3$ be a unit speed parameterized curve show that $\dot{\alpha}(t) X \ddot{\alpha}(t) \neq 0$ for every $t \in I$ .Let $T(t) = \dot{\alpha}(t), N(t) = \ddot{\alpha}(t) / \left\| \ddot{\alpha}(t) \right\|, B(t) = T(t) \times N(t)$ .Show that $(T(t), N(t), B(t))$ form an orthonormal system at $\alpha(t)$ and there exists functions k, $\tau : I \to R$ such that $\dot{T} = kN, \dot{N} = TB - kT$ and $\dot{B} = -\tau N$.

10. Let S be an oriented n surface and $p \in S$ .Let $\{k_1(p), ...k_n(p)\}$ be the principal curvatures at p with corresponding principal curvature directions $\{V_1, V_2, ...V_n\}$ show for each unit vector V in $S_p k_p(V) = \sum_{i=1}^{n} k_i(p)(V, V_i)^2$.

11. Let S be a compact connected n – surface with orientation N and Gauss Kronecker curvature no where zero. Show that the Gauss map is a diffeomorphism.

12. Let S be a 2 surface in R^3 and $\alpha : I \to S$, a geodesic on S with $\alpha \neq 0$ .Show that a vector field X tangent to S along α if and only if both $\|X\|$ and angle between X and α are constant along α .

13. Let S be a compact connected n – surface on R^{n+1} .Show that the Gauss Kronecker curvature $K(p)$ of S at p is non zero for all $p \in S$ if and only if the second fundamental form is definite for all $p \in S$.

14. Show that on each compact oriented n surface there exists a point at which the second fundamental form is definite.

15. Show that Mobius band is an unorientable 2-surface.

16. Let V be a finite dimensional vector space with dot product. Let L: V → W be a self adjoint operator on V. Then show that there exists an ortho normal basis of V consisting of Eigen vectors of L.

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. Show that the Weingarten map is self adjoint.
18. Let X be a smooth vector field on U an open set in R^{n+1} and $p \in U$ .Show that there exists an integral curve $\alpha : I \rightarrow U$ of X, where I is an open interval containing O such that (i) $\alpha(0) = p$, (ii) if $\beta : \tilde{I} \rightarrow U$ is another integral curve of X such that $\beta(0) = p$, then $\tilde{I} \subset I$ and for each $t \in \tilde{I}, \beta(t) = \alpha(t)$.
19. Determine all the geodesics on the sphere $x_1^2 + x_2^2 + x_3^2 = 1$ and on the cylinder $x_1^2 + x_2^2 = 1$.
20. Let S be an n surface in R^{n+1} and $p \in S$ .Show that there exists an open set V about p in R^{n+1} and a parameterized n -surface $\phi : U \rightarrow R^{n+1}$ such that ϕ is a one to one map from $\cup$ onto $V \cap S$.

(2 x 15 = 30)

Semester – III

MT3C14TM - NUMBER THEORY AND CRYPTOGRAPHY

Total Credits :4

Total Lecture Hours: 90

Aims:

The objective of the paper is to help the students to learn long and interesting history of Number Theory and its problems that have attracted many of the greatest mathematicians. Consequently the study of number theory is an excellent introduction to the development and the achievement of Mathematics. With its discrete precise nature is an ideal topic which helps the students to perform Numerical Experiments and Calculations; also to explore its wide spread applications in Cryptography.

Course Overview And Context:

This course starts by introducing some topics in Elementary number theory .It also explains about finite fields and quadratic residues. T hen it gives a brief description about public key .It also describes Primality and Factoring.

Syllabus Content:

Module 1: **(28 hours)**

Some topics in Elementary Number Theory:-Time estimates for doing arithmetic, divisibility and the Euclidean algorithm, congruences, some applications to factoring.(Chapter – I Sections 1, 2, 3 & 4 of the text)

Module 2: **(14 hours)**

Finite Fields and Quadratic Residues:-Finite fields, quadratic residues and reciprocity(Chapter – II Sections 1 & 2 of the text)

Module 3: **(25 hours)**

Public Key: - The idea of public key cryptography, RSA**,** Discrete log. (Chapter – IV Sections 1, 2 & 3 of the text)

Module 4: **(23 hours)**

Primality and Factoring: - Pseudo primes, The rho method, Fermatfactorization and factor bases, the quadratic sieve method. (chapter – V Sections 1, 2, 3 & 5 of the text)

Learning Resources

Textbooks:

Neal Koblitz, A Course in Number Theory and Cryptography, 2ndedition, Springer Verlag.

References:

1. Niven, H.S. Zuckerman and H.L. Montgomery, *An introduction to the theory of numbers*, John Wiley, 5th Edition.

2. Ireland and Rosen, *A Classical Introduction to Modern Number Theory*. Springer, 2nd edition, 1990.

3. David Burton, *Elementary Number Theory and its applications,* Mc Graw-Hill Education (India) Pvt. Ltd, 2006.

4. Alfred J. Menezes, Paul C. van Oorschot and Scott A. Vanstone, *Handbook of Applied Cryptography*, CRC Press, 1996

5. Douglas R. Stinson, *Cryptography Theory and Practice*, Chapman & Hall, 2nd edition

6. Victor Shoup, A computation Introduction to Number Theory and Algebra, , Cambridge University Press, 2005

7. William Stallings, *Cryptography and Network Security Principles and Practice*, Third edition, Prentice-hall, India.

Competencies of the Course:

C_1 Explain about Elementary Number theory

C_2 Describes finite fields and quadratic residues

C_3 Defines public key

C_4 Explains primality and factoring

MT3C14TM - NUMBER THEORY AND CRYPTOGRAPHY

	Part A	**Part B**	**Part C**
Module I	1	3	1
Module II	2	2	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

THIRDSEMESTER M.SC MATHEMATICS EXAMINATION

MT3C14TM - NUMBER THEORY AND CRYPTOGRAPHY

Time: 3hours **Max.Marks:75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Divide $(40122)_7$ by $(126)_7$
2. Describe all the solutions 3x= 4 mod 12
3. Find all Carmichael numbers of the form 3 pq(with p and q prime)
4. Use Fermat factorization to factor 4601
5. Find an upper bound for the number of bit operations required to compute n!
6. Prove that n^5 n is always divisible by 30.
7. For any integer b and any positive integer n, prove that b^{n-1} is divisible by b-1 with quotient $b^{n-1}+b^{n-2}+.........+b^2+b+1$.

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each question carries 5 marks)

8. Show that the order of any at F^*_q divides q-1, where F^*_q denotes the set of non- zero elements in the finite field F_q.
9. In F_9^* with α a root of X^2-X-1, find the discrete logarithm of -1 to the base α .
10. For each degree d≤ 6, find the number of irreducible polynomials over F_2 of degree d.
11. Describe the EIGamel cryptosystem
12. Find the discrete log of 153 to the base 2 in F^*_{181}
13. Factor 4087 using f(x)= x^2+x+1 and x_0= 2
14. Use quadratic sieve methodto factor 1046603 with P= 50 and A= 500.
15. Show that the Euclidean algorithm always gives the greatest common divisor in a finite number of steps.
16. If gcd (a.m)= 1, show that $a^{d(m)}\equiv$1mod m.

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. State and prove the Chinese Remainder theorem
18. State and prove the law of quadratic reciprocity
19. Describe the algorithm for finding discrete logs in finite fields
20. Explain Diffe- Hallman key exchange system

(2 x 15 = 30)

Semester – III

MT3C15TM - OPTIMIZATION TECHNIQUES

Credit-4

Total Lecture hours- 90

Aims:

Problems in optimization are the most common applications of mathematics. The main aim of this course is to present different methods of solving optimization problems in the three areas of linear programming, nonlinear programming, and classical calculus of variations. In addition to theoretical treatments, there will be some introduction to numerical methods for optimization problems and theory of games.

Course Overview And Context:

This course starts by explaining integer programming. Then it further explains sensitivity analysis, flow and potentials in networks. It describes about theory of games and also it describes about non linear programming.

Syllabus Content:

Module I: (20 hours)

Integer Programming : .L.P in two dimensional space – General I.L.P. and M.I.L.P problems – cuttingplanes – remarks on cutting plane methods – branch and bound method – examples –general description – the 0 – 1 variable.(Chapter 6; sections: 6.1 – 6.10 of text – 1)

Module II: (25 hours)

Sensitivity Analysis; Flow And Potentials In Networks: Introduction – changes in bi – changes in cj – Changes in aij – introduction of newvariables – introduction of new constraints – deletion of variables - deletion of constraints –Goal programming. Graphs-definitions and notation – minimum path problem – spanning tree of minimum length –

problem of minimum potential difference – scheduling of sequential activities – maximum flow problem – duality in the maximum flow problem – generalized problem of maximum flow.
(Chapter – 5 & 7 Sections 5.1 to 5.9 & 7.1 to 7.9, 7.15 of text - 1)

Module III: THEORY OF GAMES (20 hours)

Matrix (or rectangular) games – problem of games – minimax theorem, saddle point – strategies and pay off – theorems of matrix games – graphical solution – notion of dominance – rectangular game as an L.P. problem.(Chapter 12; Sections: 12.1 – 12.9 of text – 1)

Module IV: NON- LINEAR PROGRAMMING (25 hours)

Basic concepts – Taylor's series expansion – Fibonacci Search - golden section search – Hooke and Jeeves search algorithm – gradient projection search – Lagrangemultipliers – equality constraint optimization, constrained derivatives – projectgradient methods with equality constraints – non-linear optimization: Kuhn-Tuckerconditions – complimentary Pivot algorithms.(Chapter 8; Sections: 8.1 – 8.14 of text – 2)

Learning Resources

Textbooks:

Text – 1 K.V. Mital and C. Mohan, Optimization Methods in Operation Research and Systems Analysis, 3rd edition.
Text -2- Ravindran, Philips and Solberg.Operations Research Principle and Practice, 2nd edition, John Wiley and Sons.

Reference:-

1. S.S. Rao, Optimization Theory and Applications, 2nd edition, New Age International Pvt.
2. J.K. Sharma, Operations Research: Theory and Applications, Third edition, Macmillan India
3. Hamdy A. Thaha, Operations Research – An Introduction, 6th edition,Prentice Hall of India Pvt. Ltd.

Competencies of the Course:

C_1 Explains about integer programming

C_2 Describe the flow and potentials in networks

C_3 Introduce and explain about game theory

C_4 Explains about non linear programming.

MT3C15TM - OPTIMIZATION TECHNIQUES

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

THIRD SEMESTER M.SC MATHEMATICS EXAMINATION

CORE COURSE- MT3C15TM - OPTIMIZATION TECHNIQUES

Time: 3 hours **Max.Marks:75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Explain how cutting plane algorithm works.
2. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
3. Discuss the changes in the coefficients a_{ij} for the given linear programming problem.Maximize Z= CX subject to Ax= b, X≤ 0.
4. Examine the following pay- off matrix for saddle points: $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 3 \\ 6 & 2 & 1 \end{bmatrix}$
5. Examine the difference between pure strategy and mixed strategy
6. What are the primary uses of Kuhn -Tucker necessary and sufficient conditions?
7. Solve the problem using constrained derivatives

Minimize $f(x) = 7x_1 - 6x_2 + 4x_3$ subject to $a_1^{\ 2} + 2x_2^{\ 2} + 3x_3^{\ 1} = 1,\ x_1 + 5x_2 - 3x_3 = 6$

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each question carries 5 marks)

8. Describe the algorithm for branch and bound method for the profit maximization problem.
9. Find the maximum flow in the network with the following dataflows in acrs not necessarily being non negative:Arc:

(a,1) (a,2) (1,2) (1,3) (2,4) (3,4) (3,b) (4,b)

(bi,ci)(0,10) (0,5) (-2,3) (7,10) (-3,5) (-1,1) (0,8) (0,4)

10. Describe the Hooke and Jeeves search algorithm

11. Explain the complementary pivot algorithm to solve a complementary problem.
12. A factory can manufacture two products A and B. The profit on a unit of A is Rs.80 and of B is Rs 40. The maximum demand of A is 6 units per week, and of B is 8 units. The manufacturer has set up a goal of acheieving a profit of Rs.640 per week. Formulate the problem as goal programming and solve it. Also obtain the graphical solution.
13. Solve the problem using gradient projection Minimize $f(x) = 25(x_1 - 3x_2)^2 + (x_1 - 3)^2$
14. Give the algorithm to solve the generalized problem of maximum flow.
15. Explain the problem of potential difference
16. Let f(X,Y) be such that both $\max_X \min_Y$ f(X,Y) and $\min_Y \max_X$ f(X,Y) exist. Then show that $\max_X \min_Y$ f(X,Y) $\leq \min_Y \max_X$ f(X,Y). (6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. a)Use Golden section search to :Maximise f(X)= $3x^4+(x-1)^2$, $0\leq x\leq 4$ with a resolution of $\in = 0.10$

    b)Differentiate between constrained and unconstrained optimization techniques
18. Use Kuhn Tucker conditions to

    Maximise Z= $12x_1+21x_2+2x_1x_2-2x_1^2-2x_2^2$ subject to $x_2 \leq 8$

    $x_1+x_2 \leq 10$ and $x_1, x_2 \geq 0$.
19. Write down the algorithm to solve the complementary problem and outline its application.
20. a)Define (i) sub graph(ii) strongly connected graph (iii)potential difference in a network(iv) fictitious vertex

b)Show that if $\{x_i\}$ and $\{y_i\}$ are two flows in a graph, then $\{ax_i+by_i\}$ is also a flow,

where a and b are real constants.

(2 x 15 = 30)

Semester – IV

MT4C16TM -SPECTRAL THEORY

Credit-3

Total Lecture hours- 90

Aims

Functional analysis plays an increasing role in the applied sciences as well as in mathematics itself. Spectral theory is a very relevant part of Functional analysis which extend the eigenvector and Eigenvalue theory of a single square matrix to a much broader theory of the structure of operators in a variety of mathematical spaces. Consequently, it becomes more and more desirable to introduce the student to this field at this semester .

Course Overview and Context

Module 1starts by introducing the basic concepts like strong and weak convergence for sequences of elements in a normed space and three main theorems in Functional analysis like open mapping theorem, closed graph theorem and Banach fixed point theorem. In Module 2 Spectral theory in finite dimensional normed space is introduced and a discussion on the spectralproperties of bounded linear operators and Banach algebras is done. In Module 3 and in Module 4 Spectral properties of Compact linear operators ,bounded self adjoint linear operators and projection operators are discussed.

Syllabus Content

Module 1 **(25 hours)**

Strong and weak convergence, convergence of sequence of operators and functionals,open mapping theorem, closed linear operators, closed graph theorem, Banach fixedpoint theorem (Chapter 4 - Sections 4.8, 4.9, 4.12 & 4.13 - Chapter 5 – Section 5.1 of the text)

Module 2 **(25 hours)**

Spectral theory in finite dimensional normed space, basic concepts, spectralproperties of bounded linear operators, further properties of resolvant and spectrum,use of complex analysis in spectral theory, Banach algebras, further properties ofBanach algebras.(Chapter 7 - Sections 71. to 7.7 of the text)

Module 3 **(20 hours)**

Compact linear operators on normed spaces, further properties of compact linearoperators, spectral properties of compact linear operators on normed spaces, furtherspectral properties of compact linear operators, unbounded linear operators and theirHilbert adjoint operators, Hilbert adjoint operators, symmetric and self adjoint linearOperators (Chapter 8 - Sections 8.1 to 8.4 - Chapter 10 Sections 10.1 & 10.2 of the text)

Module 4 **(20 hours)**

Spectral properties of bounded self adjoint linear operators, further spectral propertiesof bounded self adjoint linear operators, positive operators, projection operators,further properties of projections

(Chapter 9 - Sections 9.1, 9.2, 9.3, 9.5, 9.6 of the text)

Learning Resources

Text Book: Erwin Kreyszig, Introductory Functional Analysis with applications,
John Wiley and sons, New York

References:-

1. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York, 1963.

2. Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi: 1989

3. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt Ltd, Madras, 1994

4. Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut: 1995-96

5. M. Thamban Nair, Functional Analysis, A First Course, Prentice – Hall of India Pvt. Ltd, . 2008

6. Walter Rudin, Functional Analysis, TMH Edition, 1974.

Competencies of the Course

C1. Discuss the Strong and weak convergence of sequence of operators and functional

C2. Introduce open mapping theorem, closed linear operators, closed graph theorem and Banach fixed point theorem

C3. Introduce Spectral theory in finite dimensional normed space

C4. Discuss spectral properties of bounded linear operators and resolvant and spectrum

C5. Explore the use of complex analysis in spectral theory

C6. Explain Banach algebra and their further properties

C7. Discuss Compact linear operators on normed spaces and their further properties

C8. Introduce unbounded linear operators and Hilbert adjoint operators

C9. Explain symmetric and self adjoint linear Operators

C10. Analyze Spectral properties of bounded self adjoint linear operators

C11. Introduce positive operators and projection operators and discuss their further properties

MT4C16TM -SPECTRAL THEORY

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M.SC MATHEMATICS EXAMINATION

MT4C16TM- SPECTRAL THEORY

Time : 3 hrs Max. Marks: 75

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Prove or disprove : A sequence in a Hilbert space converges if and only if it is weak convergent.
2. Give an example of an inner product space and bounded linear operator A on it such that the adjoint of A does not exist. Indicate a proof of your claim.
3. Give an example of a self adjoint operator on R^2. Substantiate your claim.
4. Define the eigen spectrum of a bounded linear operator on a normed space X and show by an example that it may be different from the spectrum.
5. Prove that a bounded linear operator on a finite dimensional Hilbert space is compact.
6. Give an example of a compact operator whose range is not finite dimensional. Indicate a proof of your claim.
7. Prove that a bounded linear operator on a Banach space is invertible if and only if it is Bijective (5 x 3 = 15)

Part B

(Answer any 6 questions. Each questioncarries 5 marks)

8. Prove that a linear open map from a normed linear space X to a normed linear space
9. Y is surjective.
10. Show by an example that the open mapping theorem may fail when the domain is not complete.
11. If A is a bounded linear operator on a normed space of finite rank, prove that theeigen spectrum, the approximate eigen spectrum and the spectrum of A are equal.
12. If (xn) is a bounded sequence in a Hilbert space, prove that it has a weak convergent subsequence.
13. Prove that a Hilbert space is reflexive

14. Prove that for a continuous linear functional on a subspace of a Hilbert space H, there exists a unique Hahn – Banach extension.
15. If A is a bounded linear operator on a Hilbert space and A_n is a compact operator for every positive inter n such that $\| A_n - A \|$ converges to 0, then prove that A is compact.
16. Let H be a finite dimensional Hilbert space over K and A is a bounded linear operator on H. Suppose that K = C and A is normal or K = R and A is self adjoint. Then prove that there exists an orthonormal basis for H consisting of eigen vectors of A. (6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. State and prove the closed graph theorem.
18. State and prove Banach fixed point theorem
19. State and prove open mapping theorem
20. Prove that he spectrum σ (T) of a bounded self adjoint linear operator $T: H \rightarrow H$ on a complex Hilbert space H is real.

(2 x 15 = 30)

Semester – IV

ELECTIVE COURSES

MT4E17TM - ANALYTIC NUMBER THEORY

Credit-3

Total Lecture hours- 90

Aim

Analytic number theory is a branch of number theory that uses methods from mathematical analysis to solve problems about the integers. Theorems and results within analytic number theory tend not to be exact structural results about the integers, for which algebraic and geometrical tools are more appropriate. Instead, they give approximate bounds and estimates for various number theoretical functions. The choice of topics included here is intended to provide some variety and some depth into the subject. Problems which have fascinated generations of professional and amateur mathematicians are discussed together with some of the techniques for solving them. One of the goals of this course has been to nurture the intrinsic interest that many young mathematics students seem to have in number theory .

Course overview and Context

Module 1 introduces several arithmetical functions which play an important role in the study of divisibility properties of integers and the distribution of primes.It also discusses Dirichlet multiplication,a concept which helps clarify interrelationships between various arithmetical functions and an inquiry is made about the behavior of these and other arithmetical functions f(n) for largevalues of n. In module 2 Elementary Theorems on the Distribution of Prime Numbersare studied. Module 3 discusses the concept of Congruences and in Module 4 Primitive roots and partitions are investigated.

Syllabus Content

Module 1 **(30 hours)**

Introduction, the Mobius function, the Euler totient function a relation connecting and the Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, the Mangoldt function , multiplicative e functions and Dirichlet multiplication, the inverse of

completely multiplicative functions, the Liovillie's function , the divisor function , generalized convolutions, formal power series, the Bellseries of an arithmetical function, Bell series and Dirichlet multiplication.Introduction to Chapter2 of the text, the big oh notation, asymptotic equality offunctions, Euler's summation formula, some elementary asymptotic formulas, the
average order of , The average order of the divisor function , average orderof an application of distribution of lattice points visible from the origin, averageorder of „the partial sums of a Dirichlet product. (Chapter 2 sections 2.1 to 2.17 and Chapter 3 sections 3.1 to 3.11 of the text)

Module 2 **(15 hours)**

Introduction to Chapter4, Chebyshev's functions and , relation connecting , some equivalent forms of prime number theorem, inequalities of and Shapiro's Tauberian theorem, applications of Shapiro's theorem, an asymptotic formula for the partial sum .(Chapter 4 sections 4.1 to 4.8 of the text) (15 hours)

Module 3 **(30 hours)**

Definition and basic properties of congruences, residue classes and complete residuesystems, liner congruences , reduced residue systems and Euler – Fermat theorem,Polynomial congruences modulo , Lagrange's theorem, applications of Lagrange'stheorem, simultaneous linear congruences, the Chinese reminder theorem,applications of Chinese reminder theorem, polynomial congruences with prime powermoduli (Chapter 5 sections 5.1 to 5.9 of the text)

Module 4 **(15 hours)**

The exponent of a number mod m. Primitive roots, Primitive roots and reducedsystems, The non existence of Primitive roots mod 2a for a ³3, The existence ofPrimitive roots mod p for odd primes p, Primitive roots and quadratic residues.Partitions – Introduction, Geometric representation of partitions, Generating functionsfor partitions, Euler's pentagonal-number theorem.(Chapter 10 sections 10.1 to 10.5 &Chapter 14 sections 14.1 to 14.4 of the text)

Learning Resources

Text Book: Tom M Apostol, *Introduction to Analytic Number Theory,* Springer International Student Edition, Narosa Publishing House

References:

1. Hardy G.H and Wright E.M , Introduction to the Theory of numbers, Oxford, 1981

2. Leveque W.J, Topics in Number Theory, Addison Wesley, 1961.

3. J.P Serre, A Course in Arithmetic, GTM Vol. 7, Springer-Verlag, 1973

Competencies of the Course

C1. Introduce several arithmetical functions like the Mobius function , Euler totient function

Mangoldt function , Liovillie's function etc.

C2. Analyze formal power series and the Bell series of an arithmetical function

C3. Understand the big oh notation, asymptotic equality of functions, Euler's summation formula

C4. Explain some elementary asymptotic formulas

C5. Describe the average order of the divisor function , average order of an application of distribution of lattice points visible from the origin, average order of the partial sums of a Dirichlet product.

C6. Introduce Chebyshev's functions

C7. Identify some equivalent forms of prime number theorem

C8. Understand inequalities of and Shapiro's Tauberian theorem and its applications

C9. Identify an asymptotic formula for the partial sum

C10. Define basic properties of congruences, residue classes and complete residue systems

C11. Obtain Euler – Fermat theorem

C12. Understand Lagrange's theorem and its applications

C13. Describe simultaneous linear congruences

C14. Explain the Chinese reminder theorem and its applications

C15. Understand polynomial congruences with prime power moduli

C16. Introduce and analyze the Primitive roots

C17. Define Partitions

C18. Describe Geometric representation of partitions

C19. Understand Generating functions for partitions

C20. Obtain Euler's pentagonal-number theorem.

MT4E17TM - ANALYTIC NUMBER THEORY

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	1	3	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M SC MATHEMATICS EXAMINATION

MT4E17TMAnalytic Number Theory

Time : 3 hrs **Max. Marks: 75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Find all integers n such that $\phi(n) = n/2$
2. Determine the quadratic residues and non residues modulo 7.
3. Determine whether 73 is a quadratic residue or non residue of the prime 383.
4. Prove that the Dirichlet product of two multiplicative functions is multiplicative.
5. State Euler's summation formula.
6. Find the average order of $\phi(n)$
7. Give the density of the set of lattice points visible from the origin .

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each questioncarries 5 marks)

8. State and Prove Abel's identity.
9. Assume gcd (a, m) = 1. Then Prove that the linear congruence ax $\equiv$ b (mod m) has exactly one solution.
10. State and Prove Euler-Fermat theorem.
11. State and ProveWolstenholme's theorem.
12. If p is an odd prime and a > 1 then prove that there exist odd primitive roots g modulo p^a. Also show that each such g is also a primitive root modulo $2p^a$.
13. lf m has a primitive root g then m has exactly $\phi(\phi(m))$ incongruent primitive roots
14. State and prove Mobius inversion formula.
15. Calculate the highest power of 10 that divides 1000!
16. Define the Legendre symbol (n/p) and show that it is a completely multiplicative function of

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. a) Assuming the Gauss's lemma and the formula for the integer m in the Gauss's lemma, prove quadratic reciprocity law.

    (b) Determine those odd primes p for which 3 is a quadratic residue andthose for which it is a nonresidue.
18. a) State and prove Legendre's identity.

    b) Prove that two lattice points (a, b) and (m, n) are mutually visible iff a-m and b – n are relatively prime
19. State and prove the prime number theorem. Prove that for every $n > 1$, there exist n consecutive composite number
20. Derive the Bell series of a completely multiplicative function

(2 x 15 = 30)

Semester – IV

MT4E18TM -COMBINATORICS

Credits :3

Total Lecture hours : 90

Aim

Combinatorics is the branch of mathematics studying the enumeration, combination, and permutation of sets of elements and the mathematical relations that characterize their properties. The basic principles and techniques taught in this course have found many applications in other fields like computer science and Operations Research. Therefore we introduce a course on Combinatorics to the post graduate students.

Course overview and Context

Module 1 starts with the introduction of basic counting principles in Permutations and Combinations. In Module 2the Piegeonhole Principle and Ramsey Numbers are discussed. Module 3 explains the Principle of Inclusion and Exclusion and in Module 4 Generating Functions and Recurrence Relations are analyzed along with some modeling problems.

Syllabus Content

Module 1 **(20 hours)**

Two basic counting principles, Permutations, Circular permutations, Combinations, The injection and bijection principles, Arrangements and selection with repetitions ,Distribution problems (Chapter I of the text)

Module 2 **(20 hours)**

Introduction, The piegeonhole principle, More examples, Ramsey type problems and Ramsey numbers, Bounds for Ramsey numbers (Chapter 3 of the text)

Module 3 **(25 hours)**

Introduction, The principle, A generalization, Integer solutions and shortest routesSurjective mappings and Sterling numbers of the second kind, Derangements and ageneralization, The Sieve of Eratosathenes and Euler j -function.

(Chapter -4 Sections 4.1 to 4.7 of the text) (25 hours)

Module 4 **(25 hours)**

Ordinary generating functions, Some modelling problems, Partitions of integer,Exponential generating functions, Recurrence Relations Introduction, Two examples, Linear homogeneous recurrence relations, General linear recurrence relations, Two applications (Chapter 5, 6 Sections 6.1 to 6.5) (25 hours)

Learning Resources

Text Book: Chen Chuan -Chong, Koh Khee Meng, Principles and Techniques in Combinatorics, World Scientific,1999.

References:-

1. V Krishnamoorthy, Combinatorics theory and applications, E. Hoewood,1986
2. Hall,Jr, Combinatorial Theory, Wiley- Interscinice, 1998.
3.Brualdi, R A, Introductory Combinatorics, Prentice Hall,1992

Competencies of the Course

C1. Introduce two basic counting principles

C2. Explain Permutations, Circular permutations and Combinations

C3. Explain the injection and bijection principles

C4. Describe Arrangements and selection with repetitions and Distribution problems

C5. Introduce the pigeonhole principle with some examples and Ramsey type problems

C6. Explain Ramsey numbers and Bounds for Ramsey numbers

C7. Obtain shortest routes

C8. Understand Surjective mappings and Sterling numbers of the second kind

C9. Describe The Sieve of Eratosathenes and Euler j -function

C10. Explain Ordinary generating functions and Partitions of integer,

C11. Describe Exponential generating functions and Recurrence

C12. Explain Linear homogeneous recurrence relations

C13. Understand General linear recurrence relations using applications

MT4E18TM -COMBINATORICS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	2	2	1
Module IV	1	3	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M SC MATHEMATICS EXAMINATION

MT4E18TMCombinatorics

Time : 3 hrs **Max. Marks: 75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. How many 4-digit numbers (with repetition allowed) can be formed using digits 0, 2, 4, 6, 8?
2. Find the number of non-negative integer solutions of theequation : $x + y + z + w = 17$
3. If a school has 100 students with 50 students taking Hindi, 40 students taking Sanskrit, and 20 students taking both the languages, how many students take neither Hindi nor Sanskritlanguage ?
4. Given a collection of 2n objects, n identical and the other n all distinct, how many different subcollections of n objects are there ?
5. How many n-digit ternary (0, 1, 2) sequences are there with at least one 0, at least one 1 and at least one 2 ?
6. Let $A = \{a_1, ., a_n\}$ and $B = \{0, 1\}$. Determine the number of onto functions from A to B.
7. State the pigeon hole principle and its generalized version.

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each question carries 5 marks)

8. Find the characteristic equation of the recurrence relation: $a_n = 3a_{n-1} - 2a_{n-2}$.
9. Find the number of ways of distributing n distinct objects into m distinct boxes so that no box is empty.
10. How many four digit numbers are there formed from the digits 1, 2, 3, 4, 5 (with possible repetition) that are divisible by four
11. Find a formula for an satisfying the relation : $a_n = - 2a_{n-2} - a_{n-4}$ with $a0 = 0$, $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$.

12. Given a group of n women and their husbands, how many people must be chosen from this group of 2n people to guarantee that the set contains a married couple ?
13. There are eight persons A, B, C, D, E, F, G, H. Determine the number of ways in which they can be seated at a round table so that A and B do not sit in adjacent seats.
14. Solve the recurrence relation an = an–2, where n ≥ 2 and a0 = a1 = 1.
15. Find the number of non-negative integer solutions of the equation : x + y + z = 15 where x< 6, y < 7, z < 8
16. Define generating function for a sequence (a_r) of numbers. Obtain the generating function for the sequence (1,1,1,……)

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. a) Find a recurrence relation for the number an of ternary sequences of length n that contains two consecutive digits that are the same. What are the initial conditions ? Find a6.

    b) Show that any subset of n + 1 distinct integers between1 and 2n (n ≥ 2) always contains a pair of integers withno common divisor
18. a) Find the exponential generating function for the sequence (1, 1.3,1 .3.5, ……)

    b) Prove that the number of partitions of r into distinct parts is equal to the the number of partitions of r into odd parts
19. a) State and prove Hanson , Seffarth and Weston theorem on derangements

    b)Show that among any group of seven people , there must be at least 4 of the same sex
20. Define the Ramsey number R(p, q). Prove that for all positive integers p and q R (p, q) = R (q, p). Find R (3, 4) and R (5, 9)

(2 x 15 = 30)

Semester – IV

MT4E19TM -MATHEMATICAL ECONOMICS

Credits -3

Total lecture hours - 90

Aim

Mathematical economics is the application of mathematical methods to represent theories and analyze problems in economics. Mathematics allows economists to form meaningful, testable propositions about wide-ranging and complex subjects which could less easily be expressed informally. This course aims to teach students the language of mathematics that will allows them to make specific and positiveclaims about controversial or contentious economic theory by constructing convincing mathematical economic models.

Course overview and Context

The course starts by introducing the theory of consumer behavior. In module2 different types of production functions are explained . Input – Output Analysis is done in Module 3 and in module 4 difference equationsare studied.

Syllabus Content

Module:-1 **(20 hours)**

Introductory, Maximization of utility,Indifference cure approach, Marginal rate of substitution, Consumer's equilibrium,Demand curve, Relative preference theory of demand, Numerical problems related tothese theory part.
(Chaper – 13 .Sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6 & 13.13 of text – 1

Module:-2 **(30 hours)**

Meaning and nature of production function, The law of variable proportion, Isoquants, Marginal technical rate of substitution,Producer's equilibrium, expansion path, The elasticity of substitution, Ridge linesand economic region of production, Euler's theorem, Cobb Douglas production function, The CES Production function, Numerical problems related to these theory parts.

(Chapter – 14. Sections 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9, 14.10 & 14.11 of text - 1)

Module:-3 **(20 hours)**

Meaning of input – output, main features of analysis, Assumptions, Leontief's static and dynamic model, limations, Importance and

Applications of analysis, Numerical problems related to these theory parts..(Chapter – 15. Sectios 15.1, 15.2, 15.3, 15.4, 15.5,15.6, 15.7, 15.8 & 15.9 of text - 1)

Module:- 4 **(20 hours)**

Introduction, Definition and Classification of Difference equations, Linear Difference equations, Solution of Difference equations, Linear First-Order Difference equations with constant coefficients, Behaviour of the solution sequence, Equilibrium and Stability, Applications of Difference equations in

Economic Models, The Harrod Model, The General Cobweb Model, Consumption Model, Income – Consumption – Investment Model.

(Chapter 6 Sections 6.1 to 6.5 of text 2) (20 hours)

Learning Resources

Text – 1:- Singh S.P, Anil K.Parashar, Singh H.P, Econometrics and Mathematical Economics, S. Chand & Company, 2002.

Text – 2:- JEAN E. WEBER, MATHEMATICAL ANALYSIS Business and Economic Applications, Fourth edition, HARPER & ROW PUBLISHERS, New York.

References:-

1. Allen.R.G..D, Mathematical Economics, 1959.

2. Alpha C Chiang, Fundamental methods of Mathematical Economics.

3. Koutsoyiannis. A, Modern Microeconomics, Macmillen.

4. Samuelson. P.A, Foundation of Economic Analysis.

5. Josef Hadar, Mathematical theory of economic behaviour, Addison-Wesley

Competencies of the Course

C1. Introduce Maximization of utility

C2. Understand Indifference cure approach and Marginal rate of substitution

C3. Recognize Consumer's equilibrium

C4. Define Demand curve

C5. Obtain Relative preference theory of demand

C6. Discuss Numerical problems related to these theory part.

C7. Recognize Meaning and nature of production function

C8. Obtain the law of variable proportion

C9. Define Isoquants and Marginal technical rate of substitution,

C10. Understand Producer's equilibrium and expansion path

C11. Bring out the elasticity of substitution

C12. Discuss Ridge lines and economic region of production

C13. Analyze Euler's theorem and Cobb Douglas production function

C14. Obtain The CES Production function

C15. Understand Meaning of input – output and main features of analysis

C16. Discuss Leontief's static and dynamic model

C17. Bring out limations, Importance and Applications of analysis

C18. Introduce the definition and Classification of Difference equations

C19. Recognize Linear Difference equation

C20. Obtain Solution of Difference equations

C21. Describe Linear First-Order Difference equations with constant coefficients

C22. Investigate the behaviour of the solution sequence

C23. Understand Equilibrium and Stability,

C24. Bring out the applications of Difference equations in Economic Models

C25. Understand the Harrod Model and the General Cobweb Model

C26. Investigate Consumption Model and Income – Consumption – Investment Model.

MT4E19TM -MATHEMATICAL ECONOMICS

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	2	2	1
Module IV	1	3	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M SC MATHEMATICS EXAMINATION

MT4E19TM -Mathematical Economics

Time : 3 hrs **Max. Marks: 75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. What are the assumptions of input-output model ?
2. Explain Prisoner's dilemma.
3. Explain consumer's surplus.
4. From the demand function $X = 80 - 4P - P^2$, determine price elasticity ofdemand when $P = 5$.
5. If the total cost is $TC = 50 + 10q + 25q^2$. Find the Average Cost and Marginal Cost when $q = 2$
6. Determine which type of returns to scale is the following production function $Q = 2K + 3L + KL$.
7. Explain price effect (5 x 3 = 15)

Part B

(Answer any 6 questions. Each questioncarries 5 marks)

8. Explain CES production function.
9. Explain economic interpretation of time margin function
10. The cost function of a monopolist is $TC = 500 + 20q^2$. The demand function is $P = 400 - 20q$. What is the profit maximising output ?
11. Explain Ridge Lines.
12. Explain dynamic input-output model.
13. State and prove Adding-up Theorem.
14. The supply function for a commodity $P = 2 + Q^2$. Find the producer's surplus when price is Rs. 18.
15. Explain Cobweb Model.

16. Find firm's equilibrium and cost function if the production function is Cobb- Douglas function

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. a) Explain CES production function in detail.

    b) State and prove Euler's theorem

18. a) What are isoquants ? Draw a rough Sketch and explain

    b)Explain the methods of maximizing a utility function

19. a) Formulate Harrod's model.

    b) Discuss cobweb theorem

20. Explain Leontief's static and dynamic models and its importance. What are the limitations ?

(2 x 15 = 30)

Semester – IV

MT4E20TM- OPERATIONS RESEARCH

Credit-3

Total Lecture hours- 90

Aims

Operations Research (OR) applies scientific method to the management of organized systems in business, industry, government and other enterprises. Employing techniques from other mathematical sciences, such as mathematical modeling, statistical analysis, and mathematical optimization, operations research arrives at optimal or near-optimal solutions to complex decision-making problems. This course provides a wide range of problem-solving techniques and methods that can be applied in the pursuit of improved decision-making and efficiency.

Course overview and Context

This course starts by introducing Inventory Models . In module 2 Queuing Systems are explained and in module 3 the method of Dynamic Programming is discussed. Network sequencing and Simulation Modeling are described in module 4.

Syllabus Content

Module 1: (20 hours)

Introduction – Variables in an inventory problem – Objectives of inventory control – The classical E.O.Q. without shortages – The classical E.O.Q. with shortages – The Production Lot size (P.L.S) models – Nonzero Lead time – The Newsboy Problem (a single period model) – Lot size reorder point model – Variable lead times – The importance of selecting the right model. (Chapter 8; Sections: 8.1 – 8.14 of text 1)

Module 2: (25 hours)

Why study queues? – Elements of a queueing model – Role of exponential distribution (Derivation of exponential distribution; forgetfulness property) – Pure Birth and Death models – Relationship between the exponential and Poisson distributions – Generalized Queueing Models – Kendall notation – Poisson Queueing Models – Single server models and multiple server models – Machine servicing models – (M/M/R) : (GD/K/K) Model – (M/G/1) : (GD/) model – PollaczekKhintchine (P - K) formula. (Chapter 17; Sections: 17.1 – 17. 9 of text – 2)

Module 3: (20 hours)

Introduction - Minimum path problem – Single additive constraint, additively separable return – Single multiplicative constraints, additively separable return - Single additive constraint, multiplicatively separable return – Computational economy in DP – Serial multistage models – Examples of failure – Decomposition – backward and forward recursions – Systems with more than one constraint – Applications of D.P to continuous systems. (Chapter: 10; Sections: 10.1 – 10.12 of text – 3)

Module 4 (25 hours)

Problem of sequencing – Basic assumptions – Processing n jobs through two machines – OptimumSequence (Johnson Bellman) Algorithm - Processing n jobs through k machines – Processing of two jobs through k machines – Maintenance crew scheduling. Simulation – Generation of random variables – Monte Carlo simulation – Sampling from probability distributions: 1. Inverse method, 2. Convolution method (&Box- Muller method), 3. Acceptance-Rejection method – Generic definition of events. (Chapter: 12; Sections: 12.1 – 12.7 of text – 4 Chapter: 18- Sections: 18.1 – 18.6 of text – 2)

Learning Resources

Text -1- Ravindran.A, Don T Philips and James J Solberg., Operations Research Principle and Practice, 2nd edition, John Wiley and Sons.

Text – 2- Hamdy A. Thaha, Operations Research – An Introduction, 6th edition, Prentice Hall of India Pvt. Ltd.

Text – 3- K.V. Mital and C. Mohan, Optimization Methods in Operation Research and Systems Analysis, 3rd edition, New Age International Pvt. Ltd..

Text – 4 -Man Mohan, P.K. Gupta and Kanti Swarup, Operations Research, Sultan Chand and Sons.

References:-

1. Thomas L Satty, Elementary Queuing Theory, McGraw Hill Publishing Company.

2. Narasingh Deo, System Simulationwirh digital Computers, 7th edition, Prentice Hall India Pvt. Ltd., 1997.

3. Geoffrey Gordon, System Simulation, 2nd edition Prentice Hall India Pvt. Ltd, 1998.

Competencies of the Course

C1. Introduce variables in an inventory problem
C2. Design objectives of inventory control
C3. Describe the classical E.O.Q. without shortages and with shortages
C4. Design the Production Lot size (P.L.S) models
C5. Describe Nonzero Lead time
C6. Design The Newsboy Problem (a single period model) and Lot size reorder point model
C7. Recognise the importance of selecting the right model.
C8. Explore the needs of studying queues
C9. Understand elements of a queueing model
C10. Derive exponential distribution
C11. Analyze the role of exponential distribution and forgetfulness property
C12. Design Pure Birth and Death models
C13. Recognise Relationship between the exponential and Poisson distributions
C14. Introduce Kendall notation
C15. Obtain Poisson Queueing Models
C16. Analyze Single server models and multiple server models
C17. Understand Machine servicing

C18. Introduce Minimum path problem

C19. Explain Single additive constraint and additively separable return

C20. Describe Single multiplicative constraints and additively separable return

C21. Analyze multiplicatively separable return

C22. Understand Computational economy in DP

C23. Design Serial multistage models

C24. Analyze backward and forward recursions

C25. Recognise sstems with more than one constraint

C26. Recognise Applications of D.P to continuous systems

C27. Describe the Problem of sequencing

C28. Explain Processing of jobs through machines

C29. Describe OptimumSequence (Johnson Bellman) Algorithm

C30. Describe Simulation

C31. Understand Monte Carlo simulation

C32. Explain Sampling from probability distributions

C33. Describe Box- Muller method

C34. Analyze Acceptance-Rejection method

MT4E20TM- OPERATIONS RESEARCH

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	1	3	1
Module III	2	2	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M SC MATHEMATICS EXAMINATION

MT4E20TMOperations Research

Time : 3 hrs **Max. Marks: 75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. What is quadratic programming problem in Operations Research?
2. Write down the standard form of a geometric programming problem.
3. Convert the following problem in to the form of a geometric programming problem:Find the dimensions of the rectangle of maximum area inscribed in a circle of radius r.
4. Briefly explain the method of Dynamic programming to solve the minimum pathproblem.
5. Describe the Rosenbrock algorithms to locate the minimum of a function.
6. Explain briefly the general method of axial directions
7. Distinguish between sensitivity analysis and parametric linear programming

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each questioncarries 5 marks)

8. Describe the problem of scheduling sequential activities. Explain how it is converted to the problem of minimum path.
9. Explain the method of parametric linear programming in the case of variations in bi.
10. verify that Kuhn – Tucker theory fails to give solution of the problem:
    Minimize $x^2 + y^2$ subject to $(x - 1)^3 - y^2 \geq 0$, $x, y \geq 0$
11. Find the minimum spanning tree in the following undirected graph

Arc	(1,2)	(1,3)	(1,4)	(2,3)	(2,8)	(2,10)	(3,4)	(3,8)	(4,5)
Length	4	8	3	9	14	4	10	15	12
Arc	(4,6)	(4,8)	(5,6)	(5,7)	(6,7)	(6,8)	(6,9)	(7,9)	(8,9)
Length	10	4	1	10	3	19	15	6	10

12. Describe the method of quadratic interpolation.
13. Explain the method of steepest descend in multidimensional search.
14. What are the important techniques used in operations research ? Explain their limitations
15. What are the different types of models used in operations research ? Explain in detail
16. Explain Monte Carlo simulation

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. Maximize f(x) = -5x + 13y + 5z subject to 12x + 10y + 4z $\leq$ 90 , -x + 3y + z $\leq$ 20, x, y, z $\geq$ 0. Use sensitivity analysis to investigate the effects on the optimal solution if a new constraint 2x +5y+3z < 51
18. Use the method of quadratic programming to solve Minimize f(x) = -x-y-z + ½($x^2 + y^2 + z^2$) subject to x + y + z -1 < 1, 4x + 2y -7/3 < 1, x, y, z $\geq$ 0
19. Use geometric programming to find the minimum of $\frac{a}{xy^{1/2}z} + bxz + cxyz$ subject to

$$\frac{d}{x^2 + y^2} + \frac{ey^{1/2}}{z} \leq 1, a, b, c, d, e > 0, x, y, z > 0$$

20. Describe the Fibonacci search plan in one dimensional search

(2 x 15 = 30)

Semester – IV

MT4E21TM -PROBABILITY THEORY

Credits -3

Total lecture hours - 90

Aim

Probability theory is the branch of mathematics concerned with probability, the analysis of random phenomena. The central objects of probability theory are random variables, stochastic processes.As a mathematical foundation for statistics, probability theory is essential to many human activities that involve quantitative analysis of large sets of data. Methods of probability theory also apply to descriptions of complex systems given only partial knowledge of their state..

Course overview and Context

The course starts by introducing Discrete Probability and Modes of convergence. In module2 univariate andmultivariate distribution functions are explained. Estimationprocess and test of hypothesis along with some models are described in Module 3 and in module 4 .

Syllabus Content

All questions shall be based on the relevant portions of the reference books given

Module – 1 **(20 hours)**

Discrete Probability (Empirical, Classical and Axiomatic approaches), Independentevents, Bayes theorem, Random variables, and distribution functions (univariate andmultivariate), Expectation and moments, marginal and conditional distributions.Probability Inequalities (Chebychev, Markov). Modes of convergence, Weak andStrong laws of large numbers (Khintchine's Weak Law , Kolmogrov Strong Law,Bernaulli's Strong Law) Central Limit theorem (Lindeberg-Levy theorem).

Module – 2 **(20 hours)**

Standard discrete and continuous univariate distributions (Binomial, Poisson,Negative binomial, Geometric, Exponential, Hypergeometric, Normal, Rectangular,Cauchy's, Gamma, Beta,), Multivariate normal distribution, Wishart distribution andtheir properties.

Module – 3 **(25 hours)**

Methods of estimation, properties of estimators, Cramer-Rao inequality, Fisher-Neyman criterion for sufficiency, Rao-Blackwell theorem, completeness ,method ofmaximum likelihood, properties of maximum likelihood estimators , method ofmoments. Tests of hypothesis: most powerful and uniformly most powerful tests(Neyman – Pearson Lemma).

Module- 4 **(25 hours)**

Gauss-Markov models, estimability of parameters, best linear unbiased estimators,Analysis of variance and covariance. One way and two way classification with oneobservation per cell.

Learning Resources

References.

1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11th Ed., Sultan Chand & Sons, 2011.
2. V.K. Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, 2nd Ed. Wiley Eastern Ltd., 1986.
3. T.W. Anderson, An Introduction to Multivariate Statistical Analysis, 3rd Ed., Wiley Interscience, 2003
4. D.D. Joshi, Linear Estimation and Design of Experiments, Wiley Eastern Ltd., 1990.
5. C.R. Rao, Linear Statistical Inference and its Applications, John Wiely, New York, 1965.

6. W.G.Cochran and G.M. Cox , Experimental Designs, 2nd Ed., John Wiely, NewYork.

Competencies of the Course

C1. Introduce Discrete Probability

C2. Define Independent events and Bayes theorem

C3. Introduce Random variables and distribution functions

C4. Define Expectation and moments and marginal and conditional distributions

C5. Understand Probability Inequalities (Chebychev, Markov)

C6. Obtain Modes of convergence

C7. Bring out Weak and Strong laws of large numbers

C8. Define Standard discrete and continuous univariate distributions

C9. Introduce Multivariate normal distribution,

C10. Understand Wishart distribution and their properties

C11. Define Methods of estimation and properties of estimators

C12. Introduce Cramer-Rao inequality and Fisher- Neyman criterion for sufficiency

C13. Obtain Rao-Blackwell theorem

C14. Explore method of maximum likelihood and its properties

C15. Understand method of moments

C16. Recognize Tests of hypothesis

C17. Understand Gauss-Markov models

C18. Recognize estimability of parameters and best linear unbiased estimators

C19. Analyze variance and covariance.

MT4E21TM -PROBABILITY THEORY

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M.ScMATHEMATICS EXAMINATION

MT4E21TM -PROBABILITY THEORY

Time : 3 hrs **Max. Marks: 75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. List the set of all events in the experiment of tossing three coins simultaneously.
2. Define random variable. Give an example.
3. Explain discrete probability space.
4. Define signed measure with an example.
5. State Jordan decomposition theorem What are the assumptions of input-output model ?
6. Prove that if X ≥ 0 then E(X) ≥ 0
7. Give an example of events A,B,C such that {A, B, C} are pair-wise independent, but {A,B, C } are not mutually independent (5 x 3 = 15)

Part B

(Answer any 6 questions. Each questioncarries 5 marks)

8. Show that the intersection of an arbitrary collection of σ − fields is again a σ − field .
9. If X and Y are random variables, then show that min(X,Y) is also a random variable.
10. Prove that probability function is a continuous function.
11. Distinguish between pair wise independence and mutual independence of events.
12. Show that almost sure convergence implies convergence in probability.
13. Prove that Borel functions of independent random variables are independent.
14. State a set of conditions under which the weak law of large numbers holds for a sequence of independent random variables not necessarily identically distributed.
15. If X and Y are two simple random variables and a and b are real numbers, then show that E(aX+bY) =aE(X)+bE(Y).
16. Find the value of k for which f (x) = $\frac{kx^2}{e^x}$, x ≥ 0 is a probability density function

(6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. Show that a non-negative random variable X can be expressed as the limit of a sequence of non-decreasing non-negative simple random variables.
18. State and establish Jordan decomposition of a distribution function.
19. State and prove the inversion theorem for characteristic function.
20. State and prove Liapounov's theorem (2 x 15 = 30)

Semester – IV

MT4E22TM -COMMUTATIVE ALGEBRA

Credits -3

Total lecture hours - 90

Aim

Commutative algebra is the branch of algebra that studies commutative rings, their ideals, and modules over such rings. Both algebraic geometry and algebraic number theory build on commutative algebra.This course deals with elementary concepts of commutative algebra

Course overview and Context

In module 1 Maximal ideals and Jacobson Rings are introduced .Noetherian and Artinian Rings are covered in in module 2.The Zariski Topology and a A Summary of the Lexicon are described in Module 3 and in module 4 .

Syllabus Content

Module – 1 **(25 hours)**

The Algebra-Geometry Lexicon – Hilbert's NullstellensatzMaximal ideals, Jacobson Rings, Coordinate Rings, Simple problems.(Chapter1 Sections 1.1, 1.2 & 1.3 of the text)

Module – 2 **(20 hours)**

The Noether and Artin Properties for Rings and Modules, Notherian Rings andModules, Simple problems(Chapter2 Sections 2.1 & 2.2, of the text)

Module – 3 **(25 hours)**

Affine Varieties, Spectra, Noetherian and Irreducible Spaces, Simple problems.
(Chapter3 Sections 3.1, 3.2 & 3.3 of the text)

Module- 4 **(20hours)**

True Geometry: Affine Varieties, Abstract Geometry : Spectra , Simple problems

(Chapter4 Sections 4.1 & 4.2, of the text)

Learning Resources

Text Book :- Gregor Kemper, A Course in Commutative Algebra, Springer, ISSN0072-5285, ISBN978-3-642-03544-6

References.

1. William W. Adams, Phillippe Loustaunau, An Introduction to Grobner bases, Graduate Studies in Mathematics 3, American Mathematical Society, 1994
2. Michael F Attiyah, Ian Grant Macdonald, Introduction to Commutative Algebra, Addison- Wesley, Reading, 1969
3. Nicolas Bourbaki, General Topology, Chapters – 1 – 4, Springer, Berlin, 1993

Competencies of the Course

C1. Define Maximal ideals and Jacobson Rings

C2. Introduce Coordinate Rings

C3. Obtain the Noether and Artin Properties for Rings and Modules

C4. Define affine Varieties and Spectra

C5. Introduce Noetherian and Irreducible Spaces

C6. Understand a Summary of the Lexicon

MT4E22TM -COMMUTATIVE ALGEBRA

	Part A	**Part B**	**Part C**
Module I	2	2	1
Module II	2	2	1
Module III	1	3	1
Module IV	2	2	1
Total	**7**	**9**	**4**

MODEL QUESTION PAPER

FOURTH SEMESTER M.Sc. MATHEMATICS EXAMINATION

MT4E22TM - COMMUTATIVE ALGEBRA

Time : 3 hrs **Max. Marks: 75**

Part A

(Answer any **5** questions. Each question carries 3 marks)

1. Define a local ring.
2. Give an example of coprime ideals in Z
3. Define a faithful A-module
4. Define a finitely generated A-module
5. Define a multiplicatively closed set in a ring A. Give an example.
6. Give an example of a primary ideal in Z which is not a prime ideal.

(5 x 3 = 15)

Part B

(Answer any 6 questions. Each questioncarries 5 marks)

7. Give an example of a primary ideal which is not a power of a prime ideal.
8. Prove that M is a Noetherian A module if and only if every sub module of M isfinitely generated.
9. Define the tensor product of two A - modules M and N. Prove that it is unique uptoisomorphism.
10. Show that in a Noetherian ring every irreducible ideal $\neq < 1 >$ is primary.
11. Show that an Artin ring has only finitely many maximal ideals.
12. Show that any unique factorization domain is integrally closed
13. Prove that the nil radical of a ring A is the intersection of all prime ideals in A.
14. Define irreducible ideal in a ring and dimension of a ring A.
15. What is the integral closure of Z and R in Q.
16. Let B be an integral domain and K its field of fractions. What is the condition that Bis a valuation ring of K (6 x 5 = 30)

Part C

(Answer any 2 question. Each question carries 15 marks)

17. State and prove Nakayama's lemma.
18. State and prove the first uniqueness theorem in primary decomposition.
19. State and prove the Gowing up theorem.
20. State and prove the structure theorem for Artin rings.

(2 x 15 = 30)

PROJECT